INDIAN INSTITUTE OF TECHNOLOGY DELHI

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NPTEL ONLINE CERTIFICATION COURSE

Stochastic Processes

Module 8: Renewal Processes

Lecture-04

Non Markovian Queues contd.

With

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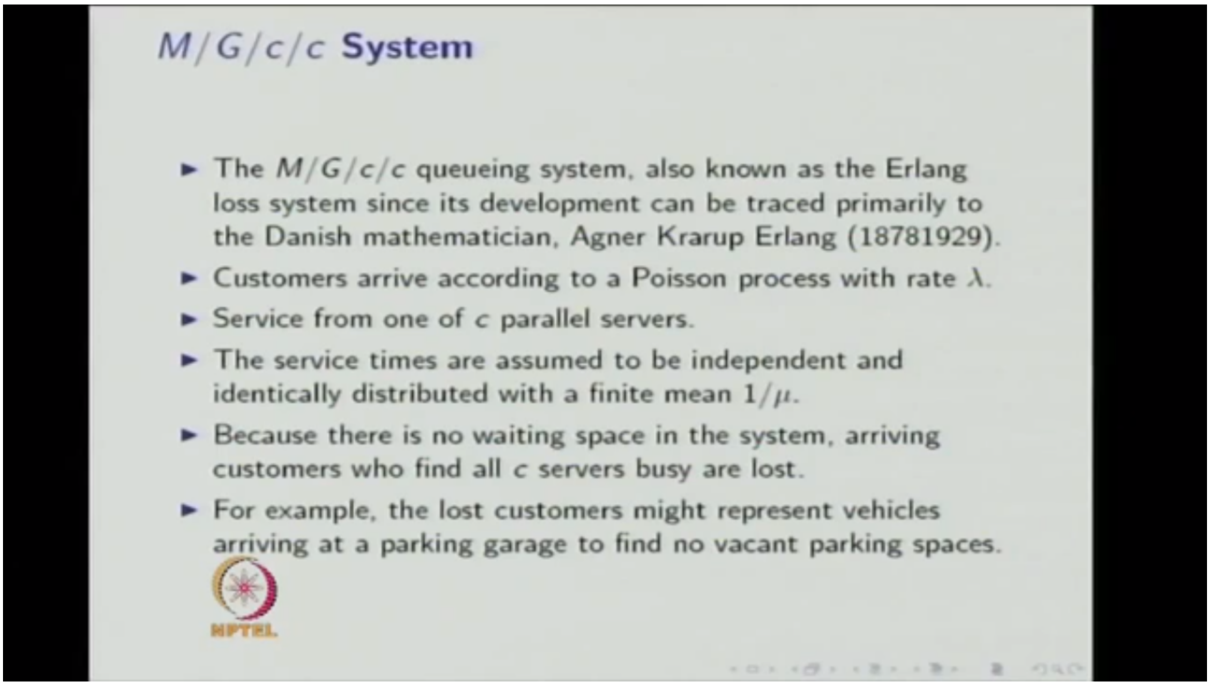
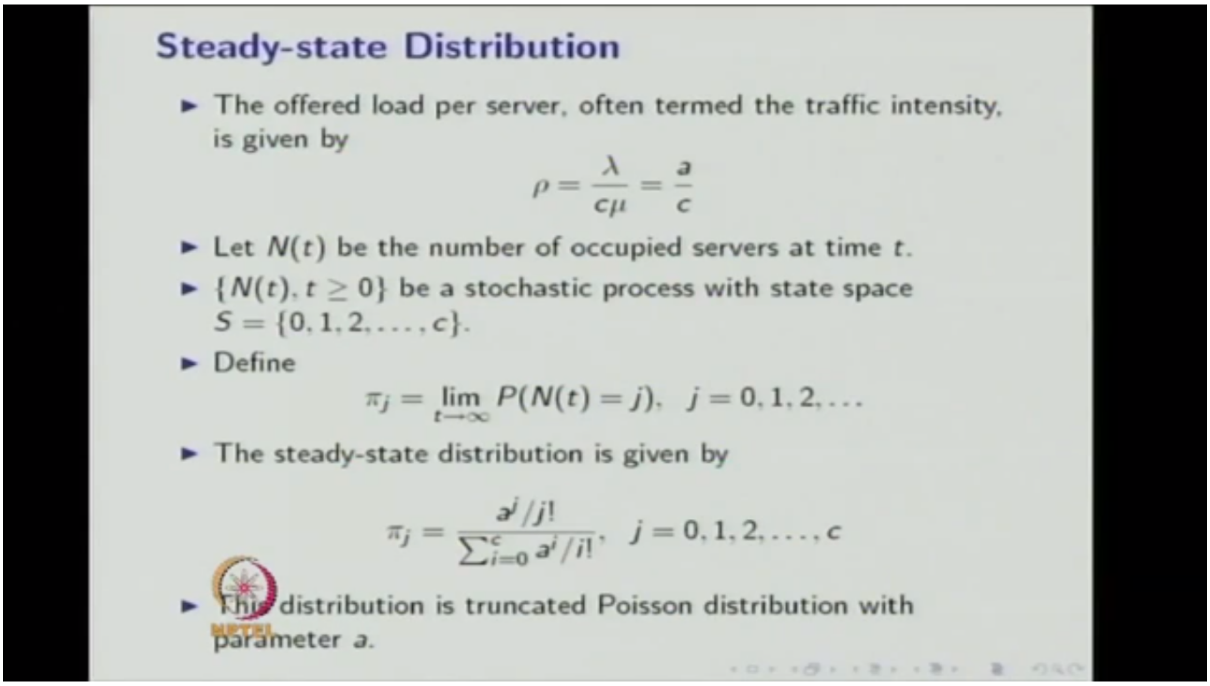


