Stochastic Processes - 1
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Lecture - 72
Jackson's Theorem, Closed Queueing Networks, Gordon and Newell Results

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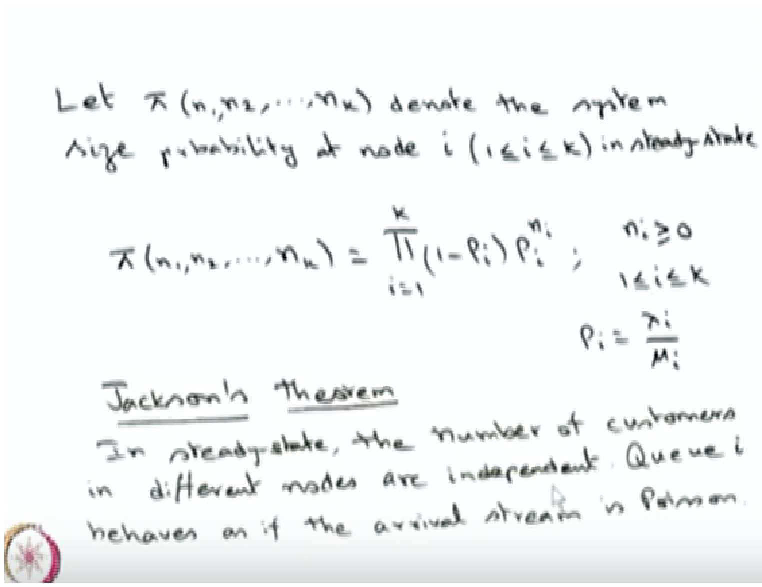

So the Jackson's theorem says in steady state the number of customers in different nodes are independent, the number of customers in different nodes are independent that means the behavior of the queuing system is a consisting of behavior of many independent nodes, the Queue i behaves as if the arrival stream is Poisson.

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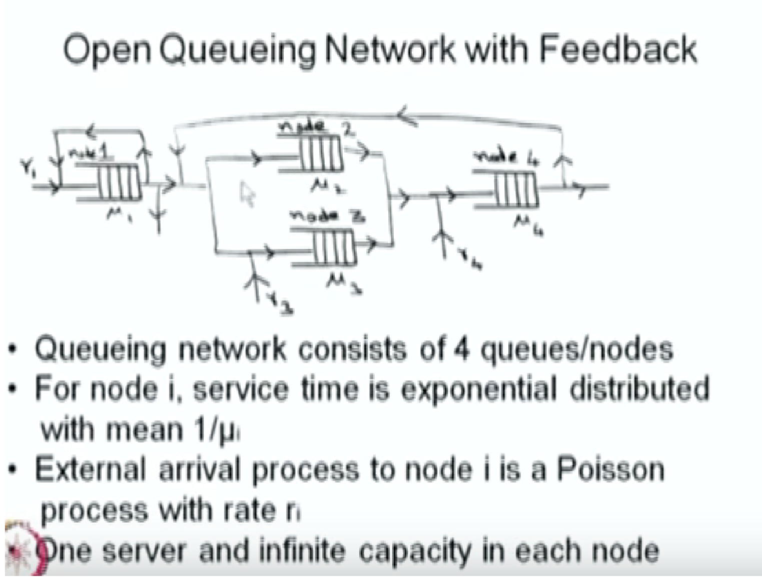

So not only each node behaves independently - not only each node behaves independently, as if it behaves the arrival is going to be a Poisson for each node, therefore this point it is a Poisson whereas this point is not a Poisson arrival process, but still in steady state the number of customers in this queue, number of customers in this queue this queue and this queue all the queue size are independent.

As well as in steady state the arrival process for the each node behaves as of - as if Poisson process, but they are not in general Poisson process.

