Stochastic Processes - 1

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Lecture – 77
Arc Extensions in Petri Net, Stochastic Petri Nets and Examples

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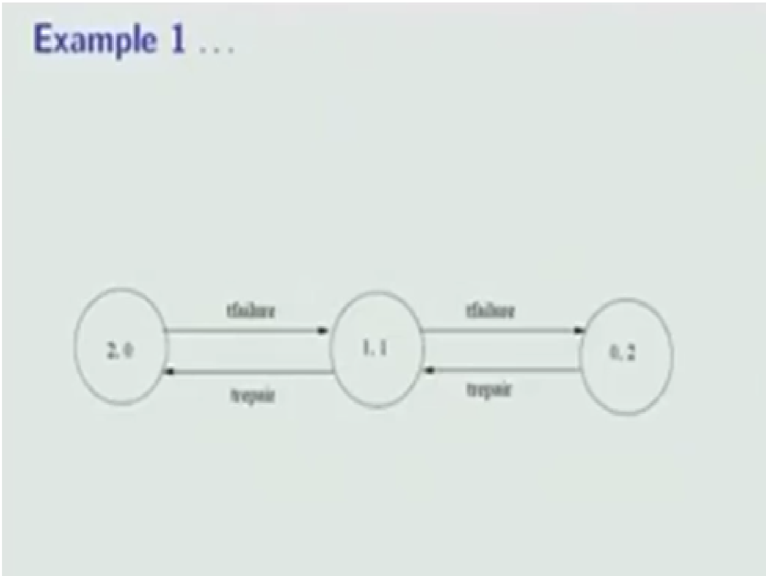

Consider this example, the example which we have considered earlier example 1.

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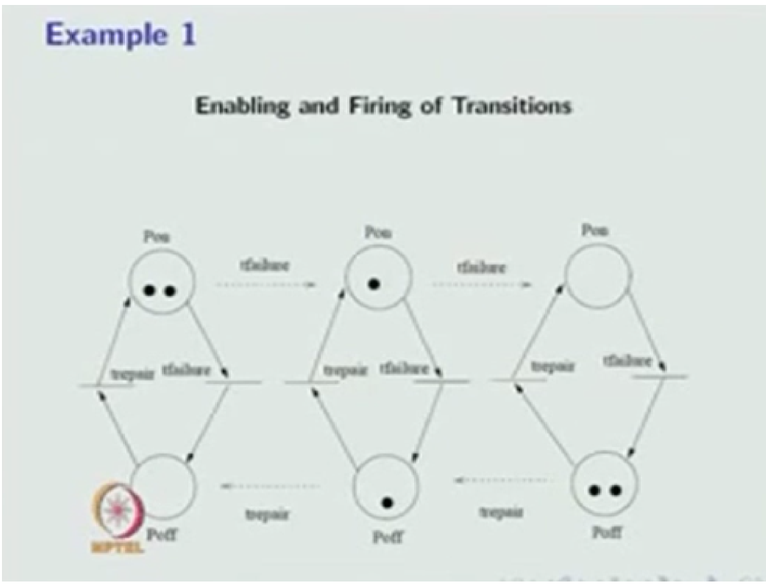

That means we have two places, place P on, the other place P off. Initially, two tokens in the place P on, no token in the place P off therefore, the marking will be a two tuple. Number of tokens in each place form a marking therefore, the first place is P on, second place is P off with that assumption, suppose if the first place is P on, second place is P off then the number of tokens are time zero is 2, 0.

After T failure firings the marking will be 1, 1. Again the T failure firing the marking will be 0, 2. From 1, 1 if T repair fires then the marking will be 2, 0. From 0, 2 if a T repair fires then the marking will be 1, 1.

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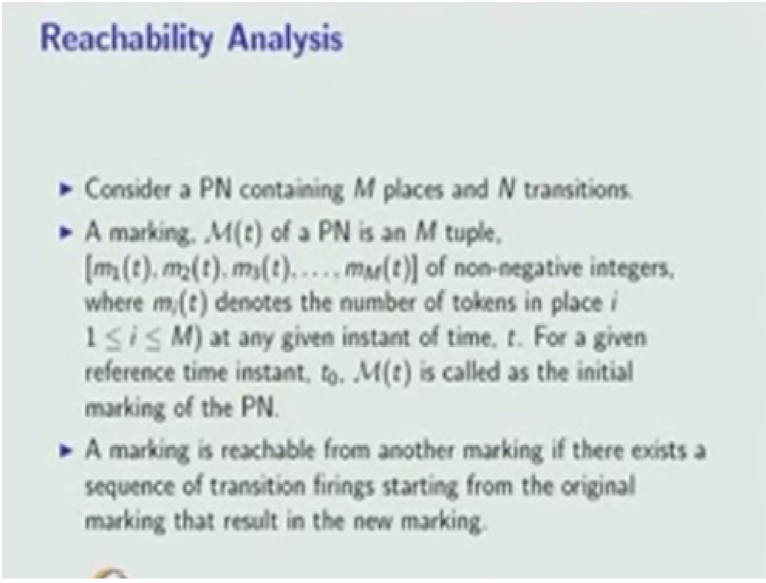

Hence, the marking is the m tuple, number of tokens in the place i at any given instant of time and the marking is reachable whenever some sequence of a transition fires.

