Stochastic Processes - 1
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Lecture – 09
Problems in Random Variables and Distribution

This is the Stochastic processes, model 1; probability theory refresher, lecture 3; problems in random variables and distributions.

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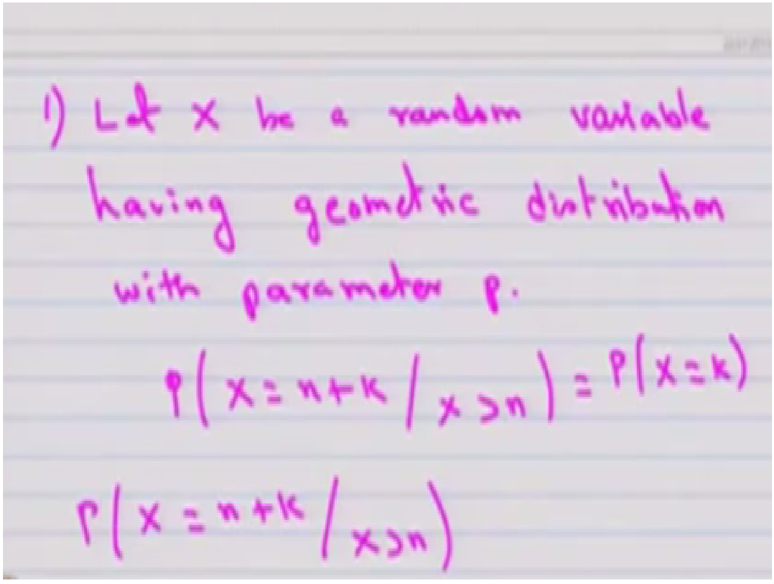

Let as a first problem; let x be a random variable having geometric distribution with the parameter p. Our interest is to find; our interest is to prove that the probability of x=n+k, given x takes the value greater than n that is as same as the probability that x takes the value k for every integers, n and k.

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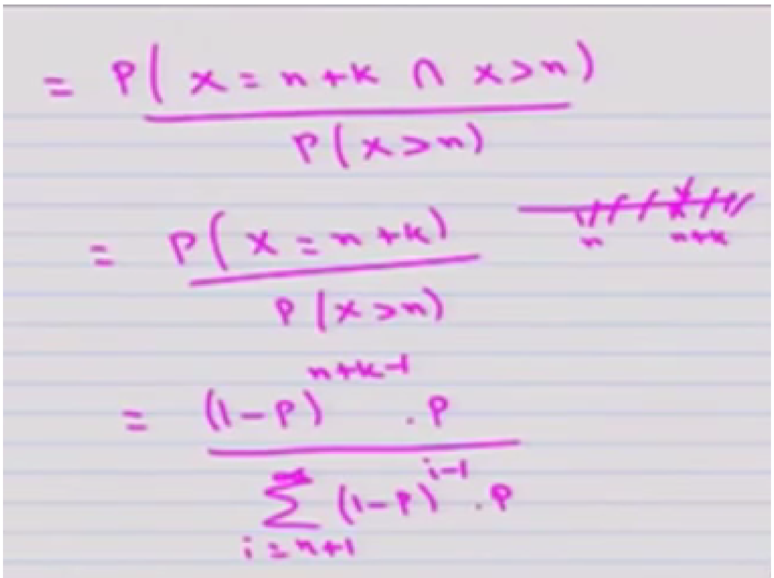

You can prove this result by starting from the left hand side that is probability of x takes the value, n + k, given x greater than n by definition this is same as probability of x = n + k intersection, x greater than n, divided by probability of x greater than n, that is same as; that is same as the numerator; x = greater than n means, all possible values, n=n + k that means that the intersection is going to be probability of x takes the value, n + k.

Whereas the denominator is the probability of x is greater than n, that is same as since x is the geometric distribution with the parameter p, the probability of x = n + k, that is nothing but 1-p time p power n+k-1* p. Whereas the denominator, probability of x is greater than n, that means summation I=n+1 to infinity 1-p power I-1 multiplied by p, that is same as numerator can keep it as it is.

