Stochastic Processes - 1

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Lecture – 75
Simulation of Queueing Systems

Now I am moving into the last part of model 5 Continuous-time Markov Chain. In this I am going to discuss the simulation of queueing systems. So the model 5 Continuous-time Markov Chain started with a definition and properties and so on then it discussed the birth death process, then I discussed the application of a birth death process in simple Markovian queueing models.

Then I also have discussed the queueing networks that is also multidimensional Continuous-time Markov Chain as application and finally I have given few practical applications in cellular networks for the performance analysis. Now I am going to discuss the discrete event simulation of a simple Markovian queueing systems.

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So in this queueing network modelling lab, one can do discrete event stimulation.

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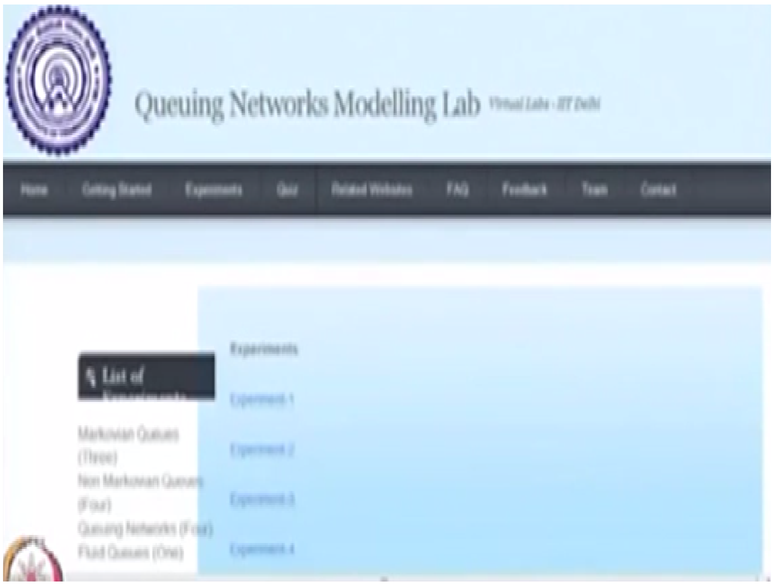

And the discrete event simulation for the queueing network involves Markovian queues, we can do some experiment over the Markovian queues and you can do the discrete event simulation for the non-Markovian queues.

And one can do the discrete event simulation for the queueing network also and finally one can do the fluid queues also. One can simulate but since I have discussed only Continuous-time Markov Chain till now I am going to do discrete event simulation for the Markovian queues. So that is the first three experiments. The other three experiments are related to the non-Markovian queues.

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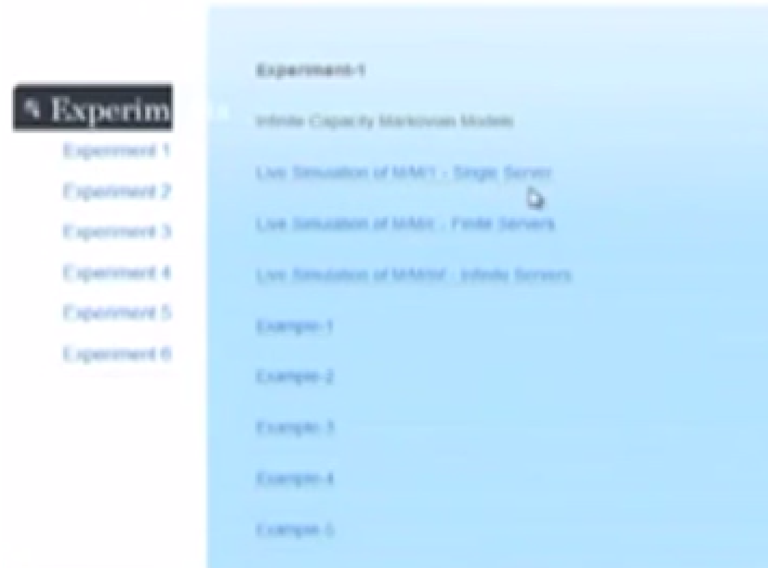

The first experiment consists of a simulation of a MM1 queue, single server queue and MMC finite server queues and the infinite server queue. So in the last sometime I have done the discrete event simulation of a MM1 queue. So let me go to MMC finite server queue.