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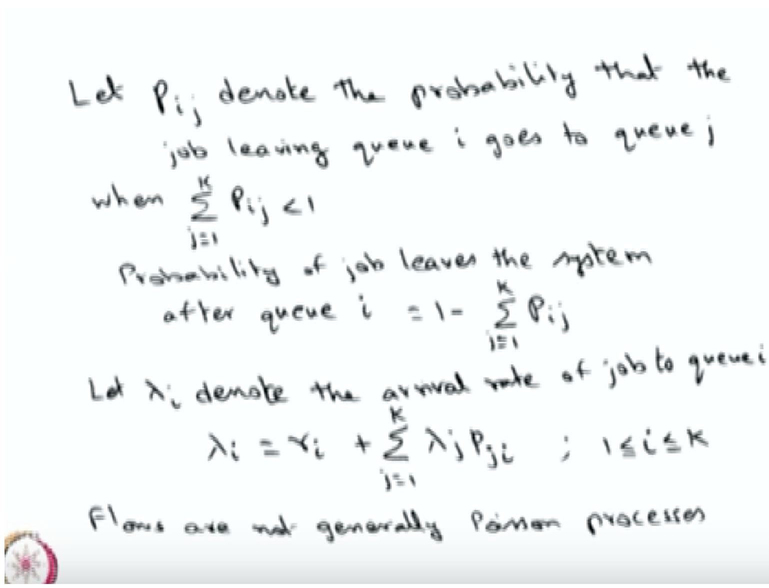

So in steady state whenever you have open queuing network with these assumptions in study state is this behaves as a independent queueing, therefore the joint probability of a n 1 customer in the first queue, n 2 customers in the second queue similarly and so on till the n k customers in the k th node, since each nodes behaves independently so the number of customers in different queues are independent.

Hence arrival or Poisson and already you made out some assumption it is infinite capacity each queue as well as only one server in each queue, therefore the arrival follows the Poisson service is exponential only one server infinite capacity that means even though it is opening queueing network with the feedback each node behaves as if MM1 queue in steady state that is important.

That means as a time dependent the system may depends on the size of the other number of customers in the other nodes but in steady state this behaves independently and each one behaves like a MM1 infinity queue, therefore you can get the joint distribution of a n i customers in i th node, that is product of n i customers in i th node, if you make a product that is going to be the joint distribution.

Because a joint distribution is going to be the product of initial probabilities if each random variables are - if each random variable is independent, therefore you can use that logic to use to get the joint probability as the product form solution and here rho i's are nothing but lambda is divided by mu i's, and which is has to be less than 1, I forgot to write each rho i has to be less than 1, it has to be stable, each queue queueing system has to be stable.

So in steady state you have a product form solution where rho i's are lambda i divided by mu i, you have to find out lambda is by solving the system of K equation, lambda i's are the unknown, r i's are given, routing probabilities are given, so using that solve for - solve for lambda i's from these K equations.

So once you know the lambda i's check whether lambda i's divided by mu i's less than 1, then the stationary distribution exists, using the Jackson's theorem the joint distribution is the joint distribution that is a stationary distribution is of the product form solution.

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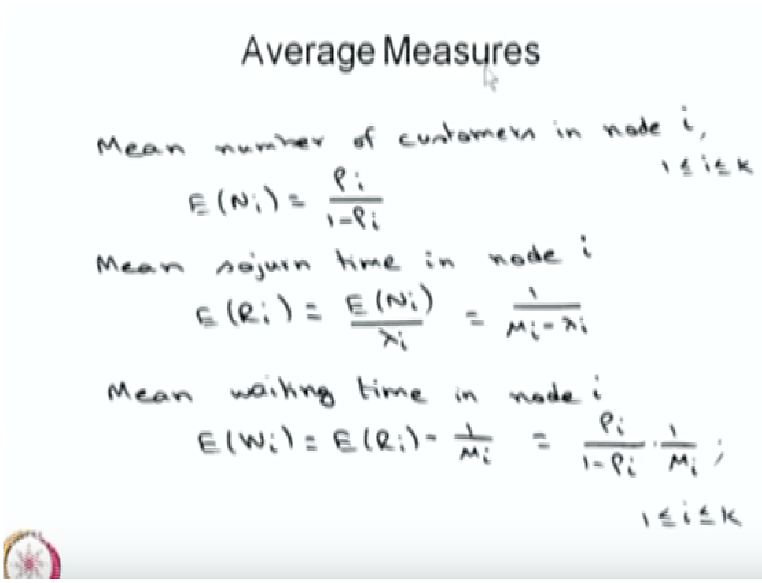

