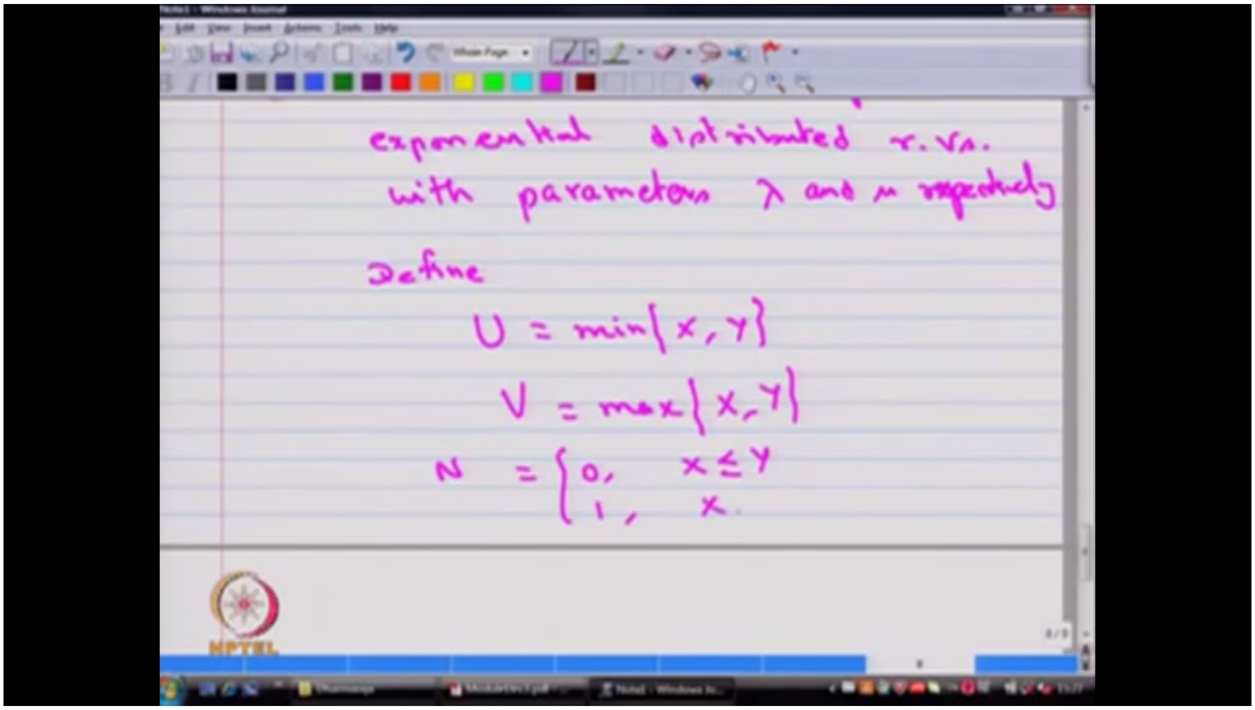
So the third example we'll discuss this one let x and y be independent exponential distributed random variables with parameters lambda and mu respectively. Define U that is nothing but minimum of X,Y and the V is nothing but maximum of the random variables X,Y. The third random variable capital N that is defined as it takes value 0 if X is less than or equal to Y. It takes a value 1 if X is greater than Y.

