

**Stochastic Processes - 1**  
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**Lecture - 74**  
**Description of 3G Cellular Networks and Queueing Model**

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### Steady-State Solutions

$$\text{Let } \lambda = \lambda_1 + \lambda_2, \mu = \mu_h + \mu_c, A = \rho = \frac{\lambda}{\mu}, A_1 = \frac{\lambda_2}{\mu}$$

$$p_n = p_0 \begin{cases} \frac{A^n}{n!}, & n \leq N - g \\ \frac{A^{N-g}}{n!} A_1^{n-(N-g)}, & n \geq N - g \end{cases}$$

where

$$p_0 = \frac{1}{\sum_{n=0}^{N-g-1} \frac{A^n}{n!} + \sum_{n=N-g}^N \frac{A^{N-g}}{n!} A_1^{n-(N-g)}}$$

G. Haring, R. Marie, R. Puigjaner, K.S. Trivedi, "Loss Formulas and Their Applications to Optimization for Cellular Network", IEEE Transactions on Vehicular Technology, 50 (3) 664-673 (2001).



Once I know the underlying stochastic process is the birth death process and that is a finite state and also it is irreducible, so therefore all the states are positive recurrent so I can solve the P Q is equal to 0 and the summation of P i is equal to 1, I can get the steady state or equilibrium stationary probabilities.

So if I solve the system of equation with the summation of pi - P i is equal to 1, I will get the P i's, where P i's in terms of initially P naught and the P naught is 1 divided by this, so this result is uptime is available in this following paper, here the lambda is nothing but the lambda 1 + lambda 2 and mu is nothing but mu h + mu c and those mu i's are n times mu and A is nothing but rho that is lambda by mu here and A 1 is lambda 2 by mu.

Because the birth death process with birth lambda's after that lambda 2, therefore you have a different expression for 0 to N minus G and N minus G to capital N you have a different expression for the P n's, so this is the steady state probability.

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## Loss Probabilities

- Dropping Probabilities

$$P_d(N, g) = p_N = \frac{\frac{A_1^{N-g}}{N!}}{\sum_{n=0}^{N-g-1} \frac{A_1^n}{n!} + \sum_{n=N-g}^N \frac{A_1^{n-(N-g)}}{n!}}$$

- Blocking Probabilities

$$P_b(N, g) = \sum_{n=N-g}^N p_n = A^{N-g} \frac{\sum_{k=0}^g \frac{A_1^k}{(k+N-g)!}}{\sum_{n=0}^{N-g-1} \frac{A_1^n}{n!} + \sum_{n=N-g}^N \frac{A_1^{n-(N-g)}}{n!}}$$



Once you know the steady state probability our interest is to get the dropping probability and blocking probability, dropping probability is nothing but the handoff calls are dropped, because of system size is full that means all N channels are busy, that means the system when the system size is in capital N, then the new calls are dropped, therefore the dropping property is equal to the probability that N customers in the system.

Here N it is nothing but the number of channels capital N is the number of total number of channels all are busy, therefore the P suffix N, will be the dropping probability. Blocking probability, blocking probability is related to the new calls, so the new calls are blocked, it is not even enter into the system, whenever the number of available channels are less than or equal to g in the system.

Therefore when the system size is from N minus g to capital N, the new calls are blocked, therefore the blocking probability is running from N minus g to N, P suffix N that will be the blocking probability, if you made the summation over the steady-state probability which we got it in the previous slide substitute those things so that will be the blocking probability.

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## Description of 3G Cellular Networks

- Homogeneous Cellular Network with identical cells
- Network traffic:
  - Voice: New and handoff
  - Video: New and handoff
    - Hard QoS (e.g., video conferencing, live program) (say *video1*)
    - Soft QoS (video on Demand) (say, *video2*)
  - Data: New and handoff
- Priority: Voice > Video1 > Video2 > Data

S. Dharmaraja, Vaneeta Jindal and Attahiru Sule Alfa: Phase Type Models for Cellular Networks Supporting Voice, Video and Data Traffic, *Mathematical and Computer Modelling*, 47 (2008) pp. 1167 - 1180.