Assuming that each queue behaves as the MM1 queue, so for any general K queue is not tandem queue open queueing network with the feedback you can get the average message also the way we have calculated for the two queues model or the tandem queue model, the same logic can be used for the open queueing networks with the feedback.

You are getting average number then sojourn time in each node, then mean waiting time in node each node by subtracting the average service time for each node you have K nodes, so therefore you are getting this measures for each node.

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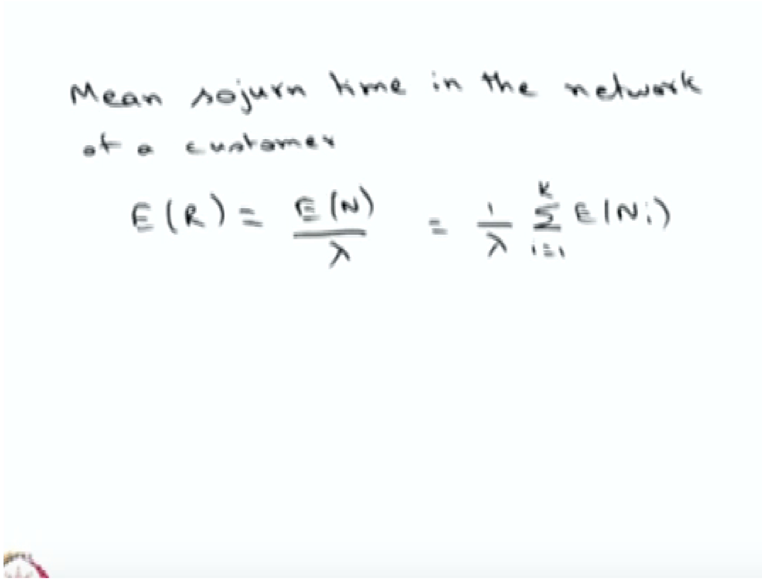

Once you know the results for the each node, you can find out the total sojourn time by using the Little's formula, because the Little's formula is valid, here the external arrival rate is lambda, so that we have to add, suppose here we have to finding out the lambda and for the open queueing network, the lambda you have to compute by adding all the external arrival rates r 1 + r 3 + r 4, that is going to be lambda in this example.

So if you add all the external arrival rates to different queue - different nodes that's summation is going to be the total arrival rate to the system, because to apply the Little's formula you think you consider the whole thing as a one system in which r 1 + r 3 + r 4, all are independent therefore the summation is going to be the arrival rate for the system, so the lambda that example r1 + r 2 sorry r 1 + r 3 + r 4.