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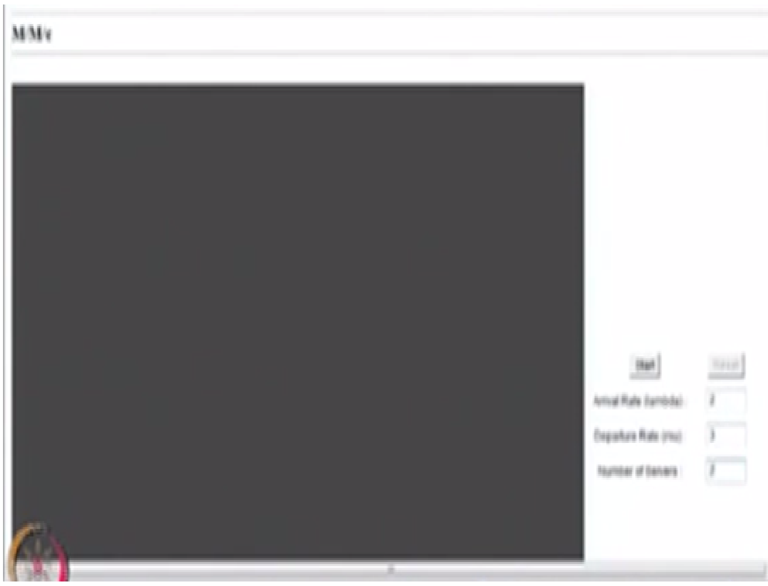

So in this you need the input of arrival rate, input of the departure rate and the number of servers, is the multi server infinite capacity model. So suppose you choose the arrival rate is 2 and the service rate is 3 and the number of servers are 2 that means it is a MM2 infinity model with arrival rate 2 and the service rate is 3.

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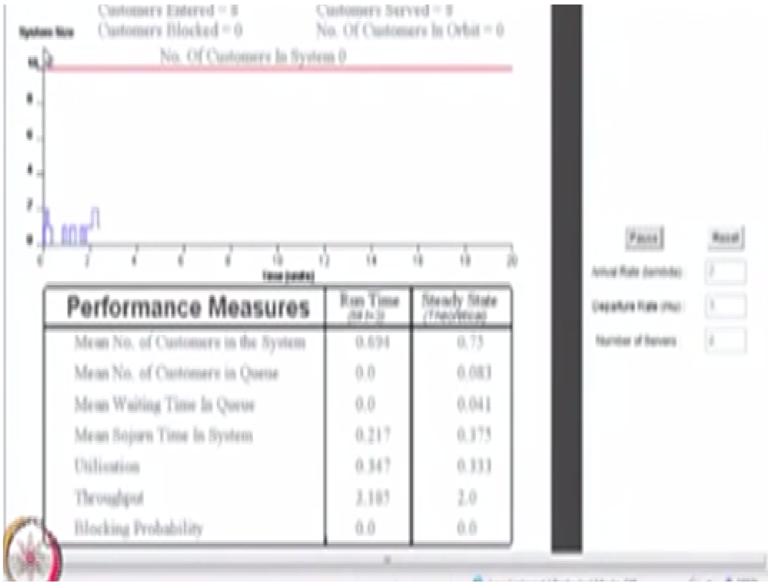

So one can start the discrete event simulation by clicking the start. Then you will get the window of, so this is the sample path over the time what is the system size. So at this time point one arrival comes, then two, then service is completed and so on. So this is a sample path over the time.

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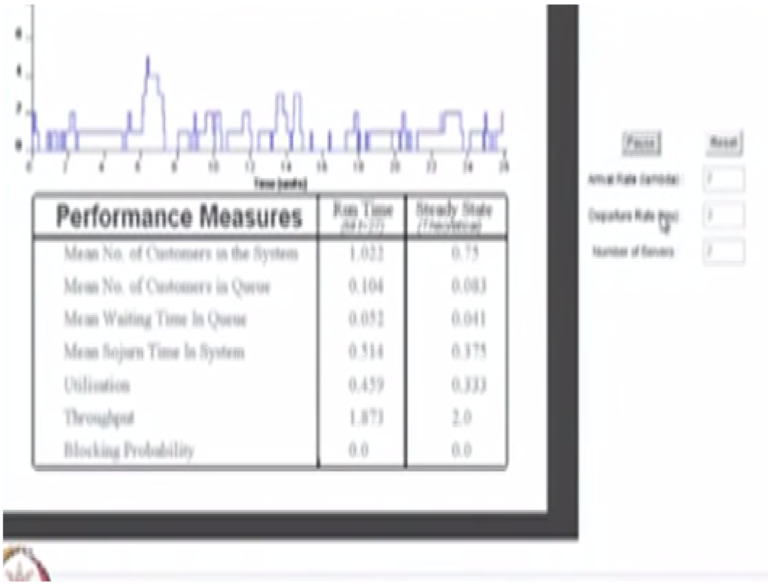

And here you can get the performance measure. So whatever I have discussed the performance measures of a steady state distribution and all other performance measure, one can get it with the theoretical results in the third column whereas the second column gives the running time till the discrete event simulation runs for 15-time unit and what is the result for the mean number of system and so on and this will converge to this value for T tends to infinity.

So I make sure that this arrival and departure rates are satisfying the condition so that the steady state distribution exist, therefore as T tends to infinity this will reaches the steady state theoretical result. If you change the value, arrival rate and the departure rate something else then there is a possibility if the conditions for the stationary distributions are not satisfied then still the runtime results you will get, but that will not converge to the theoretical and also this steady state so only possible.

