Video Course on
Stochastic Processes

by

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Module # 8
Renewal Processes

Markov Renewal and Markov Regenerative Processes contd

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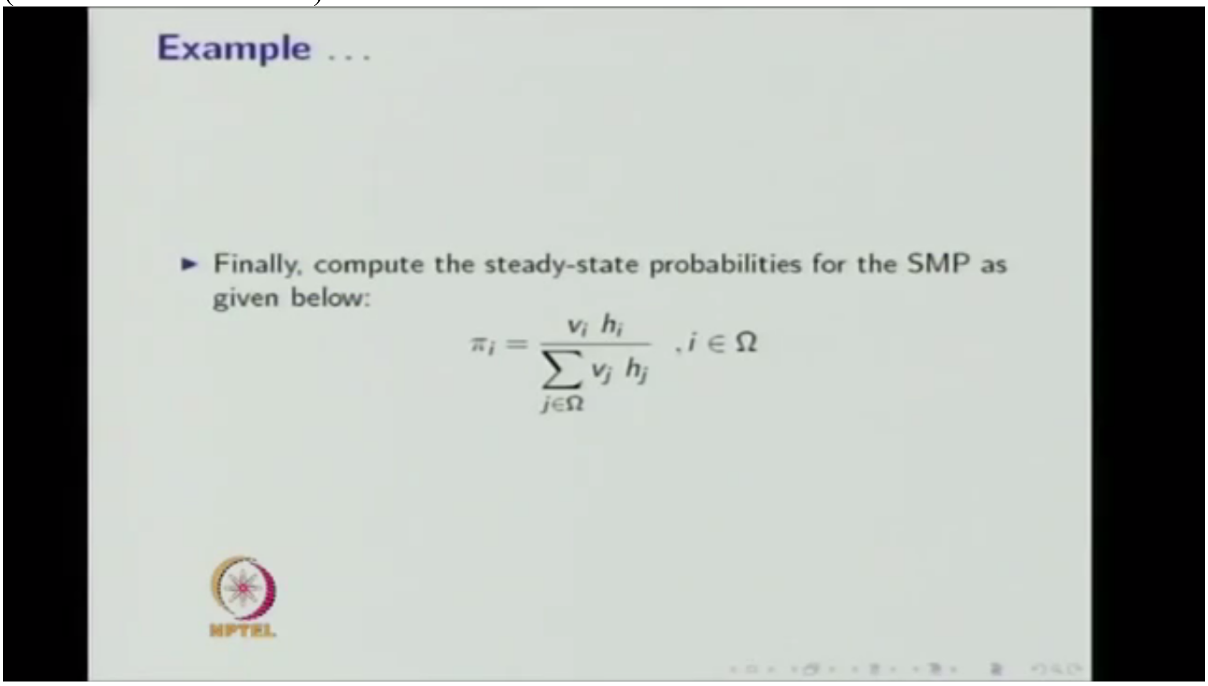

Finally, compute the steady-state probabilities of semi-Markov process using v_i's and h_i's.