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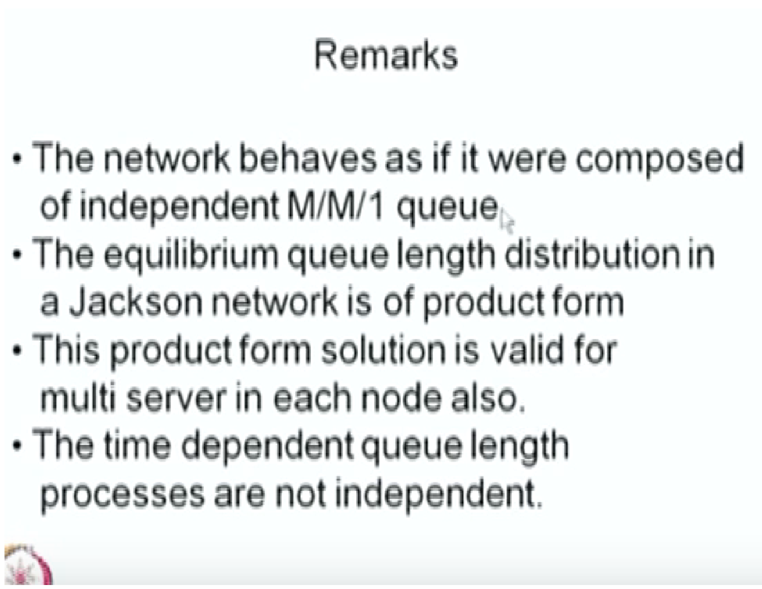

Remarks for the open queueing network, the network behaves as if it were composed of independent MM1 queues in steady state that is important I forgot to write that in steady state, in time dependent this is not the case that I am going to, I have written as a fourth remark in steady state these networks behave as if it is composed of independent MM 1 queue, so the equilibrium queueing length distribution in Jackson's network is of product form.

So the solution is called - this solution is called product form solution, for an open queueing network with the feedback that equilibrium queueing length distribution is of product form, this the previous result can be extended to the multi-server model also, in that model we have taken it as a single server infinite capacity you can think of each queue is a MMC infinity also instead of MM1 infinity, in steady state this solution is valid with the MMC infinity logic.

Whereas the time dependent queueing length process are not independent, therefore the product - product from solution won't work and the time dependent scenario is a completely different and with the steady state or the equilibrium queueing distribution you cannot discuss the behavior of time dependent, the time dependent queueing length distributions are not independent for the each queues.

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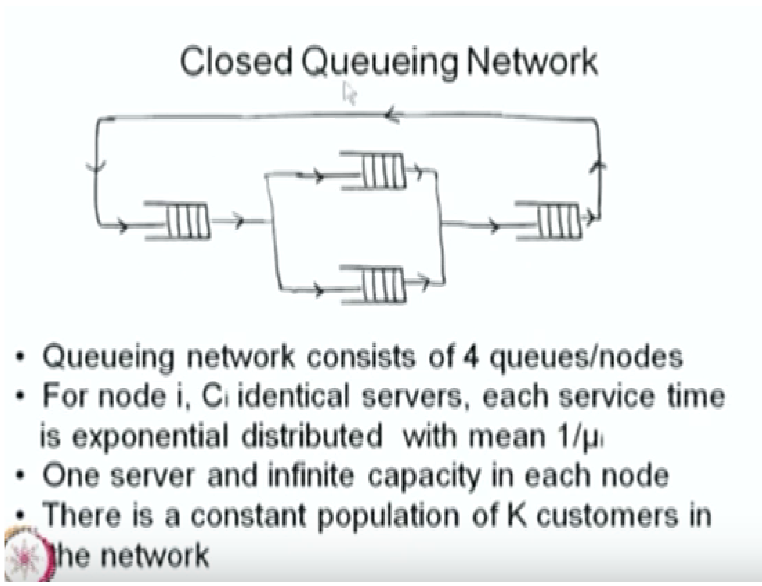

Now I am moving into the Closed Queueing Network, so here comparing with open queueing network here we have a fixed number of a population is moving around the queue's no one leaves and no one enter the system also, therefore you keep some K customers in the system in this example I have a 4 nodes and instead of either you can have a one server or more than one servers also allowed with infinite capacity queueing system.

And you make the assumption the service time is exponential distribution for each queue and all the servers are identical.