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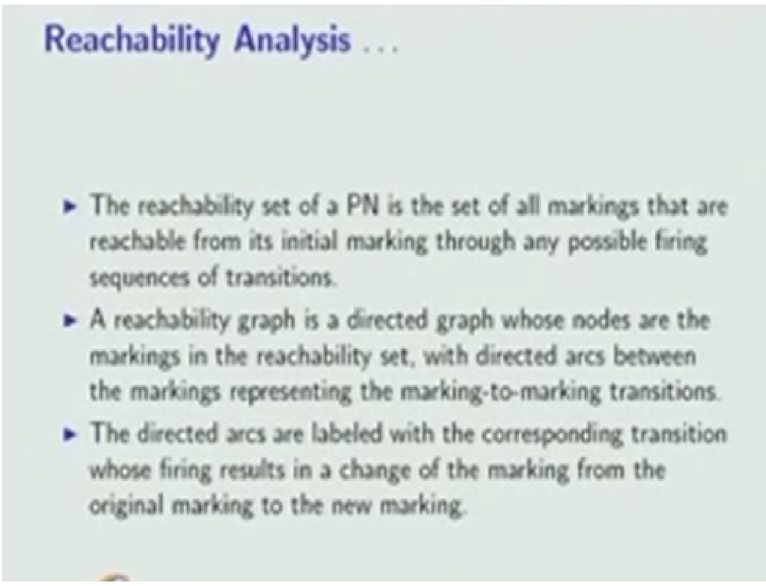

And the reachability graph is a directed graph whose nodes are the markings in the reachability set with directed arcs between the markings represent the marking to marking transitions. Hence, the same example, the markings are 2,0, 1,1, 0,2. The marking can be reachable from 2,0 to 1,1 by the transition T failure fires.

The marking 1,1 is reachable to the marking 0,2 with the T failure firing. The marking 0,2 is reachable to the state to the marking 1,1 by T repair fires. Similarly, 1,1 to 2,0 by firing T repair transition. So this is called reachability graph.

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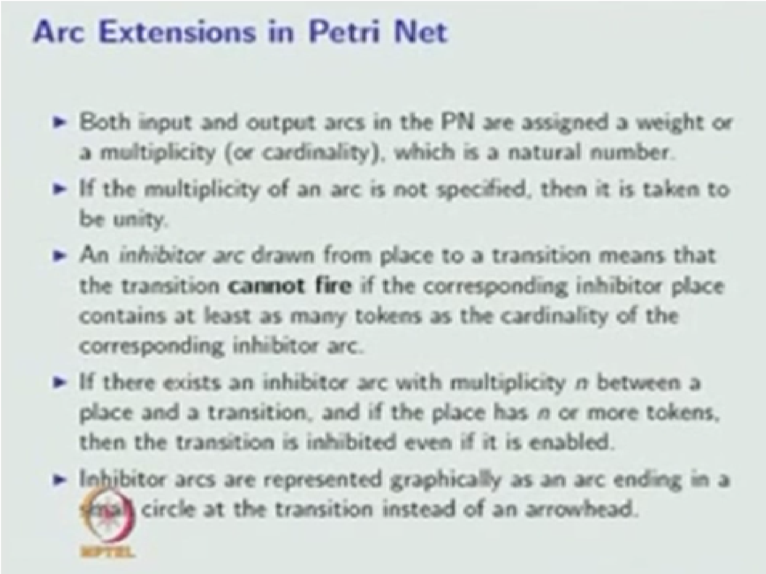

Now we are moving extensions, arc extensions in petri net. Till now we have considered very simple petri nets. Now we are going for arc extensions. Both input and output arcs in the petri net are assigned a weight or multiplicity or cardinality which is a natural number. If the multiplicity of an arc is not specified, then it is taken to be unity. So in the previous examples, we have considered as a multiplicities unity.

The next extension in the petri net is inhibitor arc. An inhibitor arc drawn from place to the transition means that a transition cannot fire if the corresponding inhibitor place contains at least as many tokens as the cardinality of the corresponding inhibitor arc. Usually, input arc and output arcs makes a transitions enabling and firing, removing the tokens depositing the tokens with the multiplicity or cardinality.