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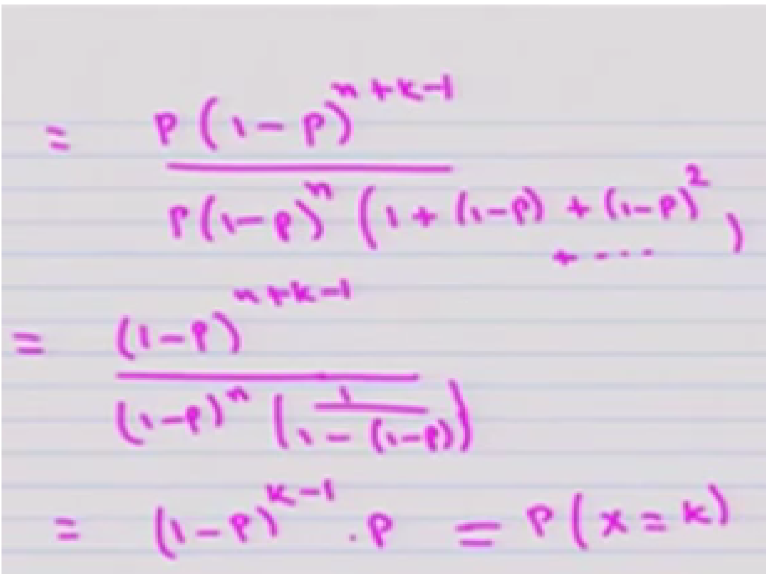

Whereas the denominator since the summation I = n + k to infinity, you can take p times 1- p power, n common outside, the remaining terms are 1+1- p, the third term will be 1-p whole square and so on. Therefore, you can still simplify you will get 1- p power n+k-1 divided by 1- p power n, keep it as it is, this series you will have the value 1-1-p. therefore if you further simplify you will get a 1- p power k-1 multiply by p.

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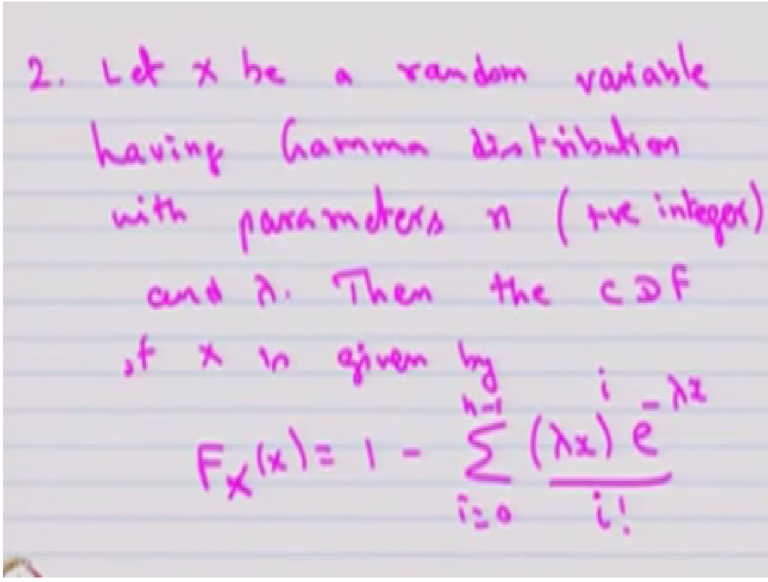

That is nothing but probability of x= k. So this results are the probability of x= n + k given x is greater than n, that is same as probability of x=k for all n at k. This is the important property of geometric distribution and this property is called a memory less property. We will move into the next problem. Let x be a random variable having gamma distribution with the parameter n, you assume that n is a positive integer, the other parameter is lambda.

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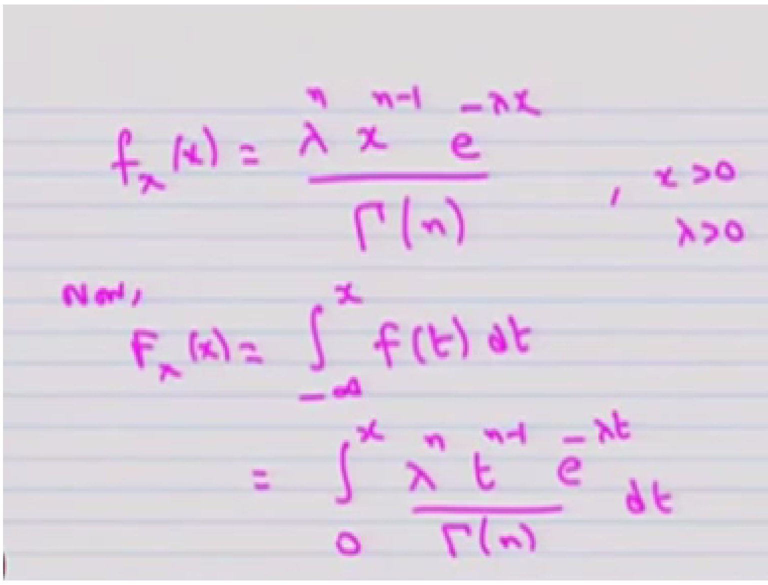

Then the cumulative distribution function CDF of x is given by capital F(x) for the random variable x that is 1-summation I=0 to n-1lambda x power I, e power –lambda times x divided by I factorial. So, whenever x is a gamma distribution with the parameters n and lambda, then the CDF can be written in this way. We know that the probability density function of the gamma distribution is lambda power n, x power n - 1 e power –lambda x divided by gamma of n.