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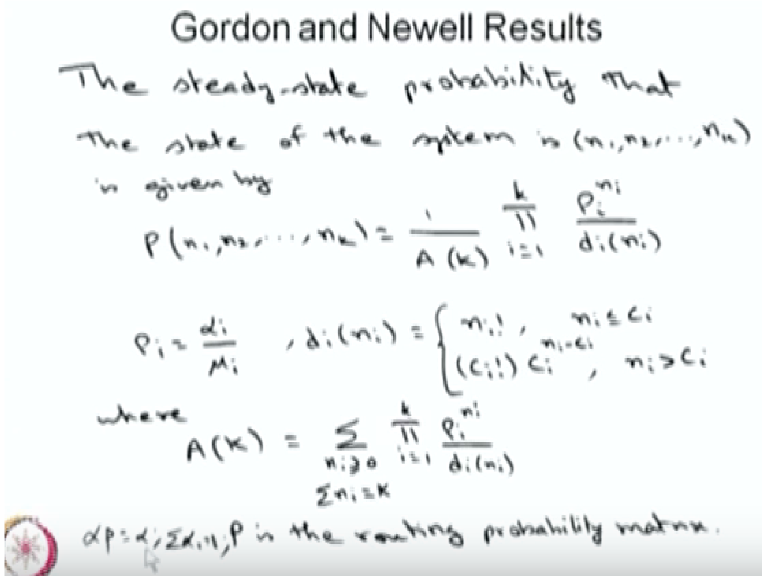

Here also we can get the product form solution at the joint distribution of the system size that is same as the product of rho i's power n i divided by d i's of n i, for the k nodes, the small k is the k nodes in the system and the capital K is the total number of population, so the here this A K's are nothing but the normalizing constant and the rho i's are in terms of alpha i divided by mu i and alpha you can calculate by solving this equation.

Where P is the routing probability matrix you solve alpha times P is equal to alpha and the summation of alpha i is equal to 1, using that you will get alpha and substitute alpha i's here, therefore you will get rho i's, then you substitute d i's here based on the number of servers in the each node is 1 or more than 1.

Accordingly, you can use this and once you know the d i n i you substitute here this product form solution will give joint distribution of system size in steady state and this result is given by Gordon and Newell, for the closed queueing network.

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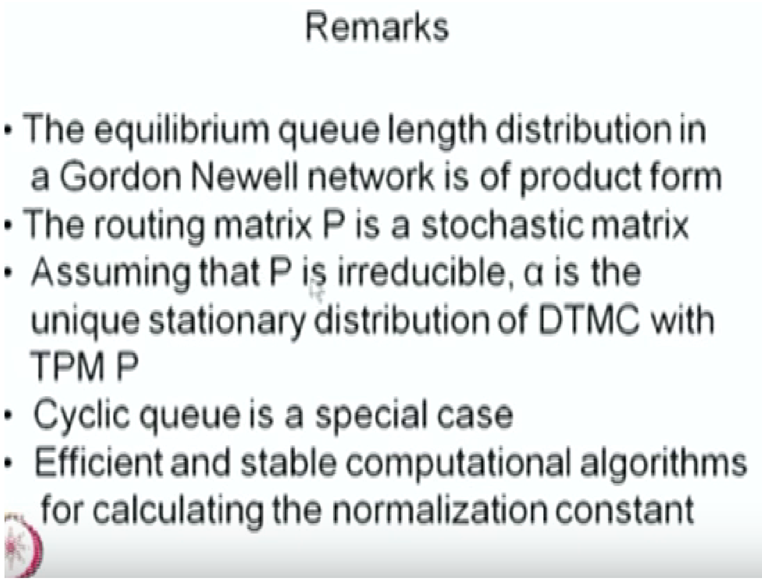

As a remark, this equilibrium solution is also product form and the routing probability matrix is a stochastic matrix and suppose you assume P is irreducible then the solving alpha - alpha P is equal to alpha and the summation of alpha is equal to 1, that is nothing but alpha's are nothing but the stationary distribution, whenever P is irreducible.