Now I am moving into extending the same logic into 3G wireless networks, so here also we are making a model for only one cell and not only - only one type of calls, so we have a three types of calls, one is the voice call, the second one is the video call and third one is the data call. In the 2G networks we have only one type of call that is a voice call and the voice calls are subdivided into two type's handoff calls and the new calls.

So here also the voice calls are divided into new calls and handoff calls, because we are considering a performance model for a one cell and calls could be a voice call could be news - new call or the handoff calls, the video calls again we are dividing into two types, one is the Hard QoS video 1 type call and the Soft QoS that is video 2 calls.

Because there are some video calls it is hard QoS, that means it won't tolerate the delay the video conferencing like program and so on, so that type of video calls are the video 1 whereas the video on demand that can tolerate the delay, so that type of call is called soft QoS calls, so this is a video call that type is called video 2, than the last data traffic that's also can be classified into new call and handoff calls.

So here the priority is the voice calls are the higher priority than the video 1 call, then video 2, then the data, so the way we have discussed the 2G cellular networks we are extending the same idea into the 3G wireless networks with the three types of calls with this priority and the

priorities are taken care with the fixed guard channel policy, earlier you have only one voice type call, now we have three types of calls therefore the reservation also changes accordingly.

And instead of only one voice call we have three types of, therefore the number of channels allocated to different calls also changes, so in this scenario we have taken voice calls uses only one channel whereas the video calls that needs more bandwidth, therefore assumption we have made assumption 4 channels are needed for the video calls and the data traffic needs only one channel, therefore voice and so this is explained in this paper in detail.

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## Fixed Guard Channel Scheme:2G Cellular Network Traffic

- Traffic consists of Voice (new and handoff) calls.  
Channels are reserved for handoff voice calls.
- Traffic characterization
  - Call Arrivals: Poisson Process
  - Call Holding Times: Exponential Distributions
  - Retrial Times (if considered): Exponential distribution

So here we made the assumption the call arrivals Poisson process the usual way is 2G networks call arrivals are Poisson process called holding times are exponential distribution and so on.

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## CAC and Resource Reservation Schemes

- **Assign**
  - One channel - Voice and Data traffic
  - Four channels - Video (1 and 2) traffic
- **Priority**
  - Order – Voice, Video 1, Video 2 and Data
- **Reserved channels**
  - $h_i$  – handoff calls (Voice, Video 1, 2 and Data)
  - $n_i$  – new calls (Voice, Video 1, 2)
- **Retry and accept**
  - New Voice, Video 2 and Data traffic



But what we are making the mod - extending the model with a one channel is needed for voice and data traffic and the 4 channels are needed for the video 1 and 2 traffic and the priorities voice, then video 1, then video 2, and data and since the priority is there we are reserving  $h_i$  number of a channels for the handoff calls in this order.

And  $n_i$  channels are reserved for the new calls of voice and video 1 and 2 and there is no reservation for the new data traffic calls, and we allow the retry of the calls of a new voice and video 2 and the data traffic.

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## The Proposed Model

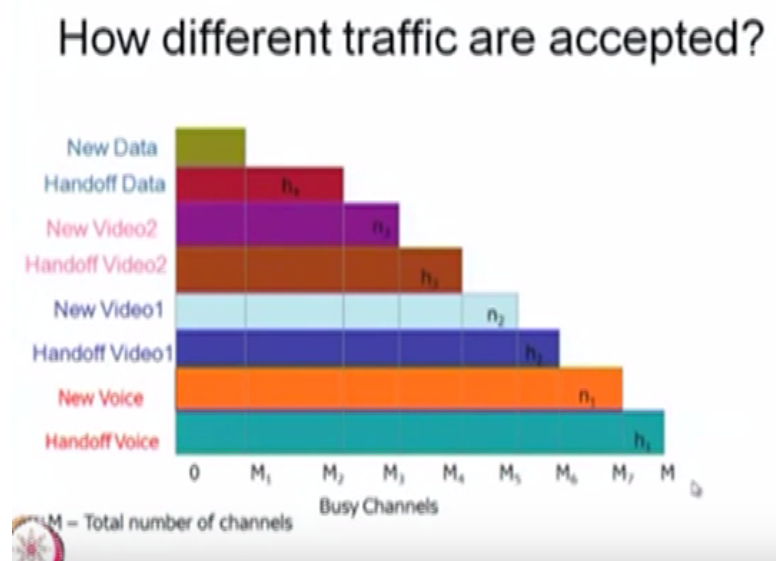
### Traffic Characterization

- Traffic arrival: Markov arrival process
- Call holding times: Phase type distributions
- Calls that are blocked joins the orbit
- Call retrial times: Phase type distributions



Therefore the proposed model instead of a Poisson we have Markov arrival process for the data arrival and instead of exponential distribution we have more general situation that is a phase type distributions for holding time call holding time, and whenever the calls that are blocked, it can go to the orbit and it can retry and retrial times also assumed to be a Poisson distribution.