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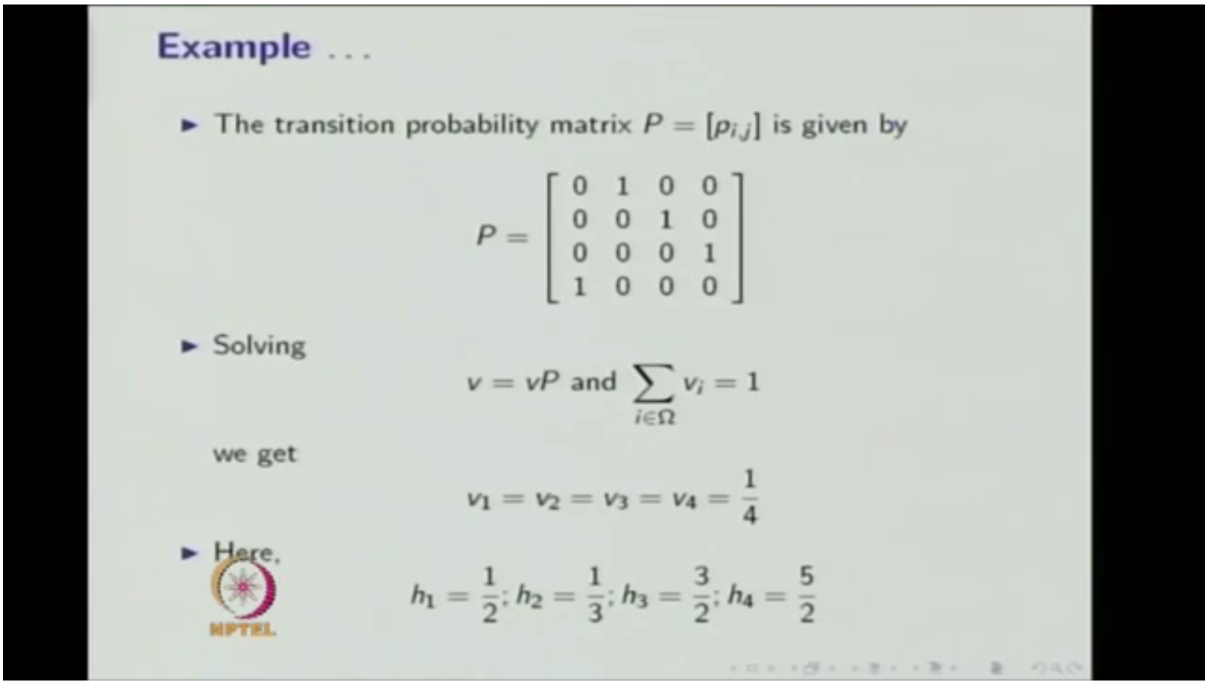

V_i's are 1/4 and h_i's are h_1 is 1/2, h_2 is 1/3, h_3 is 3/2, h_4 is 5/2.

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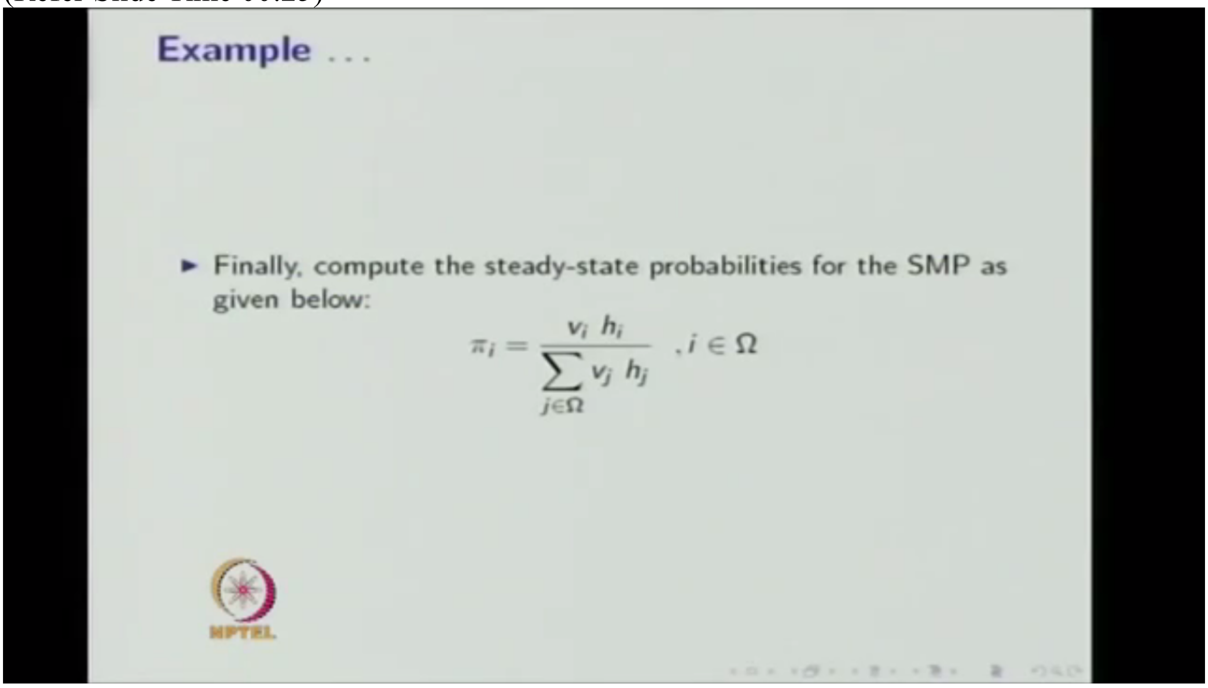

You substitute the values in this equation to get the steady-state probabilities of semi-Markov process.