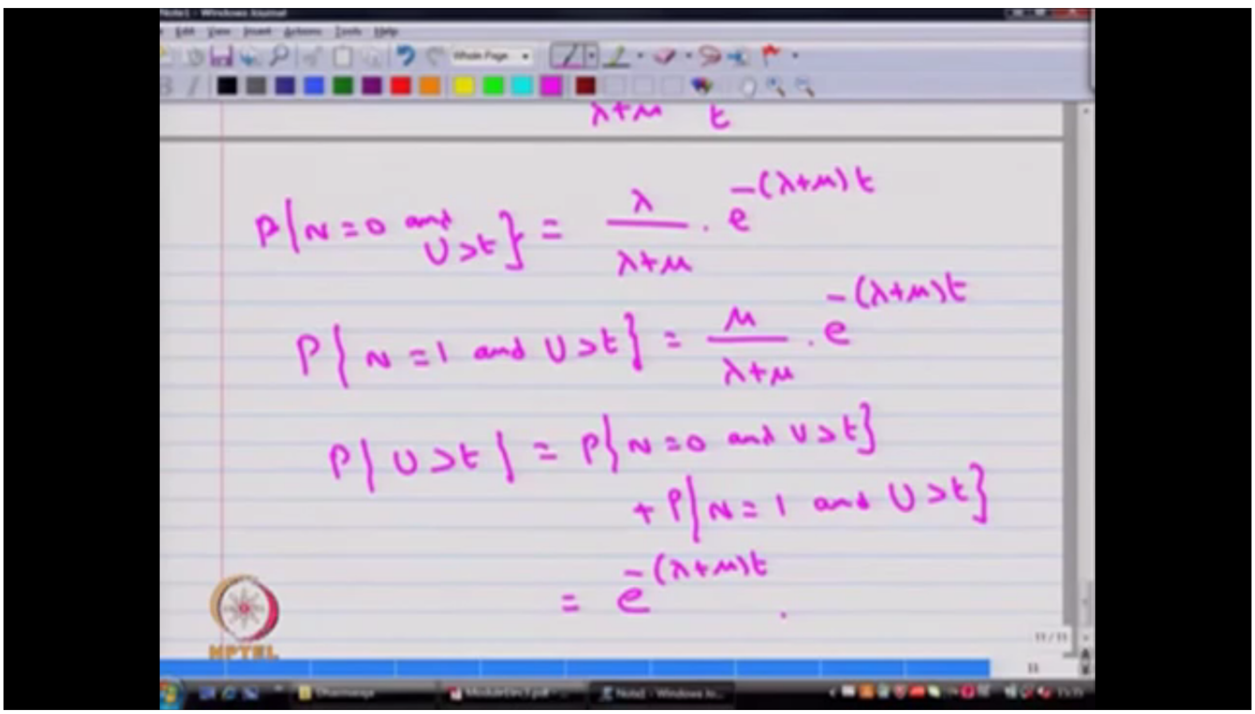


So using the two random variables X and Y we have defined three random variables U, V and N. Our interest is to find the probability of N takes value 0 and what is the probability of capital N takes value 1. The second we are interested to find out the probability of n takes the value 0 and capital U takes a value greater than sum t that t is greater than 0. So let's go for finding the second one first then we will find out the probability of equal N equal to zero and the probability of N is equal to 1. So let's start with the two first. The event N is equal to 0 and capital U takes a value greater than T that is exactly the event of T less than X less than or equal to Y.

The event n is equal to zero and the U greater than T that is same as T less than X less than or equal to Y. Therefore, the probability of finding N takes a value 0 and U takes a value greater than T that is same as probability of X takes the value T, T less than X less than or equal to Y. That is same as the double integration with the T less than X less than or equal to Y of the joint probability density function of X and Y. Since the X and Y are independent random variable the joint probability density function is a product of marginal probability sorry, this is e power minus y dy dx. So the probability of T less than X less than or equal to Y that is same as the double integration T less than X less than or equal to Y of integrant is lambda times e power minus lambda X mu times e power minus mu y dy dx. That is same as the inner integral becomes X to infinity mu times e power minus mu y dy. Then lambda times e power minus lambda X integration with respect to X between the limits T to infinity. That is same as now the inner integration you can integrate and you can substitute the limit X and infinity if you simplify you will get T to infinity e power minus mu X.