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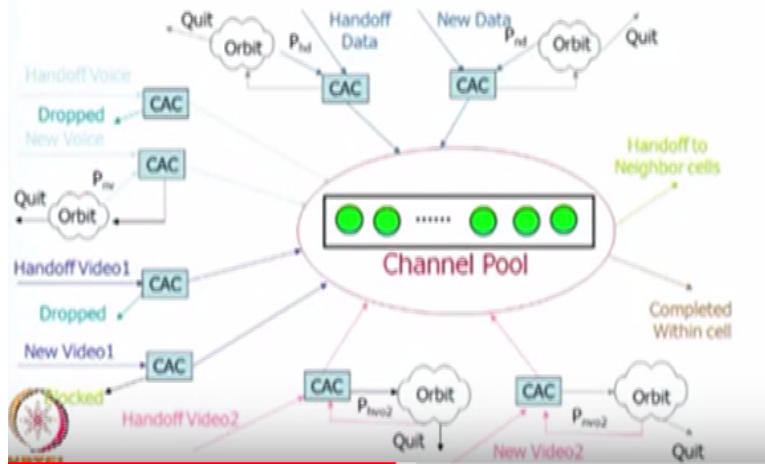


And since we have made the reservation for the calls with the  $h_i$ 's and  $n_i$ 's, accordingly this total number of channels are divided into many parts and the calls will be allowed - call girls will be allotted according to this priority that means from 0 to  $M_7$ , the handoff voice will be allotted from 0 to  $M$ , whereas the new voice will be allotted till the number of busy channels going from 0 to  $M_7$ .

Because  $h_1$  is a reservation for the handoff voice call so like that this is the - this is the higher priority, then next priority, and next priority and so on and this is the new data call and there is no priority for the, there is no priority for the new data therefore this is the last lowest priority call traffic.

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# Queuing Model



And this is the corresponding queueing model you can compare the queueing model of the 2G networks to the 3G networks, since we have a three types of calls that is a voice, video 1, video 2 and data traffic, so we have a different traffic is contending for the channels in the channel pool and after the calls are over then it will either will be completed within the cell or handoff to the neighbor cells.

The first one is the handoff voice, so the call admission control scheme allows than the call will be allotted, otherwise it will be dropped there is no waiting there is no orbit the calls blocked won't go to the orbit and wait, whereas the new voice call if there is no channel then it will go to the orbit with some probability it will again retry after phase type distributor time which I have - we have made the assumption otherwise it will quit with 1 minus this probability.

Similarly for the handoff video 1, either it will be accepted or it will be dropped, whereas the new video 1, the new video 1 also either it is the use the word dropped for the handoff and the blocking for the new type new calls, therefore new video call video 1 call will be blocked if there is no bandwidth, whereas the video 2 if there is no bandwidth then it will go to orbit with some probability it will come back with 1 minus of this probability it will quit.

Similarly, the new video 2 also either it will get accepted if the channel is available the required number of channels are available otherwise it goes to the orbit and retry, the third type that is a

data both new and the handoff data either it will be accepted or it will go to orbit and come back to, so it is similar to video 2, but the difference is the video takes a 4 channels at a time, whereas the data and voice takes only one channel at a time.

So this is the corresponding queueing model for the 3G wireless networks cellular networks for only one cell.

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### Analytical Model

- $\{X(t), t \geq 0\}$ : stochastic process to describe the state of the cell.
- It is modeled as a Quasi birth-death process.