It will be dash here, so as long as the steady state distribution those conditions are satisfied, then you will have the results in the third column, otherwise you will not have the results here. So now you can see the throughput till this much time, you are getting 2.0 whereas the steady state throughput is 2. Throughput is nothing but the number of customers served per unit time.

Let that one can get the throughput utilization average response time or mean sojourn time and mean waiting time in the queue so using Little's formula used to find out this quantity, so this quantity you can get it from the discrete event simulation at any time as well as the theoretical result and this is the mean number of customers in the system that is ease n and the mean number of customers in the queue that is EQ which we got it.

So this result, one can see the discrete event simulation as well as a T tends to infinity, what is the theoretical result if the conditions for the stationary for the stationary distribution satisfied.

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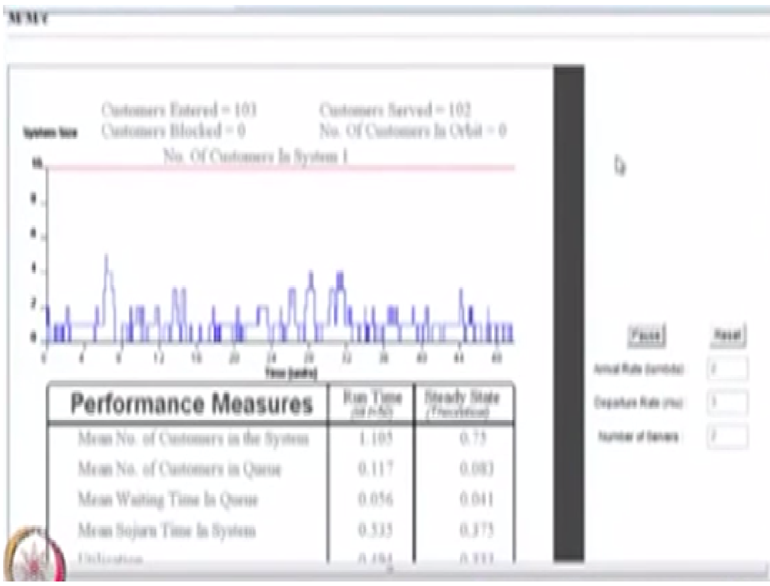

And here this information is how many calls are entered, how many customers enter into the system and how many all served and how many customers are blocked. Here there is no blocking because it is an infinite capacitive system. And we are considering the retrial orbit and so on therefore here it says number of orbit customers is zero. This is not necessary. This is irrelevant information in the MMC queueing model. Now let me go back and do the live simulation of MM infinity.

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So here we have to provide only the arrival rate and the service rate because the number of servers are infinite, this helps service system.

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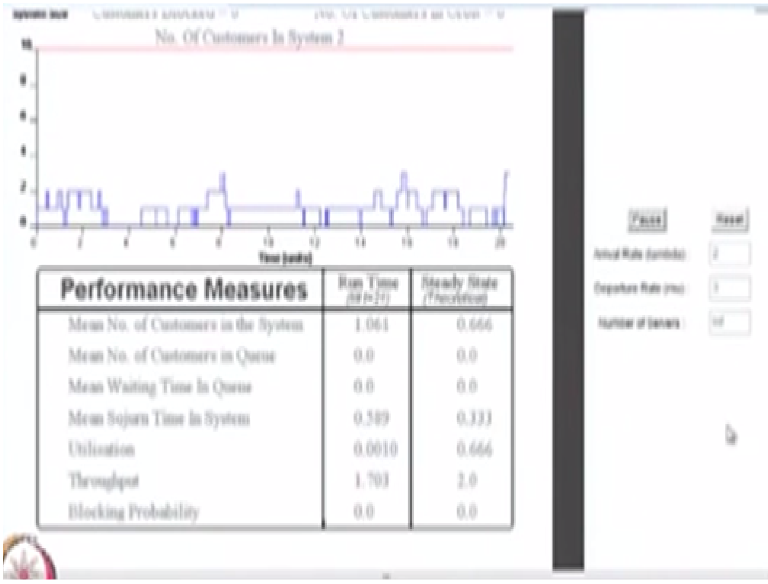

You can cross check the theoretical results are correct and so on. So here I am getting this steady state result based on the theory whereas this is a run time results. There is no blocking probability because this system infinite capacity. Mean number of customers in the queue that is zero and here also zero because it is an infinite server therefore, customers who enter into the system immediately will get the service.

Therefore, the queue average number of customers in the queue that is zero and the average time spending in the queue that is also zero, it is correct. There is no blocking and utilization is, what is the probability that the servers are utilised. Now let me go the next experiment that is the second experiment for the finite capacity queueing model whereas the first experiment is the infinite capacity queueing model.

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So in this we have, we need to give the arrival rate, service and the number of servers we can go for the multi server MMC N model, you can give one also. You can give infinite servers also can give. So here the number of servers you fix it 3 and the capacity is 4. You can do the simulation.