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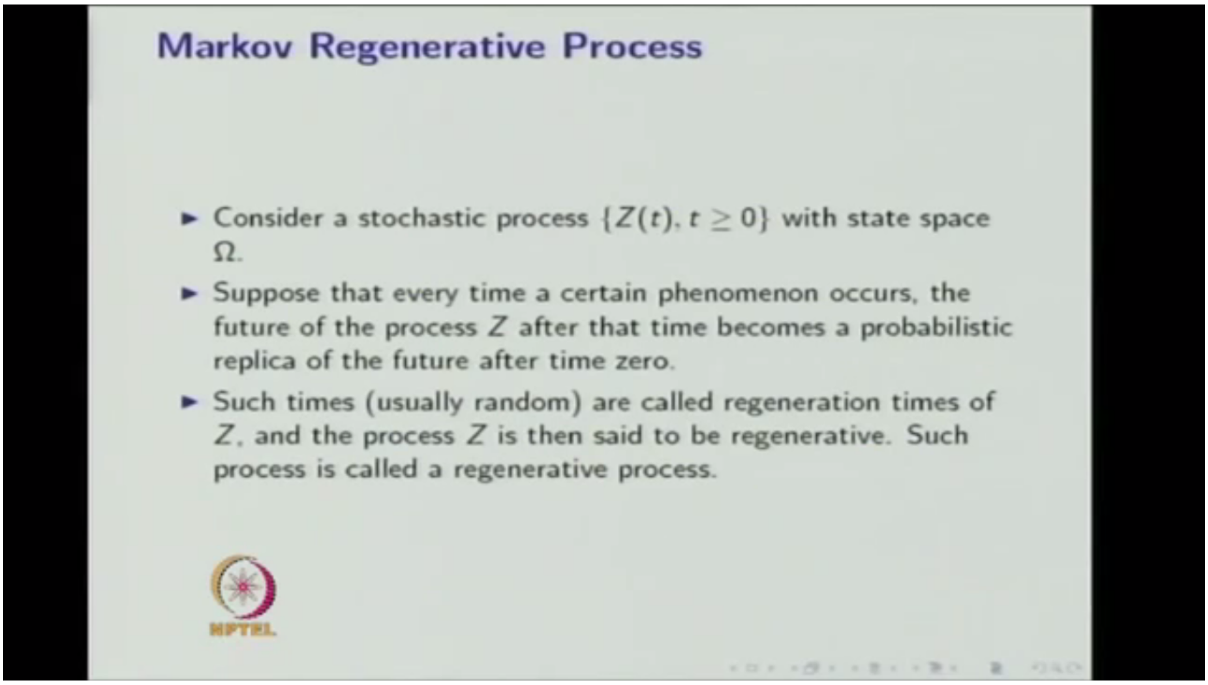

Now we are moving into the second part of Lecture 3, that is Markov Regenerative Process.

Consider a stochastic process Z(t) with the state space Ω.

Suppose that every time a certain phenomena occurs, the future of the process Z that means Z(t) after that time becomes a probabilistic replica of the future after time zero.

Such times (usually a random), such times (usually random) are called regeneration times of the stochastic process Z(t) and the process Z(t) is then said to be regenerative. Such a process is called a regenerative process.