Now we move into the second non-Markovian queuing model, not queueing model; second non-Markovian system, that is M/G/c/c/ system, because in this model, there is no queueing. The M/G/c/c queuing system also known as Erlang loss system since its development can be traced primarily to the Danish mathematician, Erlang.

Customers arrive according to the Poisson process with the rate, λ. Service from one of c parallel servers. The service times are assumed to be independent and identically distributed with the finite mean $1/\mu$. Because there is no waiting space in the system, arriving customers who finds all c servers busy are lost. Therefore, this system is called a loss system, not a queuing system.

For example, loss customers might represent vehicles arriving at a parking garage to find a no vacant parking spaces.



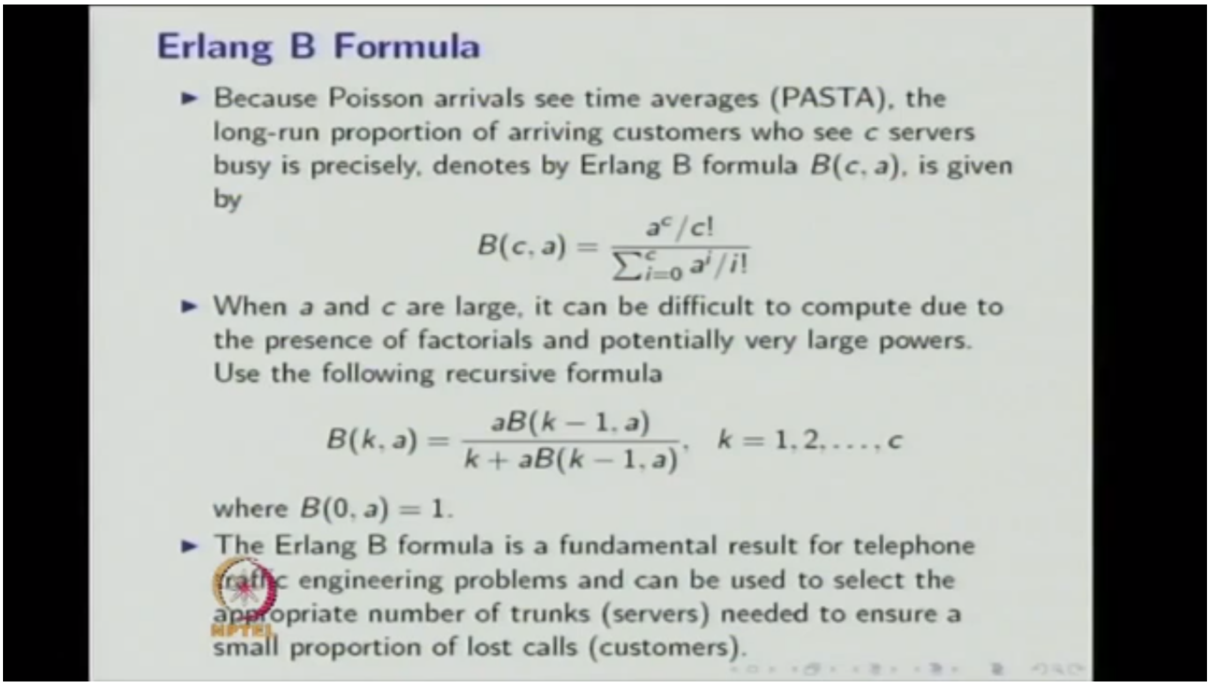
Our interest is to find out the steady-state distribution. The offered load per server often termed the traffic intensity is given by $\lambda/c\mu$. If you denote $a = \lambda/\mu$, then the ρ is nothing but a/c. Let N(t) be the number of occupied servers at time t. So N(t) is a continuous time, discrete state stochastic process with the state's space s. Define the limiting probabilities,