Not P is irreducible the underlying DTMC is irreducible with the assuming that the underlying DTMC is irreducible, then the alpha is nothing but the unique stationary distribution and this is valid for the Cyclic queue also and the toughness is how to compute A of K, where K is number of customers in the whole queueing network, so you need efficient and stable computational algorithm for calculating this normalizing constant A of K.

Here some mistake, assuming that the DTMC is irreducible - the DTMC is irreducible alpha is a unique stationary distribution, so now we will move into the summary.

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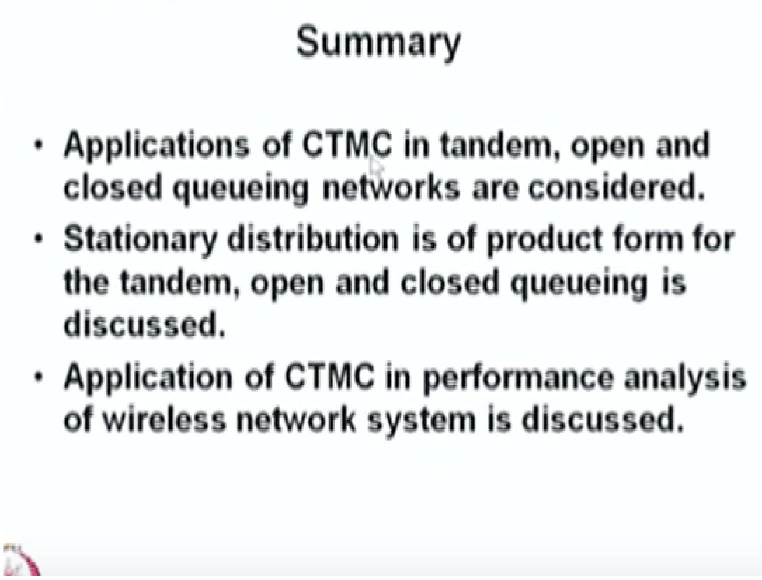

So in this lecture, we have discussed the applications of CTMC in tandem, open and queue - closed queueing networks, we have discussed the stationary distribution and other performances measures for the tandem, open and closed queueing networks with only the product form solutions, we didn't discuss the non-product form solution we have only discussed the product form solution.

And the application of CTMC in the performance analysis of wireless network system that I will discuss in the next lecture, also I'm going to discuss the simulations of a simple Markovian queueing networks in the next lecture.

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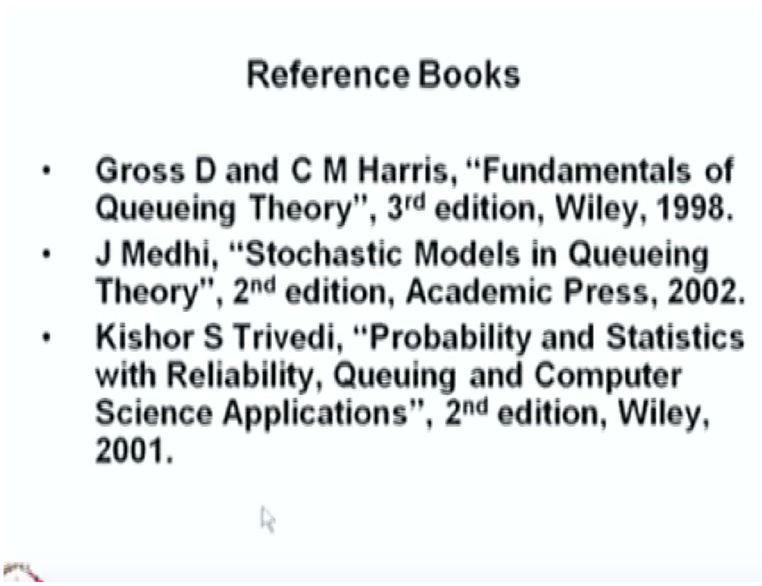

These are all the reference books. Thanks.