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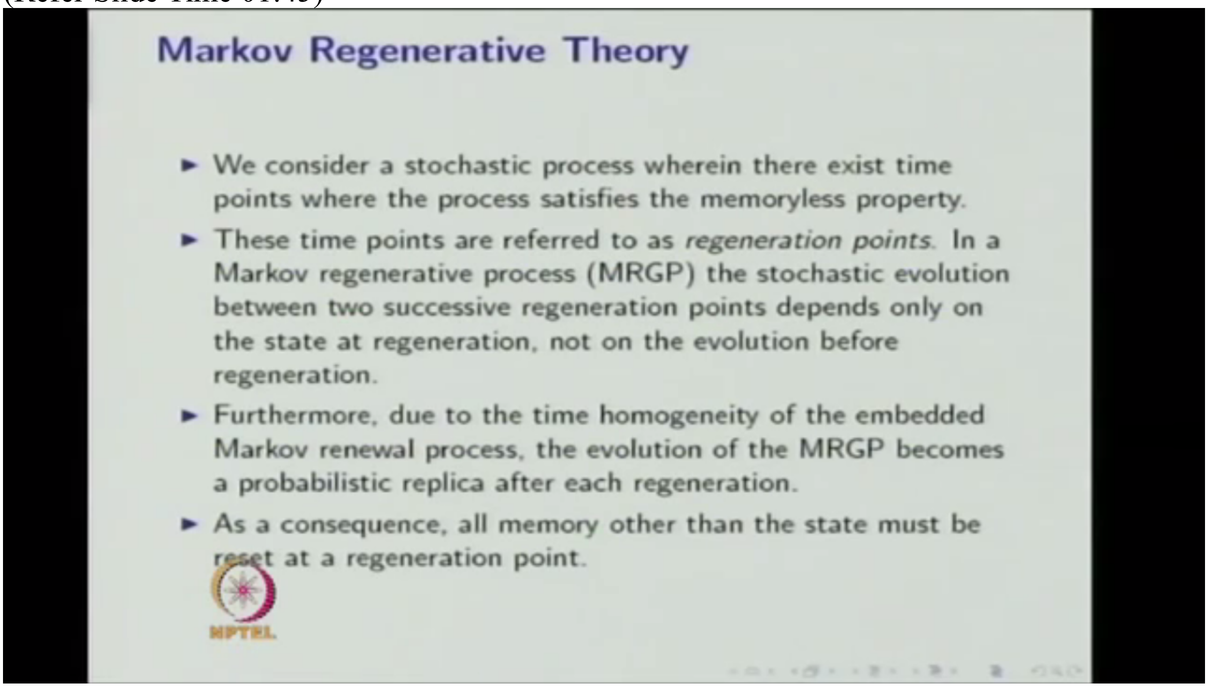

We consider a stochastic process wherein there exist time points where the process satisfies the memoryless property.

These time points are referred to as regeneration points. In a Markov regenerative process, the stochastic evolution between two successive regeneration points depends only on the state at regeneration, not on the evolution before regeneration.

Furthermore, due to the time homogeneity of the embedded Markov renewal process, the evolution of the Markov regenerative process becomes a probabilistic replica after each regeneration because of time homogeneity, time invariant.

As a consequence, all memory other than the state must be reset at the regeneration point. As a consequence, all memory other than the state, that means the future depends on the state, but not the past history.

IID property of inter-arrival time is making arrival instance independent and hence memoryless, and hence memoryless as the next arrival does not depend on how long the previous arrival took. Also since between any two arrivals, same pure death process operates, hence, the arrival points become regeneration time points.

Must be reset at a regeneration point.

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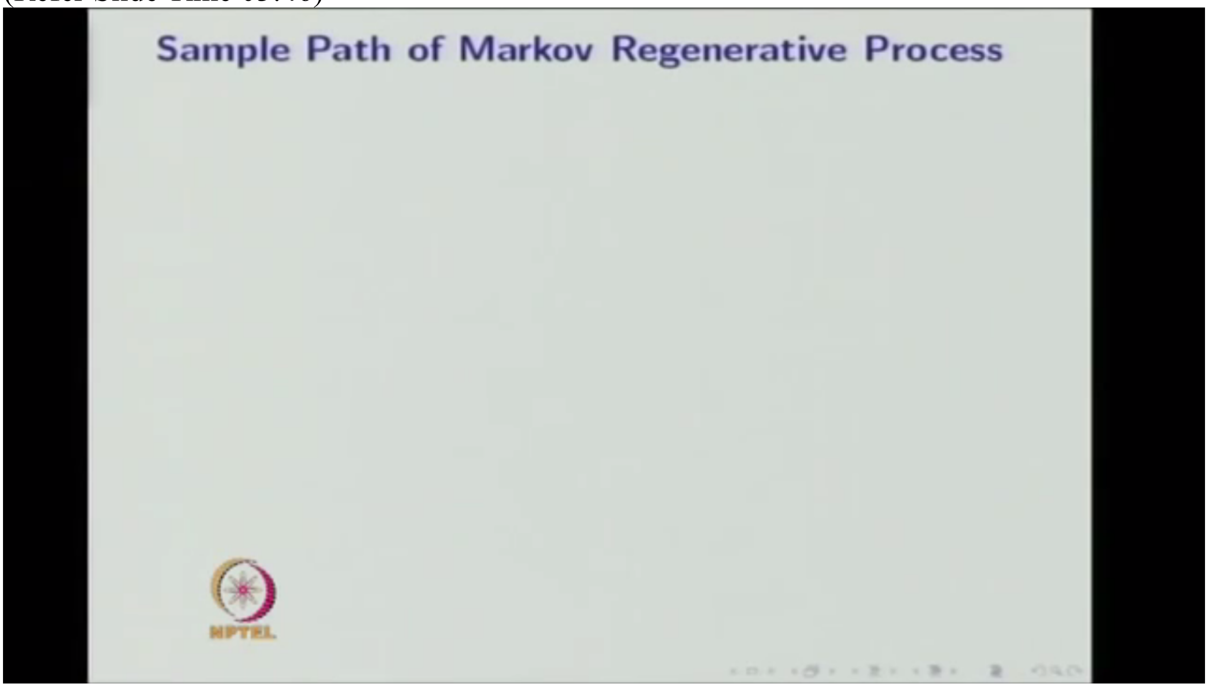