$$\lim_{t \to \infty} P(N(t) = j)$$

, that is π_j or the steady-state distribution.

You can find easily that is nothing but $(a^j/j!) / \sum_{i=0}^{\infty} a^i / i!$

So this is a steady-state distribution M/G/c/c loss system. So this distribution is a truncated Poisson distribution with the parameter, a, where a is nothing but λ/μ .

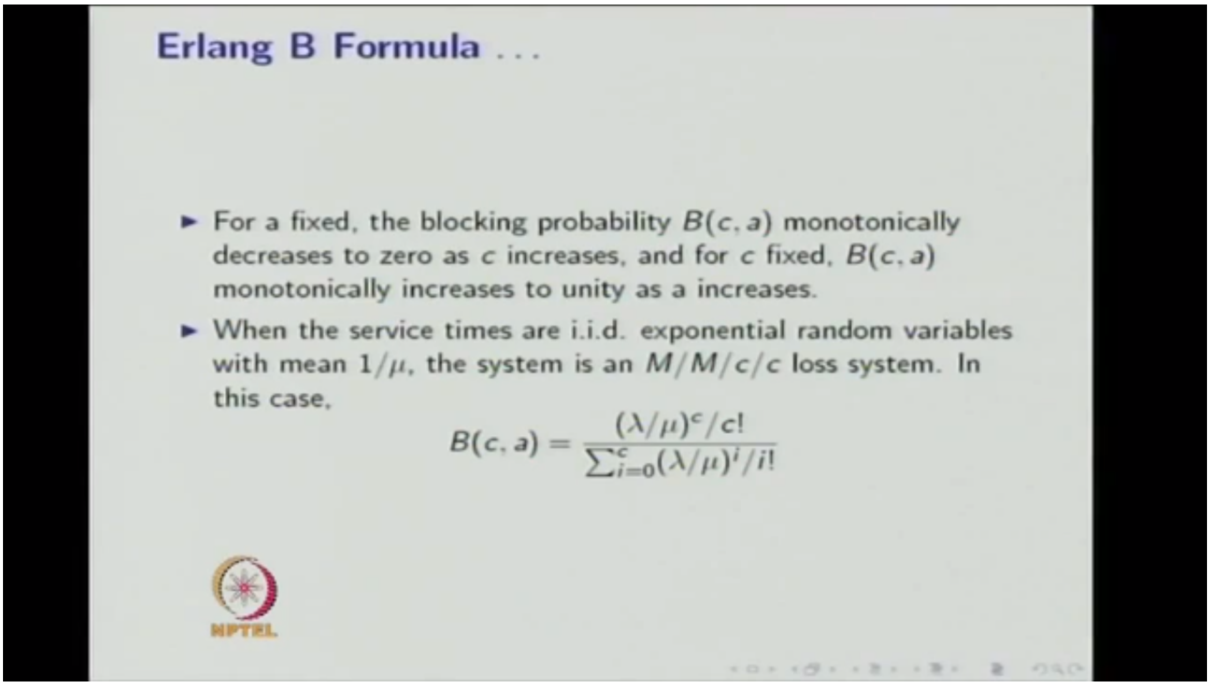


Once we know the limiting distribution, you can find the other measures. The first measure is the Erlang B formula. Because Poisson arrival sees time averages that is PASTA, we can find the long run proportion of the arriving customers who see c servers busy c that is denoted by Erlang B formula as a function of c and a where c is the number of servers in the system and a is λ/μ. That is nothing but the loss probability that is $(a^c/c!)/\sum_{i=0}^{c} a^i/i!$