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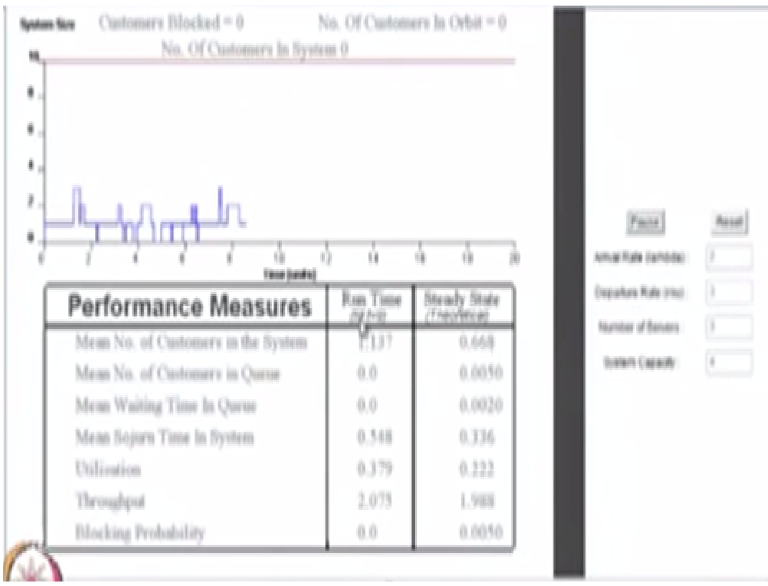

So the blocking probability the run time there is a, at this time it does not cross the number 4, therefore the blocking probability is zero is zero and the run time result whereas the theoretical steady state results is the blocking probability 0.005. So whenever the system touches 5 then you will have the blocking probability in the run time, so this is a discrete event simulation.

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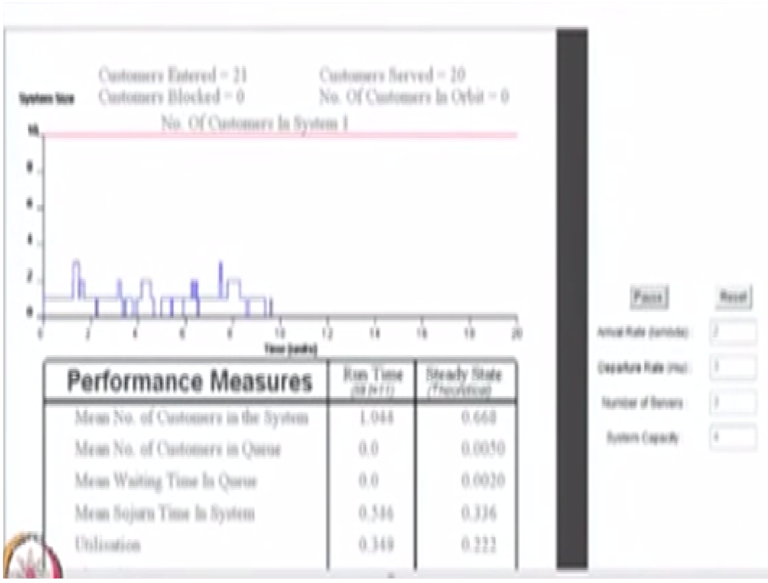

So this is the discrete event simulation. So the number of customers blocked is till now is zero therefore you are getting the blocking probability zero at this run time. Suppose you run it again let us see.

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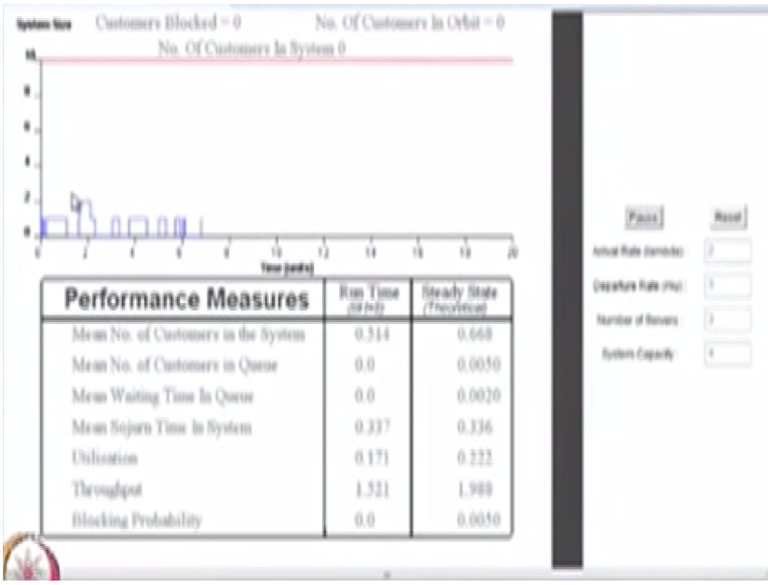

Now I will show you it does not crosses the system size by two, so maybe I can reset with the number of servers I can put 2 and capacity is 3.