Now we are going to consider the sample path of a Markov Regenerative Process. Through that I am going to explain the regeneration points.

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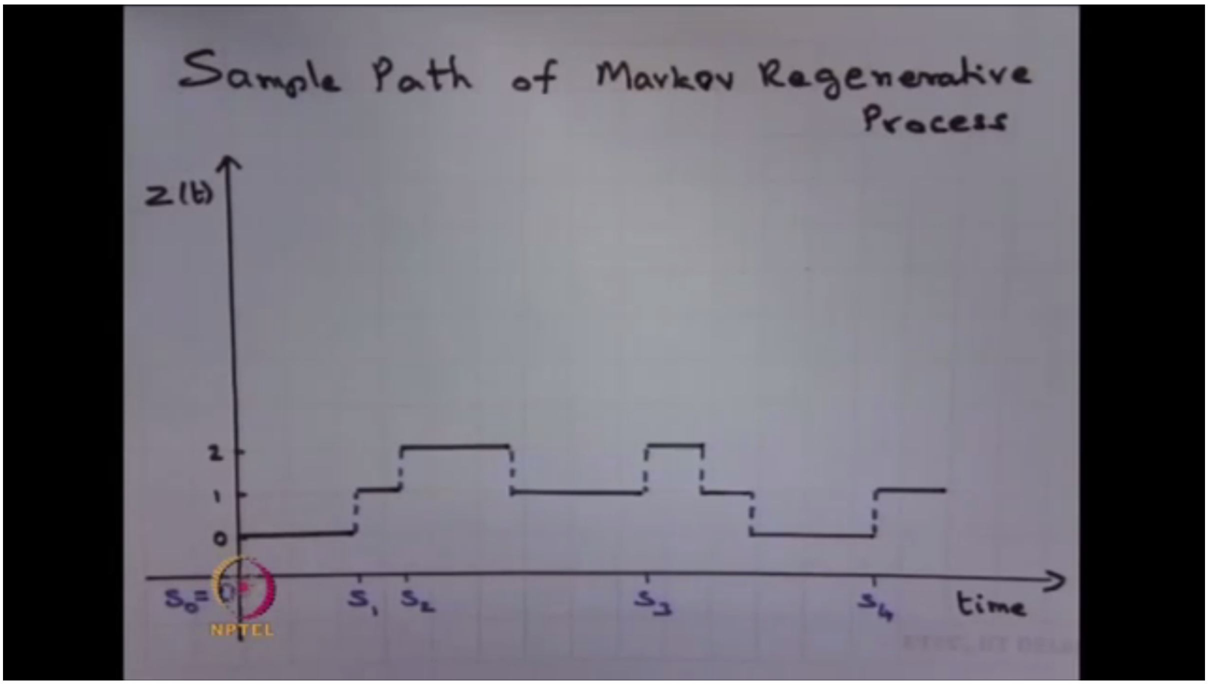

Consider this sample path of Markov Regenerative Process. In this stochastic process, the Ω is 0, 1 and 2. Consider the simple example of G/M/1/2 queuing model. That means the arrival is non-Poisson, non-Poisson process. That means the inter-arrival time distributions are not an exponential distribution whereas the service times are exponential distribution. Only one server and the capacity of the system is 2.