- a generalization of the birth-death process
- a Markov chain on the state space S, where the state space can be divided into k levels, and each level has m states (phases)

- State Space:

$$\{(i, j, k, l, u_{ve}, u_{vo1}, u_{vo2}, u_d, S_{i-j-k}^{ve}, S_j^{vo}, S_k^d, r_l)\}$$

So here the stochastic process is the state of the cell what is the number of channels are available at time t, that is going to be the stochastic process over the time and this will be modelled as a Quasi birth death process whereas the 2G wireless net cellular networks is queueing modelled as a birth death process, so here this is a Quasi birth death process so what is the meaning of Quasi birth death process? This is a generalization of birth death process.

Here the Markov chain with the state space S and the state space can be divided into k levels and each level you have a capital M states or M phases, suppose you have a M capital M phases and we have a k levels in it, so it is a generalization of birth death process means suppose the system is in some i th level and some state then there is a possibility it can go to the i minus 1 th level with the different states are i + 1 th level in the different states and so on.



Therefore here the state space is divided into k levels and each level has the some finite number of need not be the same number each level should have a some finite number of states, so it is a generalization of birth death process, accordingly you have a state space with this type of vector and you can study in detail in particular paper about how to obtain the performance measures.

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## Analytical Model

- $i$ : total no. of ongoing calls,  $0 \leq i \leq M$
- $j$ : Video calls,  $0 \leq j \leq j_m$ ,  $j_m = \min\{i, M/4\}$
- $k$ : Data calls,  $0 \leq k \leq k_m$ ,  $k_m = \min\{i-j, M/2\}$
- $l$ : No. of calls in orbit,  $0 \leq l \leq J$
- $u_s$ : phase of MAP for traffic type  $s$  ( $s=ve, vo1, vo2, d$ ),  $1 \leq u_s \leq K_s$
- $K_s$ : the number of states in the Markov chain of MAP for traffic type  $s$
- $s_{i,j,l}^v, s_{i,j,l}^{v1}, s_{i,j,l}^{v2}, s_{i,j,l}^d$ : the set of phases of service for ongoing voice, video and data calls respectively
- $r_l$ : phase of retrial of call in an orbit.

And here I am listing what is the meaning of  $i, j, k, l$  and what is the meaning of  $u_s$  and what is the meaning of  $S$  suffix  $n$  so on and the last one what is the meaning of  $r_l$  means the phase of a retrial of calls in an orbit.

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## Generator Matrix

$$A_{j,l}^i = \begin{bmatrix} A_{01}^{i,j,l} & A_{00}^{i,j,l} & 0 & 0 & 0 \\ 0 & A_{11}^{i,j,l} & A_{10}^{i,j,l} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & A_{k1}^{i,j,l} & A_{k0}^{i,j,l} \\ & & & & A_{k_{m-1}}^{i,j,l} \end{bmatrix}$$

$$A_{j,0}^i = \text{diag}(A_{01}^{i,j,0}, \dots, A_{k1}^{i,j,0}, \dots, A_{k_{m-1}}^{i,j,0})$$

So here also we use the difference between the birth death process and quasi birth death process. The queue matrix consists of block matrices, whereas in the birth death matrix - birth death process the queue matrix is a tri-diagonal matrix and each entry is an element, whereas here each entry is a matrix and the elements of that matrix also again will be a matrix based on the number of levels.

And in this model we have a 4 level quasi birth death process, therefore we have a queue matrix with block matrices in three levels - four levels and the fourth level you have the matrix with the entries are the elements.

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## Generator Matrix

$$Q_{i1} = \text{diag}(B_0^i, \dots, B_j^i, \dots, B_{jm}^i)$$

$$B_j^i = \text{diag}(B_0^{i,j}, \dots, B_k^{i,j}, \dots, B_{k_m}^{i,j})$$

$$B_k^{i,j} = \begin{bmatrix} B_{0,1}^{i,j,k} & B_{0,0}^{i,j,k} & 0 & 0 & 0 \\ B_{1,2}^{i,j,k} & B_{1,1}^{i,j,k} & B_{1,0}^{i,j,k} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & B_{l,2}^{i,j,k} & B_{l,1}^{i,j,k} & B_{l,0}^{i,j,k} \\ 0 & 0 & 0 & B_{j,2}^{i,j,k} & B_{j,1}^{i,j,k} \end{bmatrix}$$



I am just skipping this part of how the generator matrix each element each matrix exist.

