

$$P(N=0) = P\{N \geq 0 \text{ and } U > 0\}$$

$$= \frac{\lambda}{\lambda + \mu}$$

$$P(N=1) = 1 - P(N=0)$$

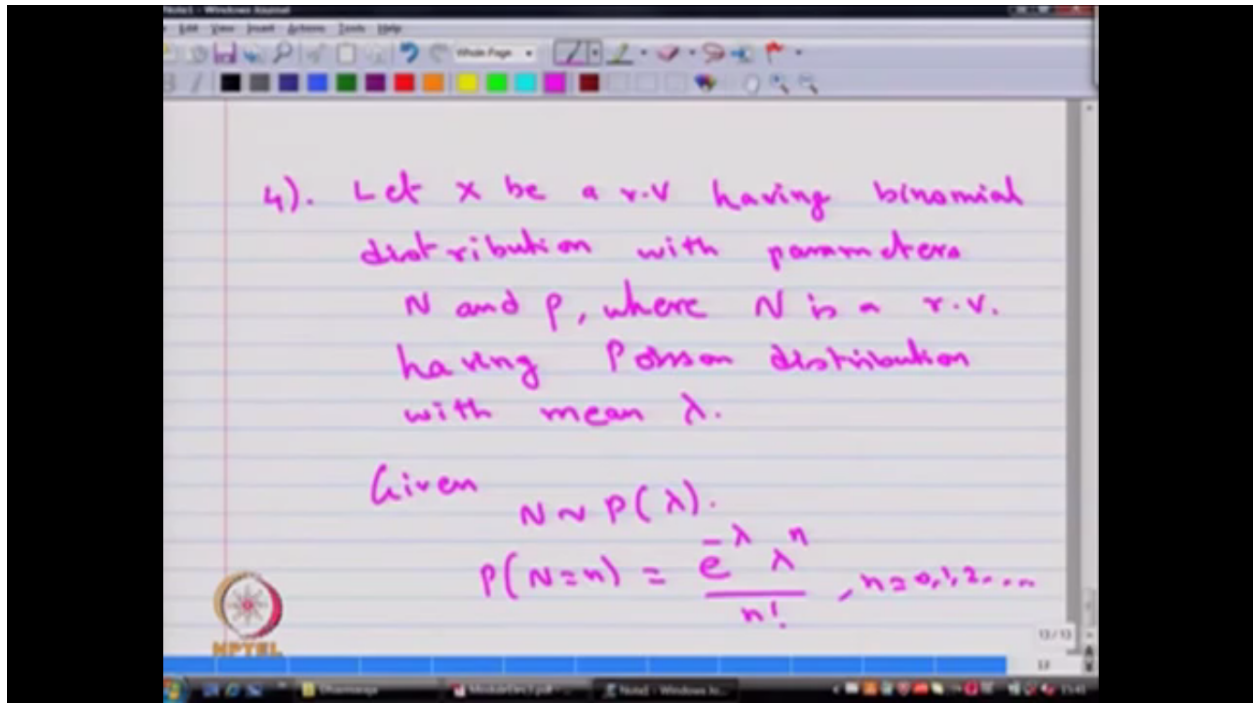
$$= \frac{\mu}{\lambda + \mu}$$

$$P(N \geq 0 \text{ and } U > t) = P(N=0) \cdot P(U > t)$$

$$P(N \geq 1 \text{ and } U > t) = P(N=1) \cdot P(U > t)$$

So in this example we observe that N and U are independent random variables and also by seeing a probability of U greater than T that is $e^{-(\lambda + \mu)T}$ you can conclude U is an exponential distribution with the parameter $\lambda + \mu$.

Now we move into the next example. Let X be a random variable having binomial distribution with parameters N and P where N is a random variable having Poisson distribution with mean λ . The question is find the marginal distribution of X or find the probability mass function of the random variable X . Given N is Poisson distribution with the parameter λ that means the probability mass function for the random variable N is $e^{-\lambda} \frac{\lambda^n}{n!}$. The possible values of N are $0, 1, 2,$ and so on. Our interest is to find out what is a probability mass function of the random variable X .



That is same as n is equal to 0 to infinity what is the conditional probability of the variable X takes the value K given the other random variable N takes a value n multiplied by probability of N takes a value n . That is same as the n takes the value from K to infinity n factorial divided by K factorial into n minus K factorial and P power K 1 minus P power n minus K multiplied by λ power n e power minus λ divided by n factorial. So no need of n is equal to 0 to K minus 1 because the capital N takes a value n therefore the running index from K to infinity. That is same as you can take some terms outside that is λ power K e power minus λ P power K divided by K factorial.

The image shows a digital whiteboard with the following handwritten derivation in purple ink:

$$\begin{aligned}
 P(X=K) &= \sum_{n=0}^{\infty} P(X=K/N=n) P(N=n) \\
 &= \sum_{n=K}^{\infty} \frac{n!}{K!(n-K)!} P^K (1-P)^{n-K} \frac{\lambda^n e^{-\lambda}}{n!} \\
 &= \frac{\lambda^K e^{-\lambda}}{K!} P^K \sum_{n=K}^{\infty} \frac{[\lambda(1-P)]^{n-K}}{(n-K)!} \\
 &= \frac{(\lambda P)^K e^{-\lambda}}{K!} e^{\lambda(1-P)}
 \end{aligned}$$

The whiteboard also features an NPTEL logo in the bottom left corner and a Windows taskbar at the bottom.

The remaining terms that is N is running from K to infinity this can be written in the form of lambda times 1 minus P power n minus K divided by n minus K factorial. That is same as lambda P power K multiplied by e power minus lambda by K factorial.

The summation N is equal to K to infinity and so on that becomes e power lambda times 1 minus P. Therefore, further you can simplify. Therefore, the probability of X takes the value K is same as lambda P power K e power minus lambda lambda P divided by K factorial where K takes a value 0, 1, 2, and so on.

Hence, the conclusion is the random variable X which is Poisson distributed with the parameter lambda times P. So this problem occurs in many situations of stochastic modeling. Therefore we have discussed this example in this lecture.