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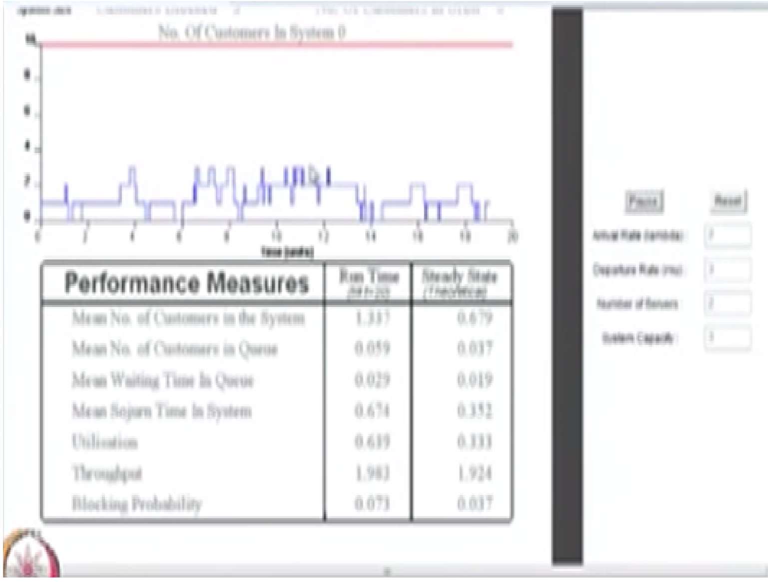

So the blocking probability is this much, the capacity of the system. So there is no arrival after crossing the first still you are getting the blocking probability zero whereas the steady state theoretical results is 0.037. so one can simulate with the different parameters and you can see the sample path. So this is a sample path for MM2, 3 queueing system with the arrival rate 2 and the service rate is 3. Now we are getting the blocking probability because the system crosses, it touches capacity 3 therefore some calls are blocked, some customers are blocked therefore the blocking so you can find out from here.

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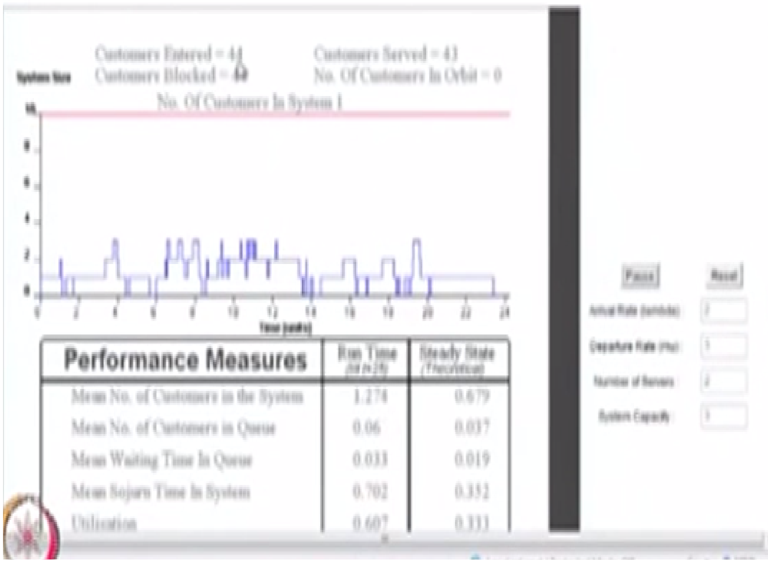

Number of customers 44 entered and 4 are blocked therefore that ratio will be the blocking probability because it is a discrete event simulation. Now let me go back and go to the experiment 3.

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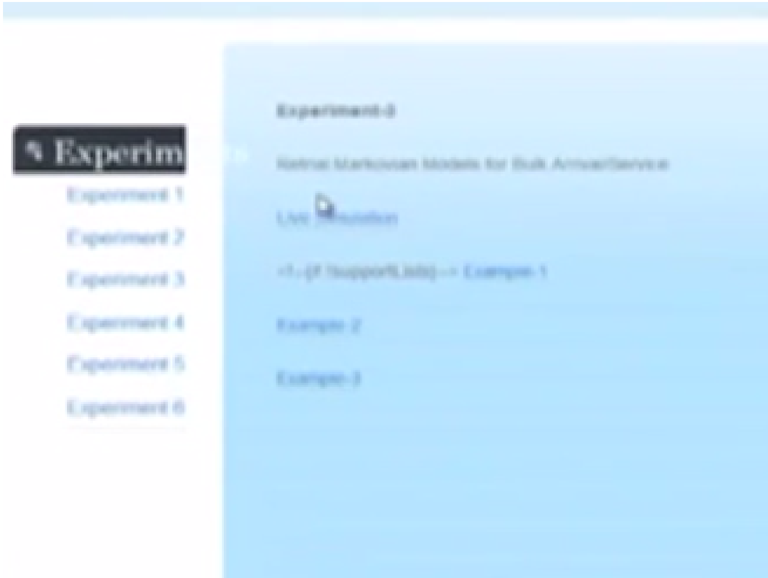

Experiment 3, we have a retrial model with the bulk arrival and bulk service. So now it is no more birth death process simulation. This is a non-birth death process because you have bulk arrival and the bulk service as possible and the retrial therefore, let me show the simulation.

