Stochastic Processes - 1
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Lecture - 70
Tandem Queueing Networks

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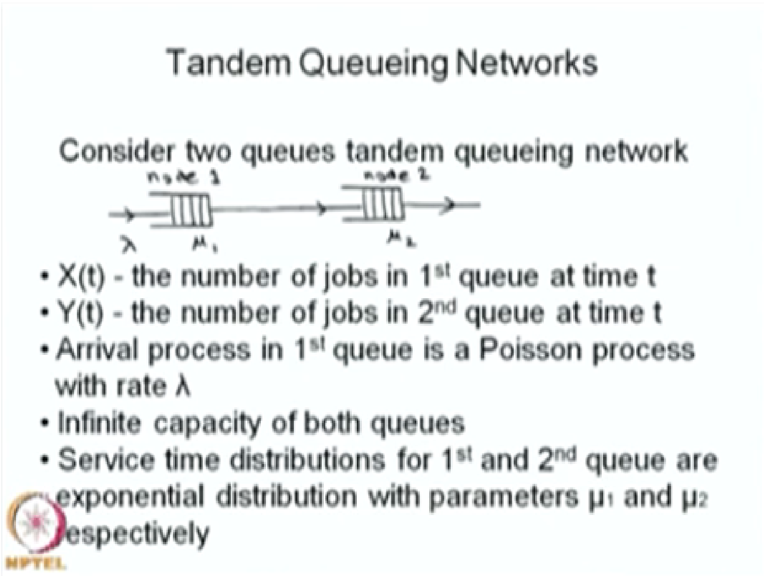

The tandem queueing networks is a special case of open queueing network in which all the nodes or queues are interconnected in series. Let me start with a very simplest tandem queueing networks with two nodes or two queueing system connected in series. We are going to relate the queueing network with a stochastic process and so on.

Therefore, let me start with random variable x of t and y of t, x of t denotes number of jobs or customers in the node 1 at any time t. Node 1 or first queue both are the same. yt another random variable that denotes the number of jobs in the second queue at time t. Therefore, together x of t and y of t, you can think of a vector as T is equal to x of t and y of t, that vector is a random vector at any time t.

And if you collect over the t, then that is going to be a stochastic process. So this stochastic process instead of one random variable, it has two random variables x of t and y of t, that with the vector. So that is a stochastic process, x of t, y of t for t greater than or equal to zero, that is stochastic process. Since x of t or y of t are the number of jobs in the first queue and the second queue and you are observing over the time.

How many customers in the first node and how many customers in the second node at any time t, therefore this is a discrete state continuous time stochastic process. Also, you assume that infinite capacity in both the nodes, both the queues. That means after the service is over in the first node, it will immediately arrive into the second node and if no customer in the system at that time, then the service will be started immediately.

Otherwise the customer who comes after completing the service in the first node, he has to wait till his service starts. So I am making one by one assumption, so that the underlying stochastic process is going to be a continuous time Markov chain. That is my objective. So for that, now I am making the first assumption, the inter-arrival time of a customer entering into the first node.

That is exponentially distributed with a parameter lambda or the arrival process into the first queue is a Poisson process with a parameter lambda. So the population is infinite, the customers, or jobs or packets for example, packets for the telecommunication system or any communication system, so the customers are entering into the node 1, their process is Poisson's process with a parameter lambda.

Now I am making the assumption for service. I make the assumption for the service time that is also exponentially distributed with the parameter, mu 1 for the first node and mu 2 for the second node, other than the inter-arrivals are exponential distribution, I made the other assumptions that is the service time for the first node and service time for the second node, both are exponentially distributed with the parameters, mu 1 and mu 2 respectively.

So ultimately I want this has to behave as a MMQ and this also has to be as MM1Q independently. Therefore, I make all the assumptions the inter-arrival times are independent with the service for the first node, similarly this one is the arrival for the second node is independent of the service of the second node and so on. So with that assumption, each queue in this queueing network is going to behave as a MM1Q and the departure of the first node that will be input for the second node.

And here you can use the Buch's theorem, this is the MM1Q model, therefore using the Buch's theorem, you can conclude the departure process is also Poisson's process because the arrival process of Poisson, the service rate is A1 using the Buch's theorem, you can conclude the departure process is also Poisson process with the same parameter of the arrival process. Therefore, here the arrival process for the node 2 is also Poisson process with the same parameter lambda.

It is independent of the service. Not only that, the departure process is independent of the number of customers in this system and so on and the inter-arrival, therefore, you will have two independent MM1Q using the Buch's theorem. Now this will separately act as MM1Q because the arrival process is Poisson with the parameter lambda and already we made the assumptions arrivals are independent with the service. Service is exponentially distributed with the parameter mu 2.

After the service is over, the customers leave the system. Therefore, this is a separate MM1Q because we made an infinite capacity in both the queues. Therefore, the two queues tandem queue the underlying stochastic process, x of t, y of t going to be the continuous time Markov chain.