But the inhibitor arc drawn from place to the transition means that the transition cannot fire if the corresponding inhibitor place contains at least as many tokens as the cardinality of corresponding inhibitor arc. If there exist an inhibitor arc with the multiplicity n between a place and a transition, and if the place has n or more tokens then the transitions is inhibited even if it is enabled.

The transition can enable based on the condition through the input arcs but if there is a inhibitor arc then it may be inhibited based on the number of tokens in the corresponding input place. Input arcs are represented graphically as an arc ending in a small circle at the transitions instead of arrowhead. Usually, input arcs as well as output arcs drawn with arrow head but inhibitor arcs are represented graphically as an arc ending in a small circle.

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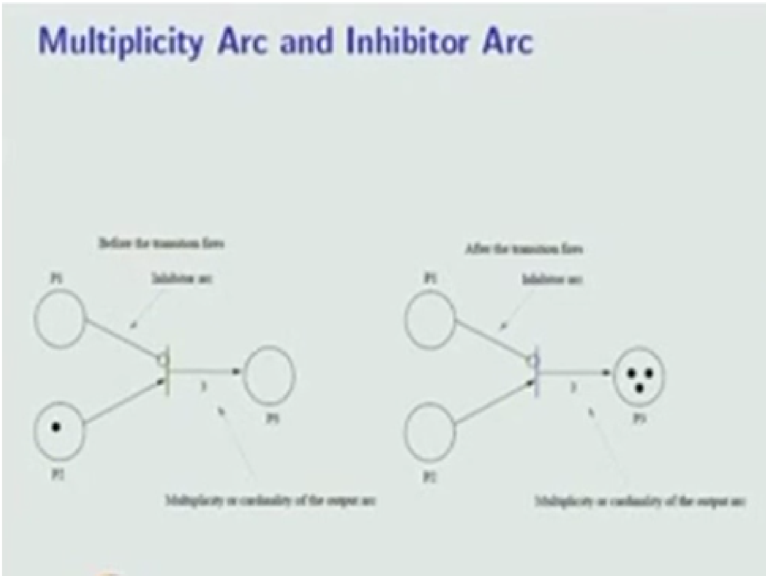

We will see the example in this slide. In this example, we have a three places p1, p2, p3, we have a one transition, one input arc, one output arc, one inhibitor arc. Here whenever the number is written next to the arcs that means the multiplicity or cardinality of the output arc is number 3. If there is no number, natural number written next to the arcs that means the default multiplicity is one.

Here also the default multiplicity is one that means if one or more token in the place p1 even the transition is enabled by the condition through the input arcs for this place input place, this transition may not fire if one or more tokens in the place p1. So here no token in the place p1 whereas one token in the place p2, no token in the place p3 hence, the transition enables and then fires by removing one token in the place p2 and one token deposited in the place p3.

Only the tokens will be removed from all the input places which are connected, the places connected with the input arcs to the transition and the tokens will be deposited to the all the output places which are connected from transition to places through output arcs. So in this example, after the transition fires, no token in the place p1, no token in the place p2 and three tokens in the place p3 because the multiplicity of the output arc is three.

So even though one token is removed from the place p2 because the multiplicity of the arc is three. Hence, three tokens will be multiplicity is three, therefore, three tokens will be deposited at same time with the place p3.

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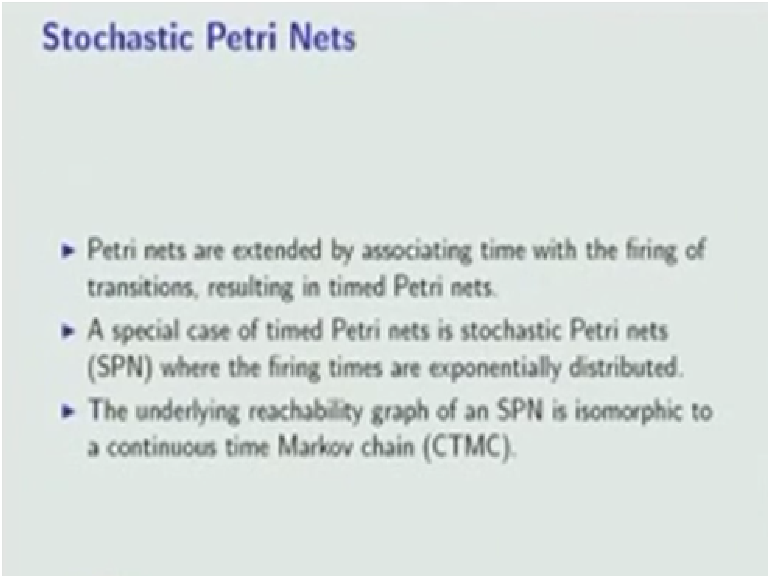

Now we are moving into the extension of petri nets into stochastic petri nets. Petri nets are extended by associating time with the firing of transitions resulting in timed petri nets. A special case of timed petri nets is stochastic petri nets in other words SPN where the firing times are exponential distribution. So whenever the firing times of all transitions are exponential distribution then the corresponding timed petri nets are called stochastic petri nets.