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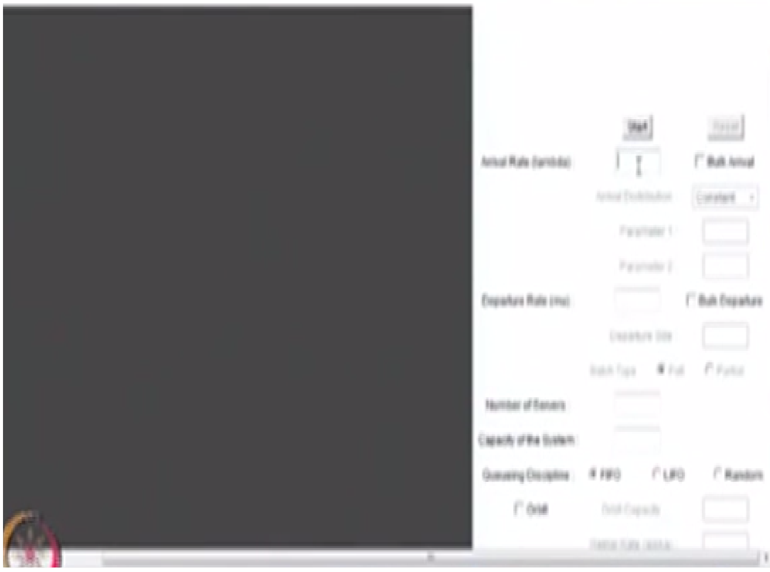

You need some more information. So what is a arrival rate we have to supply. Suppose if it is a bulk arrival and then you should say what is the distribution, whether the bulk arrival comes in a bulk or some constant number or it comes in some distribution. You can choose its geometrical distributer. And the parameter for the bulk arrival parameter you can choose some 0.5.

Then the departure rate, you can choose a departure rate, it can be a bulk departure or bulk so either you can choose bulk arrival or bulk departure or you can choose both also. The number of service in the system, suppose there are two servers in the system and the capacitive system is 4 or you can choose the infinite capacity of the system also.

And if you need orbit then you have to click for orbit and if you are changing the queue in discipline, the first come first served, last come first served are random order, you can choose the queue in discipline also accordingly the discrete event simulation goes. So this is not a birth death process, this is a Continuous-time Markov Chain simulation with a different queueing discipline also one can go for it.

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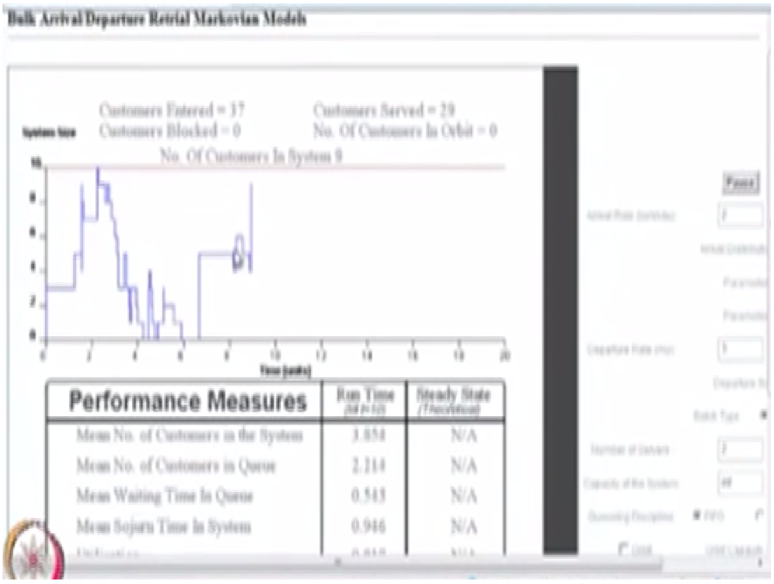

Now once you start the simulation, since it is a bulk, we have made a bulk arrival whereas we made, we did not click for the bulk departure, therefore the customers keep going by one by one whenever the service is over but since it is a bulk arrival with the arrival distribution is a the number of arrivals that is a geometrical distributer. Therefore, it just jumped with the bulk arrival whereas the departure is by one by one.