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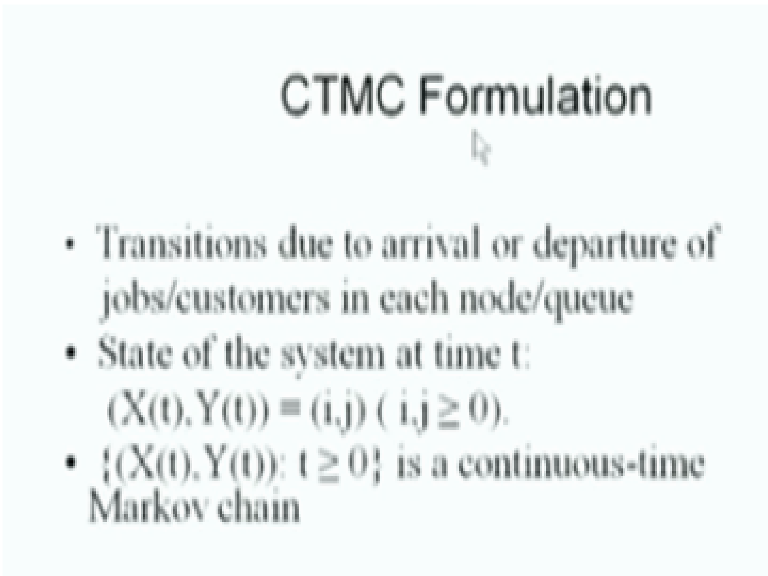

So now I am going to formulate the CTMC from that two queues tandem queueing network. The transition is due to arrival or departure of jobs in each que. Based on this, I can make straight up the system x of t, y of t and that is going to be a continuous time Markov chain. This is not going to be a birth-death process because you have two random variable x of t and y of t and each one independently MM1Q.

The MM1Q is the underlying stochastic process for the MM1Q that is a birth-death process, whereas here you have together x of t, y of t. Therefore, this is not going to be a birth-death process. It will be a general continuous time Markov chain. So once you identify this as a continuous time Markov chain because the Markov property is satisfied by the stochastic process x of t, y of t, I can go for drawing the state transition diagram.

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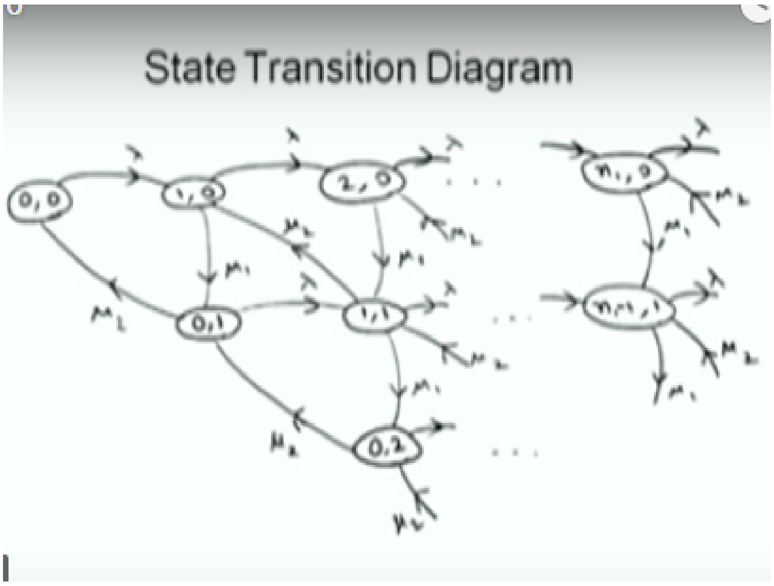

It is Markov in nature because the two queues act independently and are themselves MM1 queueing system satisfies the Markov property. The first index is for the number of customers in the first node and second index is for the number of customers in the second, so zero, zero means in both the queues, no one in the system. If one arrival comes into the first node, arrival cannot come into the second node.

Only the arrival come to the first node, the inter-arrival is exponential distribution. Therefore, the rate of moving from zero, zero to 1, zero that is lambda. Similarly, there is a possibility of one more customer entering into the system when one customer in the first que. So it will be a parameter lambda, therefore the rate will be lambda for the ox moving from zero, zero to 1, zero, 1, zero to 2, zero and so on.

Whereas after one customer already in the first node, the server in the first node would have completed the service before one more arrival into the first node. Therefore, the service is exponential distribution with a parameter mu 1, therefore the system goes from 1, 0 to 0, 1 that means, the customer was under the service. That service is over, therefore, he moved into the second que.

Now the first node has zero customer and the second node has one customer in the system and his service will start once he enter into the second node. This rare will be mu 1. Now the situations are either one more arrival, one arrival to the first node or the customer who is in the second node, he would have completed his service. So that service is over, therefore that rate is mu 2.

Then the system goes to 0, 0 or it will go to 1, 1 with the rate lambda. So the same way, one can discuss for 1, 1 also that means one customer in the first node, one customer in the second node, so the one possibility is the first customer's service is over. Therefore, it will be a 0, 2 with the rate mu 1. Or one more arrival takes place, therefore, it will be a 2, 1 with the rate lambda or the second service would have been finished. So that rate is mu 2.

Therefore, it is 1, 1 to 1, 0. So these are all the possibilities, the system can move from 1, 1 to other states 0, 2, 1, 2 or 1, 0 and so on. This is the way you can visualize the different transition ox and the corresponding rates. So this is the state transition diagram for two queues tandem queueing network. Obviously from this diagram itself we can say that this is not a birth-death process.

If it is a birth-death process, then the system has to move forward one step or backward one step, no other moves. So here you have a two-dimensional Markov chain, therefore this is not a birth-death process. It is a continuous time Markov chain.