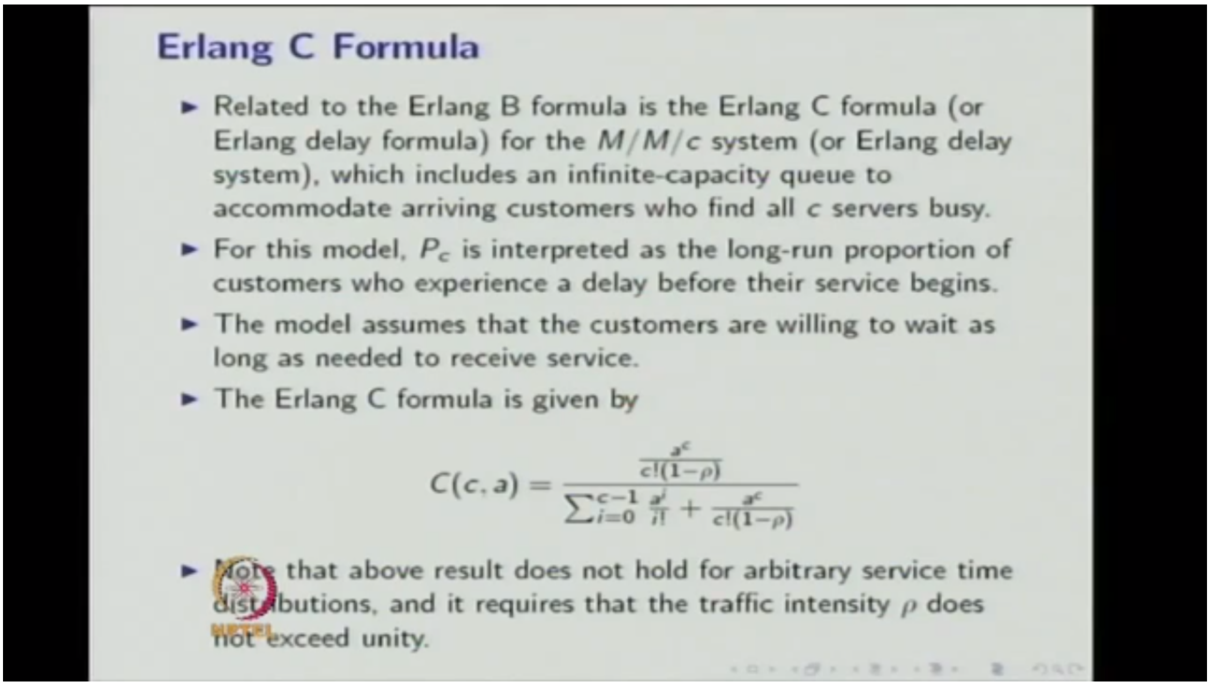
When a and c are large, it can be difficult to compute due to the presence of factorials and potentially very large powers, a^c. When c is very large then a^c as well as these factorials giving trouble. So we can use a recursive formula, we can use the recursive formula to compute the Erlang B formula. That is in terms of that is B(k,a) in terms of B(k-1,k) with the initial condition, B(0,a)=1.

So that means, to find the value of B(1,a), you use B(0,a). Then to find B(2,a), you use the value of B(1,a) and so on. So finally, you can get a B(c,a). In this recursive formula, we are avoiding the factorial as well as the large powers.

The Erlang B formula is a fundamental result for telephone traffic engineering problems and it can be used to select appropriate number of servers need to ensure a small portion of lost customers. So this is a way using Erlang formula, we can find out or we can select appropriate number of servers for the M/G/c/c loss system.



For a fixed a where a is λ/μ, the Erlang B formula or the blocking probability or loss probability monotonically decreases to 0 as the c increases, whereas for a fixed c where c is a number of servers, the blocking probability monotonically increases to unity as λ/μ increases. As a special case when the service times are i.i.d. random variables, each having exponential distribution with the mean, 1/ μ, the system becomes M/M/c/c loss system. So you can have Erlang B formula for the M/M/c/c system also. With the assumption of service times or i.i.d. random variables, each having exponential distribution with the mean, 1/μ.