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# Generator Matrix

$$Q_{i2} = \begin{bmatrix} C_{01}^i & 0 & & & \\ & \ddots & \ddots & & \\ & & C_{j2}^i & C_{j1}^i & 0 \\ & & & \ddots & \ddots \\ & & & & C_{j_{m2}}^i & C_{j_{m1}}^i \end{bmatrix} \quad C_{j1}^i = \begin{bmatrix} C_{01}^{ij1} & 0 & 0 & 0 & 0 \\ C_{12}^{ij1} & C_{11}^{ij1} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & C_{k2}^{ij1} & C_{k1}^{ij1} & 0 \\ 0 & 0 & 0 & C_{k_{n1}}^{ij1} & C_{k_{n-1}}^{ij1} \end{bmatrix}$$

$$C_{j2}^i = \text{diag}(C_{01}^{ij2}, \dots, C_{k2}^{ij2}, \dots, C_{k_{m2}}^{ij2})$$

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## Steady-state Distribution

Assuming QBD is ergodic, steady-state distribution is obtained.

Let  $x$  partitioned as  $x = x(i, j, k, l)$  with  $0 \leq i \leq M$ ,  $0 \leq j \leq j_m$ ,  $j_m = \min\{i, M/4\}$ ,  $0 \leq k \leq k_m$ ,  $k_m = \min\{i-j, M/2\}$ ,  $0 \leq l \leq J$  denote the steady-state probability vector for the generator matrix  $Q$  i.e.,  $x$  satisfies  $xQ = 0$ ,  $x\mathbf{e} = 1$ .

Each  $x(i, j, k, l)$  is a vector ordered in the lexicographical order.

Using matrix-geometric technique, steady-state probabilities are obtained.

And our interest is to get the steady state distribution for the quasi birth death process also, so here I am making the assumption the quasi birth death process is ergodic, therefore the steady state distribution exists this generalization of birth death process therefore the Markov chain whatever the steady-state distribution condition you need the same condition you can cross check.

So here this QBD is ergodic, therefore steady state distribution exist, then you can solve the  $x$  times  $Q$  matrix equal to 0 with the  $x$  times  $\mathbf{e}$  vector that is equal to 1 using that you can get the

steady-state probability vector and since it is a QBD you can use the matrix geometric technique that is well known technique for the QBD we can use that and get the steady-state probabilities.

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## Performance Measures

- Dropping probability of handoff voice calls:

$$P_{hv} = \sum_i \sum_j \sum_{k=0}^{k_m} \sum_{l=0}^J x(i, j, k, l) e$$

- Dropping probability of handoff video1 calls:

$$P_{hvo1} = \sum_i \sum_j \sum_{k=0}^{k_m} \sum_{l=0}^J x(i, j, k, l) e.$$



And these are all the, once you know the steady-state probability you can get the blocking and dropping probabilities, since we have a three types of calls we can analyze the blocking and dropping probabilities for the various traffic.

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## Performance Measures

- Blocking Probability for new video 1 calls

$$P_{nvo1} = \sum_i \sum_j \sum_{k=0}^{k_m} \sum_{l=0}^J x(i, j, k, l) e$$

- Total Carried Traffic: Average no. of channels occupied

$$TCT = \sum_{i=0}^M \sum_{j=0}^{j_m} \sum_{k=0}^{k_m} \sum_{l=0}^J (i + 3j) x(i, j, k, l) e.$$



And also we are discussing the retrial and orbit and so on, so we can find out the probability that the new voice handoff video 2, new video 2, handoff data and the new data calls join the orbit

what's the probability for that you can find out after getting the steady state probabilities, so you can one can see the paper to understand the complete paper.

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## Special Cases:

- Poisson arrivals, exponential service and retrial times, four types of calls
- MAP arrivals, exponential service and retrial times, one type of call
- Poisson arrivals, phase type service and exponential retrial times, one type of call
- Poisson arrivals, exponential service, no retrials and one type of calls.



And also one can discuss the special cases by making the simpler assumptions, the last assumptions is a Poisson arrival exponential service and no trial and one type of call that will land up into the 2G cellular networks which we have got it, so if you make a subsequent special cases you will land up with the blocking probability and dropping probability which we have discussed in the second generation cellular networks.