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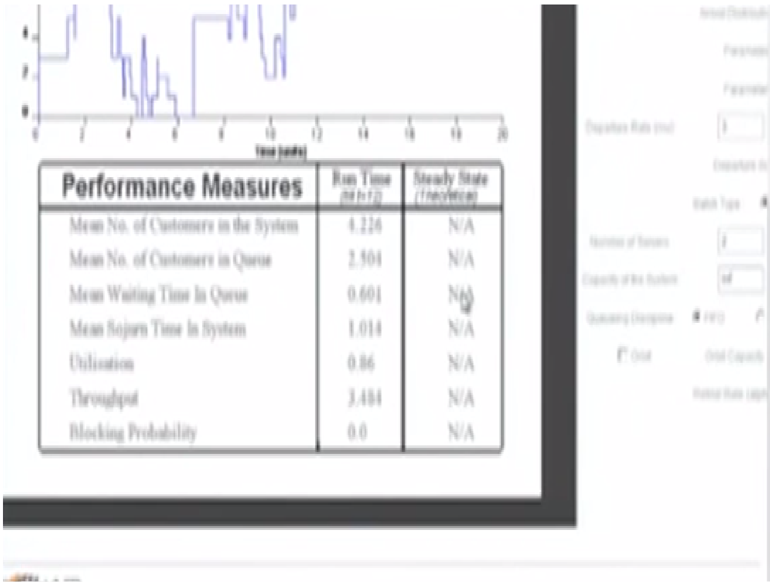

And here is the performance measures is the run time performance measures and till now we did not supply the theoretic steady state or the theoretical results for this model. And these are all the different results with over the run time. And similarly one can go with the bulk departure and you can change the queue in discipline also. So this is a discrete event simulation sample path for this scenario.

So I can reset and if I do not want the bulk arrival, and if I choose the capacity of the system is 4, so this will be a no bulk arrival, no bulk departure, therefore this is a MM24 system at the first come first out. And I can go for instead of first come first out I can for the last come first out also.

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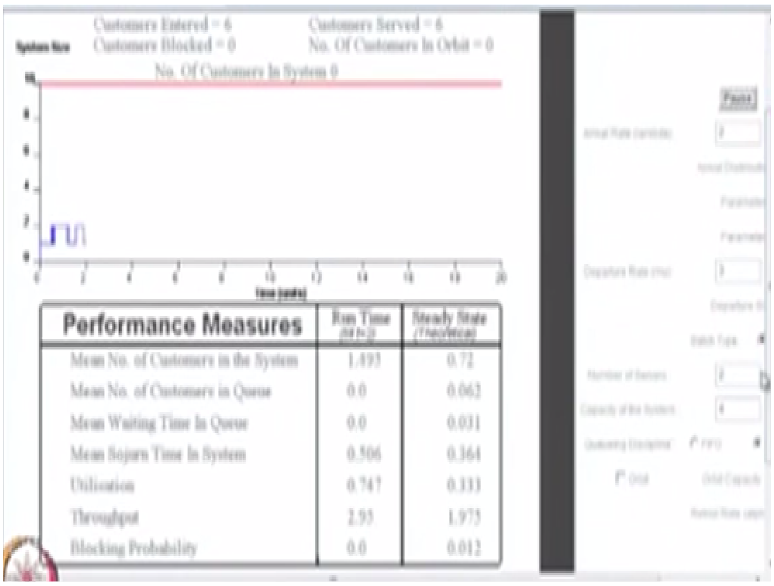

The customers are last entered, he is getting the service first. So this is a steady state and this is a time dependent result. And I can change again this until that means it is a two service infinite capacity model with the first come first out.

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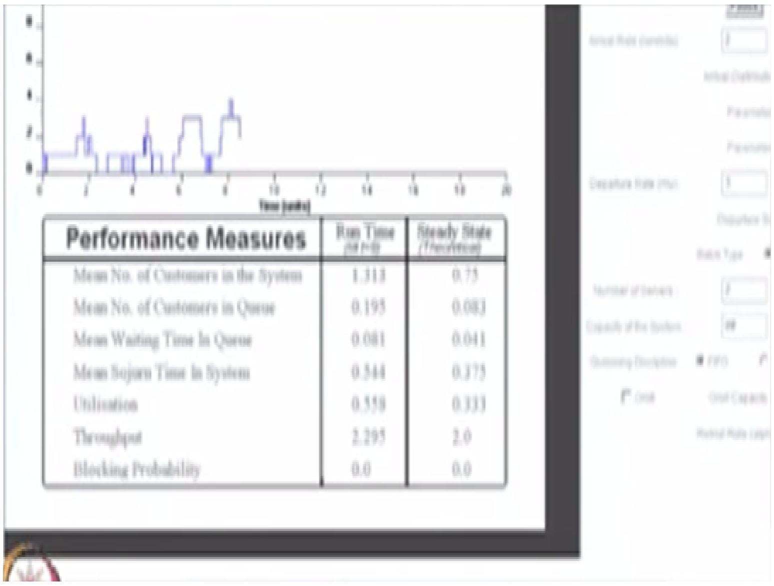

Still this is the testing phase, so some of the things should be removed, some of the things has to be edited. So is still it is in the testing phase. So this is the performance measures for the MM2 infinity model. So that means in this experiment also you can remove the bulk arrival and the bulk departure part and so on you can try the simplest, simple Markovian queueing model. Also one can do the discrete event simulation of that.