Therefore, the state transition diagram is like this. This is a CDF of the system spending in the state 0 before moving into the state 1. The system spending in the state 1 before moving into the state 2, the CDF will be F(2), F(t). Because the capacity of the system is 2, there is a possibility the arrival can come but the system will be in the state 2 again.

Therefore, I made a self loop with the dotted arcs whereas if the service is completed with the assumption of a service is exponential distribution with the parameter μ, then the system can move from the state 2 to 1. 1 to 0 also the rate will be μ.

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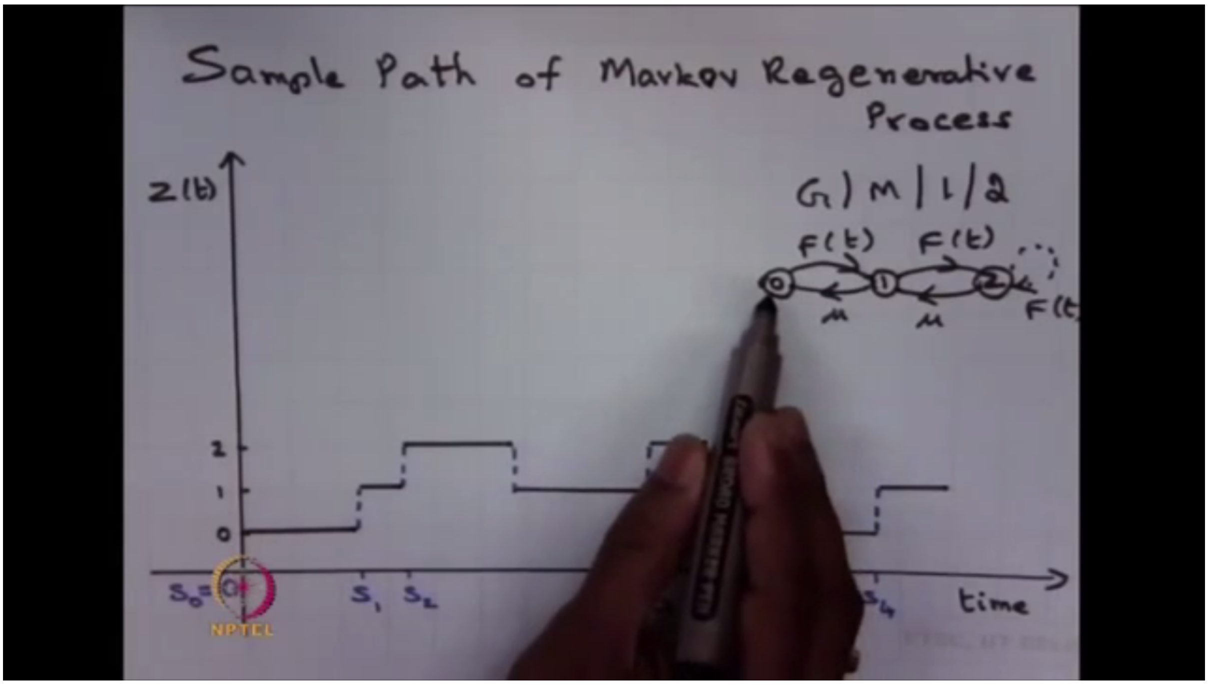

So now you see the sample path. At time 0, the system is in the state 0. The inter-arrival time is any distribution, need not be an exponential distribution. At time S_1, at time instant S_1, the first arrival enter into the system. Therefore, the system size is now 1. So system move to the state 1.

Now there are two possibilities. Either the service would have been completed before the next arrival or the arrival occurs, the next arrival occurs first before the service completion of the first arrival, first server, first customer who is under service.

So suppose you assume that second arrival occurs first before the completion of the -- service completion of the first customer. Therefore, system moved to the state 2 at the time point S_2.