Every petri net one can get the reachability graph. The underlying reachability graph of a stochastic petri net is isomorphic to a Continuous-time Markov Chain. For a stochastic petri net the underlying reachability graph is isomorphic to a Continuous-time Markov Chain.

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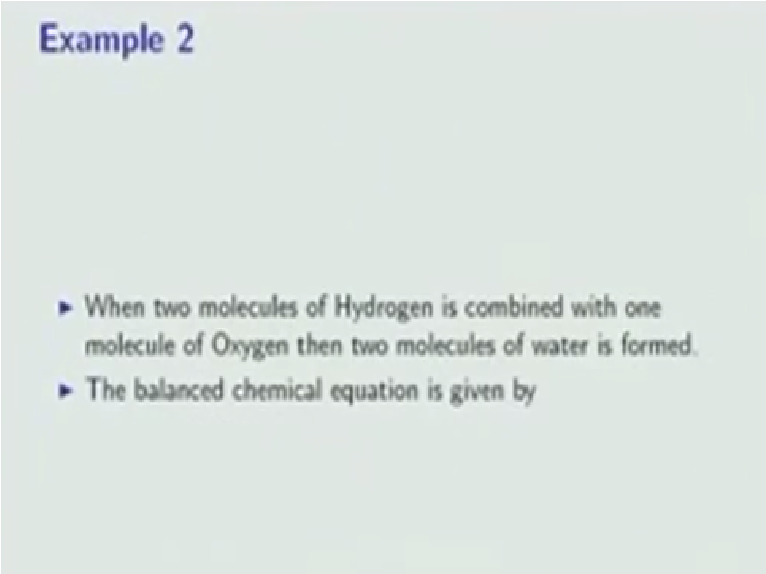

Let us see through the examples. When two molecules of hydrogen is combined with one molecule of oxygen then two molecules of water is formed.

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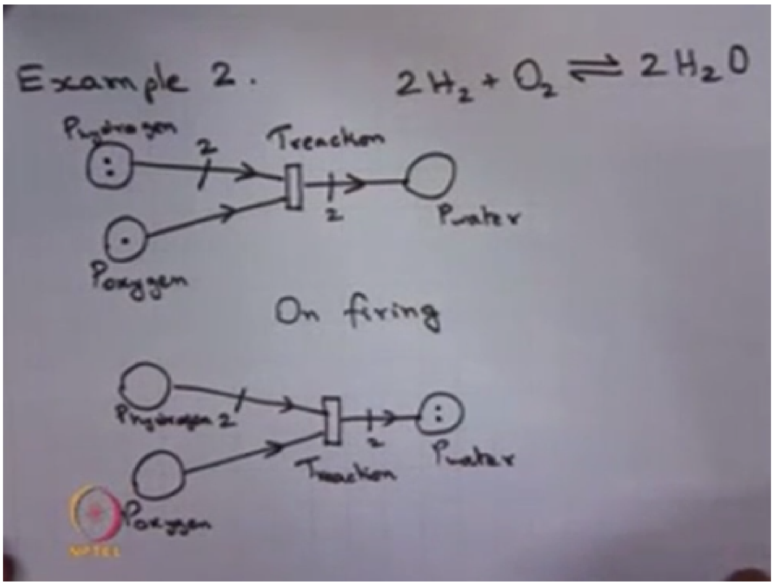

The balanced equation is written; the balanced chemical equation is given by 2H2 plus O2 gives two times H2 O2. So this is the balanced chemical equation. For this scenario, we have made three places, p hydrogen, p oxygen, p water and we have one transition that is called a T reaction.

The multiplicity of an input arc from the place of p hydrogen to the T transition that is two whereas the multiplicity of the arc from p oxygen to the T reaction that is one. Whereas the multiplicity of an output arc from the transition to the place T water that is two. On firing of the transition T reaction, two tokens because the multiplicity is two. Two tokens will be removed from the place p hydrogen.

The multiplicity is one therefore; one token will be removed from the place p oxygen. Since, it is a rectangle bar that means the timed transition follows the time of timed transition follows exponential distribution. So after the, so two tokens will be removed from the place p hydrogen, one token will be removed from the place p oxygen on firing of the reaction T reaction, two tokens will be deposited in the place p water because the multiplicity is two.

And here we made the assumption, the reaction time is exponential distribution, hence we made a stochastic petri net and we make the model using stochastic petri net of this chemical reaction.