Since n is a positive integer, gamma of n is a n-1 factorial. Now you can find out the CDF of this random variable that is nothing but –infinity to x, the probability density function that is same as since the f(x) is the; this is valid for a x is greater than 0 and lambda is greater than 0. So this integration is valid from 0 to x lambda power n t power n-1 e power – lambda times t divided by gamma of n dt.

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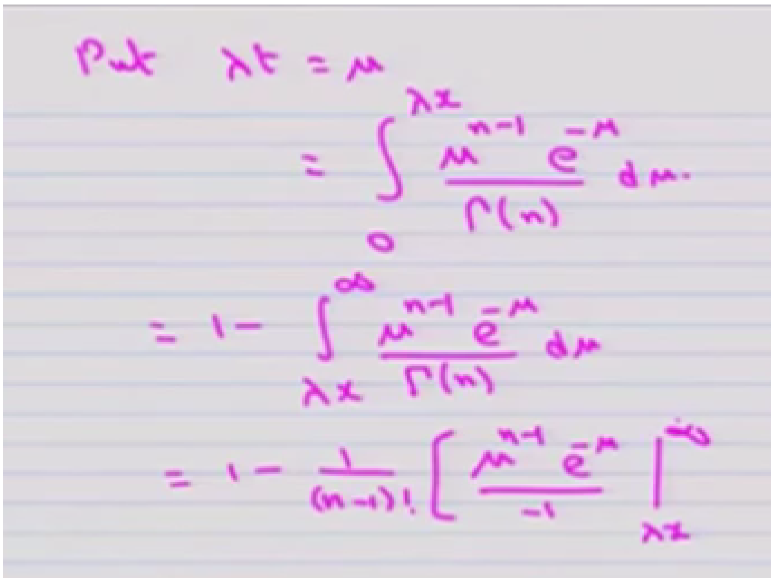

So now you have to integrate this one and get an expression for this CDF of the random variable x. So what we can do, make a substitution lambda times t, that is same as; we make it as some Mu. Therefore, these integration becomes the integration from 0 to lambda times x, Mu power n-1 e power – Mu, divided by gamma of n * d (Mu). That is same as 1- integration goes from lambda x to infinity Mu power n-1 e power – Mu divided by gamma of n d Mu.

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$$- \int_{\lambda x}^{\infty} \frac{(n-1)\mu^{n-2}}{-1} e^{-\mu} \, d\mu \Bigg]$$

$$= 1 - \frac{1}{(n-1)!} (\lambda x)^{n-1} e^{-\lambda x} - \frac{(\lambda x)^{n-2} e^{-\lambda x}}{(n-2)!} - \cdots - \frac{(\lambda x)^{0} e^{-\lambda x}}{0!}$$

$$= 1 - \sum_{i=0}^{n-1} \frac{(\lambda x)^{i} e^{-\lambda x}}{i!}$$

That is same as 1-; since n is a positive integer gamma of n is n-1 factorial so you can take it outside. You can do this integration by parts so you will get a; Mu power n-1 e power - Mu divided by -1 between the limits lambda x to infinity – integration from lambda x to infinity n-1 times Mu of n-2 e power –Mu divided by -1 d Mu. So the whole thing is multiplied by n-1 factorial.

Now you can integrate the second term again by integration by parts and when you substitute the limits for Mu is infinity and as well as Mu = lambda x and subsequently if you do the integration by parts, you will get a 1- n-1 factorial lambda x power n-1 e power – lambda x. Then the next term will be –lambda x power n-2 e power - lambda x by n-2 factorial. Similarly, the other terms.

The last term will be by doing integration by parts again and again, the last term you will get minus- of lambda x power 0 e power –lambda x by 0 factorial. These we can write it in the form 1-summation I=0 to n-1 lambda x power I e power –lambda x by I factorial. So here we are finding this CDF of the gamma distribution when one of the integer is the positive integer, one of the parameters is a positive integer.

This result will be useful in finding the total time spending the queuing system, that will be discussed in the later models.