Now consider a scenario. The first customer who is under service his service completed first before the third arrival. Therefore, the system size now it will be 1. That means earlier the system was in the state 2. Since the service is completed, the customer's -- the system size becomes 1 for some time.

At this duration, the third customer entered into the system at the time point S_3. Now the system size is 2 again. That means initially the system's -- system state was zero. From state 0, the system moved to the state 1 because of F(t). Then one more arrival. Therefore, system size 2. First customer service completion. System size is -- system state is 1. Then the third arrival. Now the system state is 3, sorry, 2. Then the service completion. Therefore, it goes to the 1. One more service completion. Therefore, the system goes to the state 0. Then the arrival S_4.

Note the time points S_1, S_2, S_3, S_4, these are all the time points in which the arrival occurs. That is nothing but the arrival epochs whereas this is a time point in which the service completes. This is a time point in which the service completes. This is a time point in which the service completes. We are not noting down the service completion time points whereas we are note down the arrival time epochs because at the time of service completion epochs, still we should know how much elapsed time of the next arrival because inter-arrivals are now it is any distribution, not an exponential distribution. If it is an exponential distribution, then the memoryless property satisfied. Therefore, the residual or the elapsed or the remaining inter-arrival time is also exponential if inter-arrival times are exponential distribution, but here (silence 10:23).

Therefore, at the time of service completion, we should remember the elapsed or remaining inter-arrival time. Therefore, at the service completion time epochs, the memoryless property won't be satisfied whereas at the time instants S_1, S_2, S_3, S_4 and so on, these are all the time points in which the arrival occurs.

At those time points, the memoryless property satisfied even though the system is moving into the different states in between the arrival time epochs. For instant, between the time epochs S_2 to S_3, the system is moved from the state 2 to 1. At the time point 2, at the time point S_2, the system was in the state 2 whereas at the time point S_3 also the system is in the state 2, but in between the system was in the state 1 for some time.

Similarly, in between the time instants S_3 and S_4, the system was in the different states in between the -- these two time epochs and in between the -- even though the transition occurs at those time points, the memoryless property was not satisfied whereas at the arrival time epochs, the memoryless properties are satisfied.

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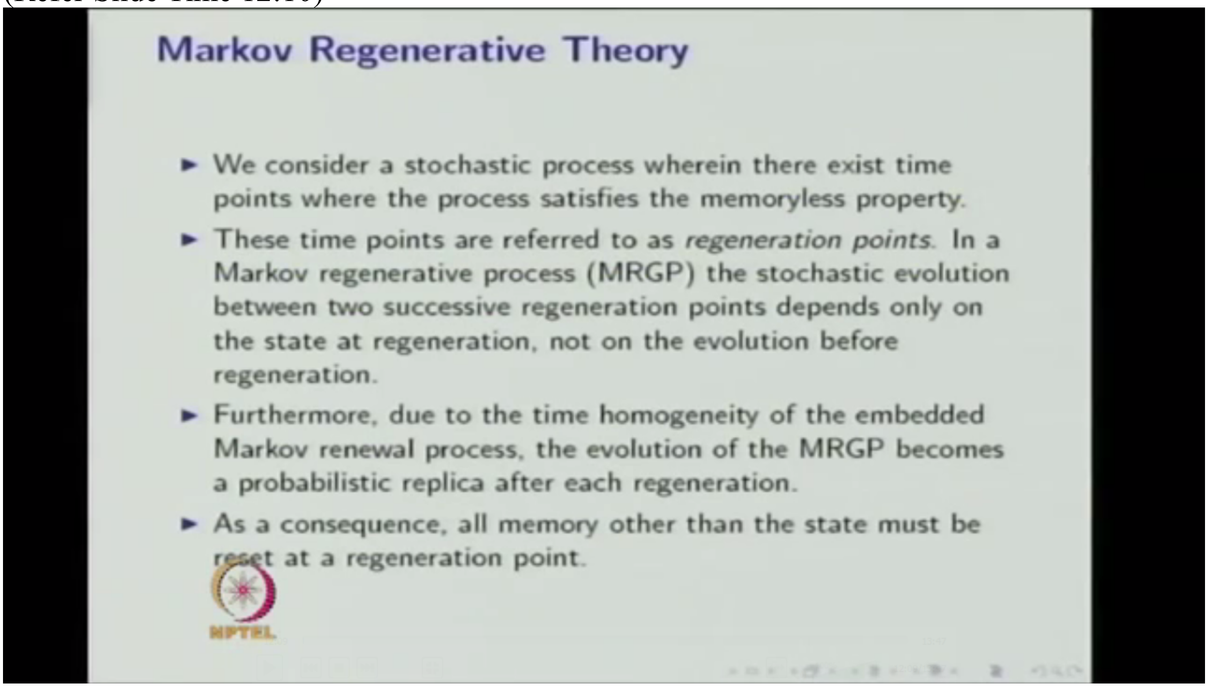