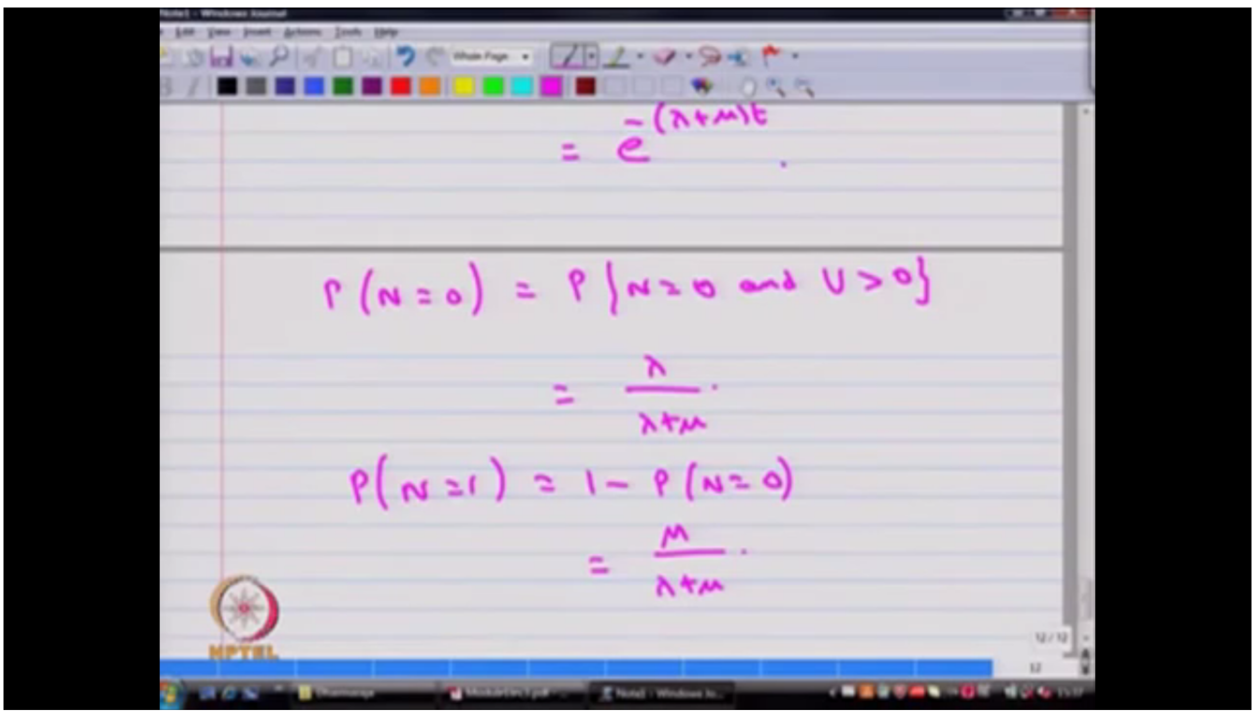
Then multiplied by lambda times e power minus lambda X dx. If you do the integration the interior integration you will get e power minus mu X then the remaining things are as it is. That is same as you can keep lambda by lambda plus mu outside. This becomes integration from T to infinity of lambda plus mu times e power minus lambda plus mu X dx. You know how to do the integration for this. If you simplify you will get the answer that is lambda divided by lambda plus mu times e power minus lambda plus mu times T. So this is a result for probability of N takes value 0 and U takes value greater than T where T is greater than 0.

Similarly you can work out the probability of n takes a value 1 and U takes a value greater than T that will be mu divided by lambda plus mu multiplied by E power minus lambda plus mu times T. So with this the second part is over. Now we have found the probability of N is equal to 0 and U greater than T. Also we've got the probability of N is equal to 1 and U greater than T now we will go for finding the first result but before that you can find what is the probability of U greater than T also. That is nothing but finding out the probability of N is equal to 0 and U greater than T plus the probability of N takes a value 1 and U greater than T. That is a way you can find out the probability of U greater than T.



You know the result. If you add you will get the result that is e power minus lambda plus mu times T. That is a meaning of the minimum of two random variable takes a value greater than T that is e power minus lambda plus mu times T. You can find out the probability of N is equal to 0 in using probability of N is equal to 0 and U is greater than 0. The T is a great nor equal to 0 therefore probability of N is equal to 0 that you can compute from probability of N is equal to 0 and U is greater than 0, that means you substitute T equal to 0 in the previous result you will get a probability of N is equal to 0. So that is nothing but if you recall probability of N is equal to 0 and U greater than T that is lambda divided by lambda plus mu e power minus lambda plus mu T here the T can be greater or equal to 0 so substitute T equal to 0 that will give probability of N is equal to 0. Therefore this will become lambda divided by lambda plus mu.

Similarly you can find out the probability of N is equal to 1 in probability of N is equal to 0 and probability of N is equal to 1 and U is greater than 0 or you can find out probability of N is equal to 1 by making one of a probability of N is equal to 1 minus probability of N is equal to 0. So two ways you can find the probability of N is equal to 1. So here I am using 1 minus probability of N is equal to 0 that will be mu divided by lambda plus mu. So in this problem even though we ask only two things but here we have got the probability of N is equal to 0, probability of N is equal to 1 also probability of U is greater than T. If you observe probability of N is equal to 0 and U greater than T that is same as probability of N is equal to 0 multiplied by probability of U greater than T.



Similarly you will get the observation probability of N is equal to 1 and U greater than T that will give probability of N is equal to 1 multiplied by probability of U greater than T. So hence N and U are independent random variables.

So this example is useful in a birth-death processes. Therefore I am explaining this problem as a illustrative example.