And since it is a infinite capacity model, the blocking probability is zero. And the steady state throughput is 2, one can find out from the formula also whereas the run time is this. So if this discrete event simulation runs for a longer time then this will converge to like that you can discuss all other results also. All the results will become converges to the steady state theoretical probability with the theoretical results.

So with this let me complete the discrete event simulation of a simple Markovian queueing model because we have discussed in this under the title of a application of Continuous-time Markov Chain. Therefore, I discuss only the Markovian queues. There are some non-Markovian queues and so on.

So I am not discussing the non-Markovian queues at this stage. After I discussed the renewal theorem and Markov regenerative process and a semi Markov process and so on, I will be discussing the non-Markovian queueing systems also. So here we have discussed only a simple Markovian queueing system that is nothing but the applications of a Continuous-time Markov Chain.

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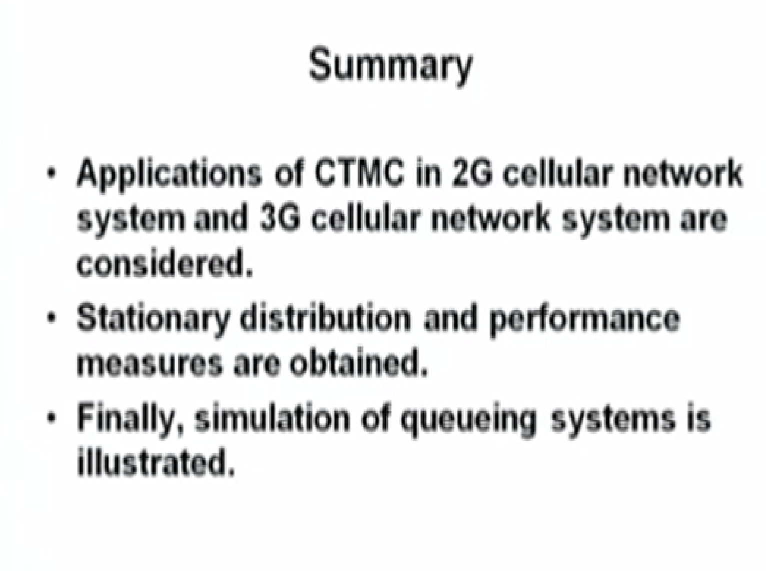

As a summary, the last seven lectures we have discussed the Continuous-time Markov Chain from the definition and properties and a few simple (()) (17:33) Continuous-time Markov Chain starting with the Poisson process, birth death process, pure birth process, pure death process, then application of CTMC in the queueing models starting with the MM1 queue and other simple Markovian queues.

Then we discussed a few queueing networks which has the product solution. Then in the last this lecture we have discussed the applications of CTMC in second generation cellular networks as well as the third generations cellular networks and application of CTMC in the 2G network is the birth death process whereas the applications of CTMC in 3G cellular networks is the quasi birth death process.

Even though I have not discussed in detailed the complete modelling, one can see it from the paper. My intention here is to explain the cause of birth death process through the applications and we discussed the stationary distribution and all other performance measures for the birth death process comes in the second generation networks and the cause of birth death in the third generation networks.

And finally I have given the discrete event simulation for simple Markovian queueing systems only whereas some more non-Markovian queueing systems so that I will discuss with the other models.

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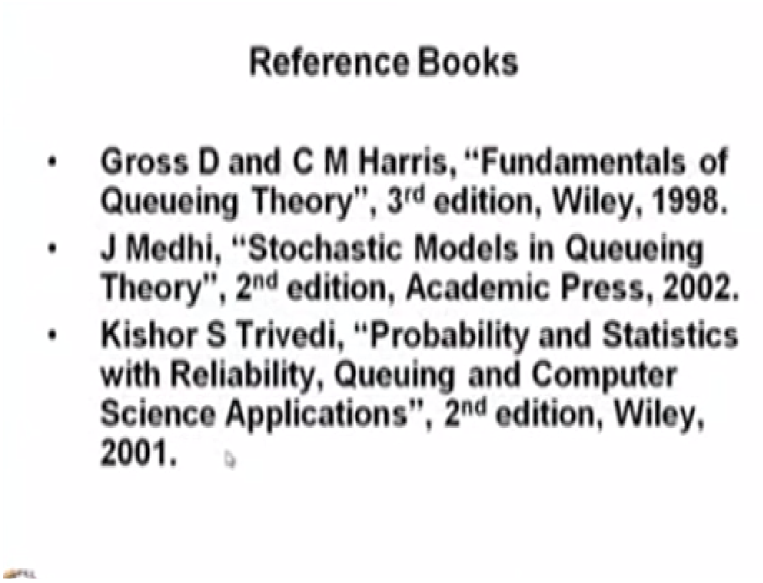

The references are Gross and Harris book and Medhi book, Kishor Trivedi’s book, Thanks.