Therefore, these time points are called -- go back to the definition, the stochastic process where there exists a time points where the process satisfies the memoryless property. The time points are referred as regeneration time points. So here the S_1, S_2, S_3, S_4 are the regeneration time points.

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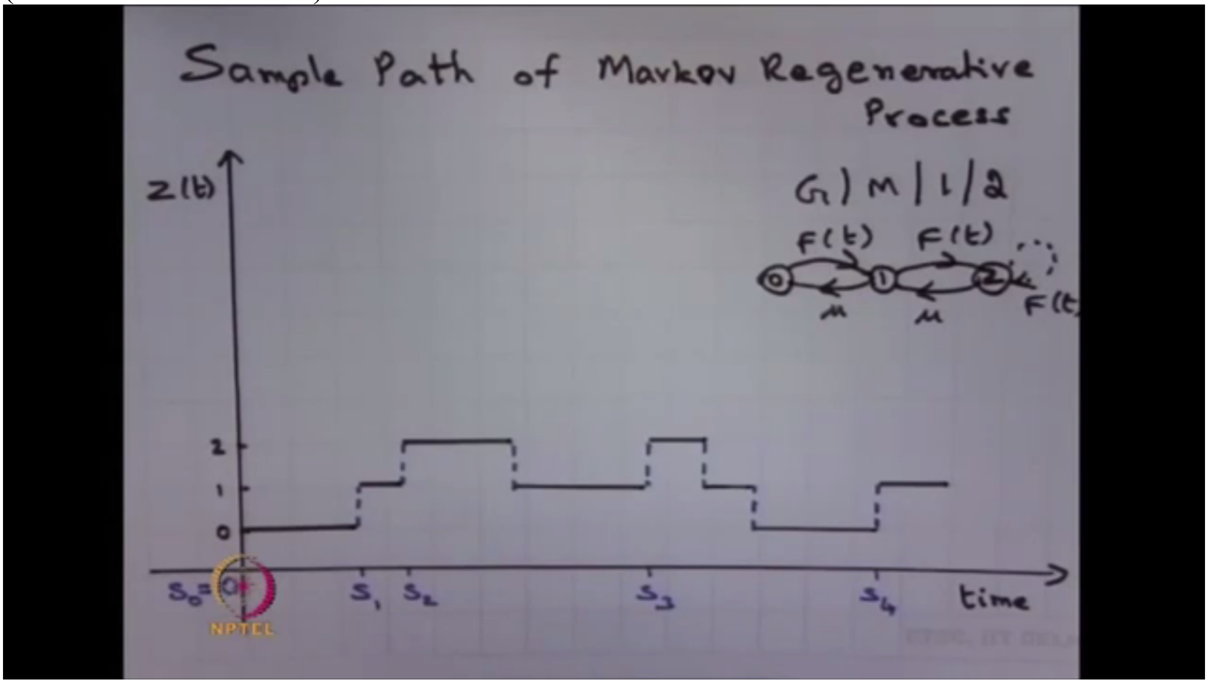

In a Markov Regenerative Process, the stochastic evolution between -- between two successive regeneration time points, that means in between S_2 to S_3 or S_3 to S_4, it depends only on the state at the transition and at regeneration, not on the evolution before regeneration. That means you should remember where the system was at the time point S_2 as well as where the system was in the state at the time point S_3 and you don't want before the regeneration time.

As a consequence, all memory other than the state must be reset at the regeneration point. Therefore, this stochastic process is called a Markov Regenerative Process and I have made a sample path for a Markov Regenerative Process with this example because we are going to consider the same example later. So here S_1, S_2, S_3 are the regeneration time points, not the time instants at which the service completion.