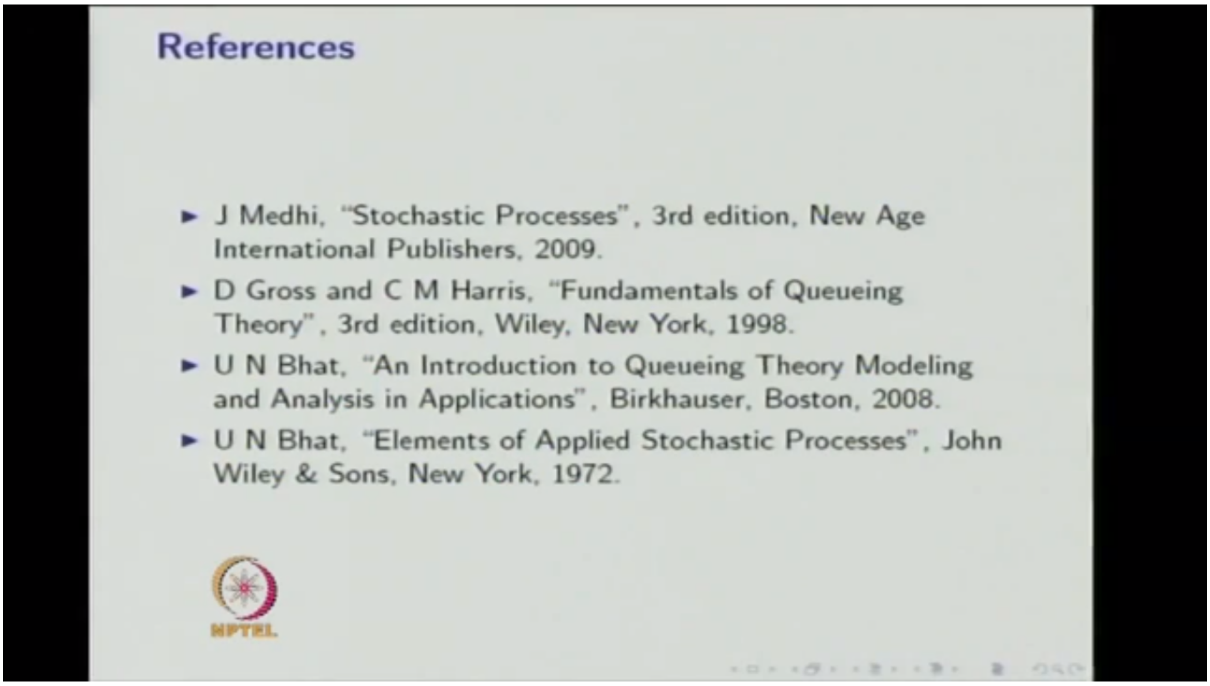


Related to the Erlang B formula, we are going to discuss the other one called Erlang C formula. This is for the M/M/c, not for M/G/C. This is for the M//M/c system which includes a infinite capacity queue to accommodate arriving customers who find all c servers are busy. That means this is a queuing system, queuing and delay system. We have seen servers and infinite capacity queue to accommodate arriving customers who find all c servers are busy.

So corresponding to M/M/c queuing and delay system, we have the formula called Erlang C formula or Erlang delay formula. In this model, the P_c is interpreted as a long-run proportion of customers who experience a delay before the service begins. The model assumes the customers are willing to wait as long as needed to receive service. So the Erlang C formula is nothing but the blocking probability for the M/M/c queuing and delay system that is in terms of that is written in this form, $(a^c/c!(1-\rho))/ \sum_{i=0}^{c-1} \frac{a^i}{i!} + \frac{a^c}{c!(1-\rho)}$.

Since it is a queuing and delay system, you need additional condition to have the Erlang C formula. The additional condition is it requires the traffic intensity ρ does not exit 1. That means that as long as $\rho<1$, the system is stable. The corresponding M/M/c queuing and delay system will be stable. Hence, the steady-state probabilities exist, and once the steady-state probabilities exist you can find the loss probability and that loss probability is same as Erlang C formula. So to have Erlang C formula it requires the traffic intensity ρ has to be less than 1.

Note that the above result does not hold for arbitrary service time distribution. So this Erlang C formula is valid only for service times are exponentially distributed, not for arbitrary service time distribution, whereas Erlang B formula is valid both for M/G/c/c loss system and M/M/c/c loss system. Erlang C formula is valid only for M/M/c queuing and delay system with the restriction ρ has to be less than 1, whereas Erlang B formula, the value of a is λ/μ need not be less than 1 because that is a finite capacity and loss system.



In these we have discussed the non-Markovian queues, in particular M/G/1 queuing system, M/G/c/c loss system, M/M/c/c loss system, Erlang B formula for M/G/c/c loss system as well as M/M/c/c. And finally, we have discussed Erlang C formula for the M/M/c queuing and delay system.

With this Lecture 4 is completed here is the reference for Lecture 4.

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