Stochastic Processes - 1
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Lecture - 69
Queueing Networks Characteristics and Types of Queueing Networks

This is stochastic process module 5, continuous time Markov chain, lecture six queueing networks. In the last five lectures, we started with the definition of continuous time Markov chain, then Kolmogorov differential equation, Japan Kolmogorov equation, infinite decimal generate matrix in the lecture one. The lecture two discussed the birth-death process. The lecture three, we have discussed the special birth-death process, Poisson process.

In the fourth lecture, we have discussed the application of continuous time Markov chain in queueing models that is the first special case special Markovian queueing model MM1Q. Then in fifth lecture, we have discussed the simple Markovian queueing models other than MM1Q. Now we are moving into sixth lecture. That is for the application of continuous time Markov chain in queueing networks.

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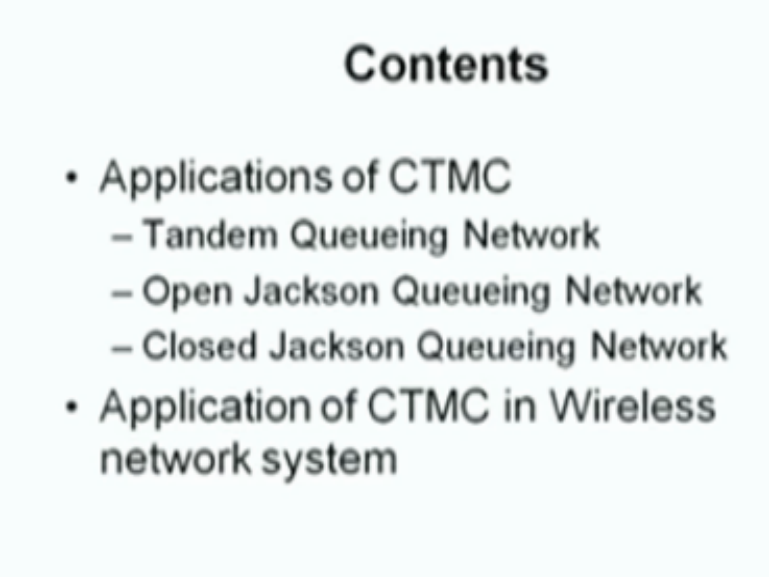

Basically the queueing networks is nothing but a network consisting of several interconnecting queues. Either we can say networks of queues or queueing network. In today's lecture, we are going to cover the queueing network as the application of continuous time Markov chain. In the last two lectures, we have discussed the application of CTMC in particular birth-death process.

But now we are moving into more general set up of a CTMC. That is coming in the queueing networks. That means the underlying stochastic process of queueing network is a more general continuous time Markov chain whereas all the simple Markovian queues, the underlying stochastic process is the birth-death process. The study of queueing network is going to be a very vast area.

So we are going to cover here only very simple queueing networks, the first one is tandem queueing network, then we are going to discuss the open queueing network or we can say open Jackson queueing network and also we are going to discuss the closed Jackson queueing network, then followed by that we are going to discuss about the application of CTMC in performance analysis of wireless network system.

If time permits, I will do in this lecture or I will do that in the next lecture. Now we are going to discuss the queueing network. The queueing network is nothing but a network consisting of several interconnecting queues. So we have discussed already the queues in the fourth lecture.

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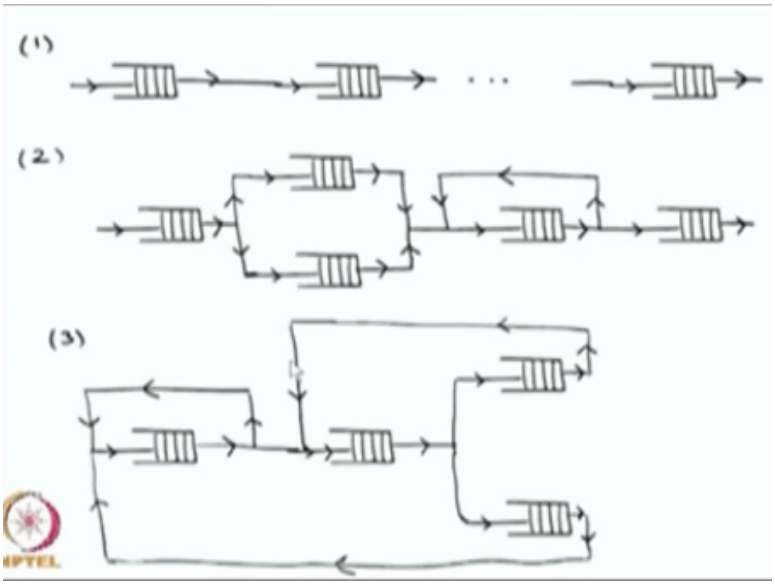

I am giving the three different examples for the queueing networks. So we can say this as the first que or we can say the queueing network has the node one, which consists of a separate one que where the arrival comes waiting for the service. After the service is over then it moves in to the second que. This is the separate queues in which the arrival waits for the

service. After the service is over then it moves into the next que. Like that it has many queues.

So this is the one simple example of queueing networks and this queueing network is called a tandem queueing network. Because the output of first que is the input for the second que. There is no feedback. The output of second que is the input for the third que and so on like that it is a system in which it has some finite number of queues in series. Therefore, this is called as tandem queueing network.

The second type this also consisting of many queues interconnecting queues whereas here the output of the first que split in some probability, the summation of this probability and this probability is one, move into the input for the second que as well as third que. If you label this as the node 1, node 2, node 3, node 4 and node 5, therefore and this is the five-nodes queueing network.

The output of second que and third que is the input for the third que whereas after the service completion of the third que with some probability the jobs or customer can move again come to the third que. Therefore, this type is called queues with feedback. We call it as queues with feedback. Then, the other proportion it moved into the input for the fifth que after the service is completed, it goes away from the system.

So this type of queueing network in which the arrivals comes from the outside from source and it departure from the system. There is a possibility the customers who finishes the service in fourth que, it can depart from the system. So this is the example for the open queueing network. The third type here also we have four queues or four nodes, there is no output of queues from the system as well as there is no input from outside the domain.

That means there is constant number of customers or jobs will be moving from one node to other nodes with these probabilities and these probabilities are called as routing probabilities, in which after the service completed in second que, the customer or jobs move into the third que with some probability till move into the fourth que. Therefore, this probability is called as routing probability of a jobs or customers moving from one node to other nodes with some probability. This type is called as closed queueing network.

So even though I have started with some three examples and these three examples are related to the different types of queueing network.

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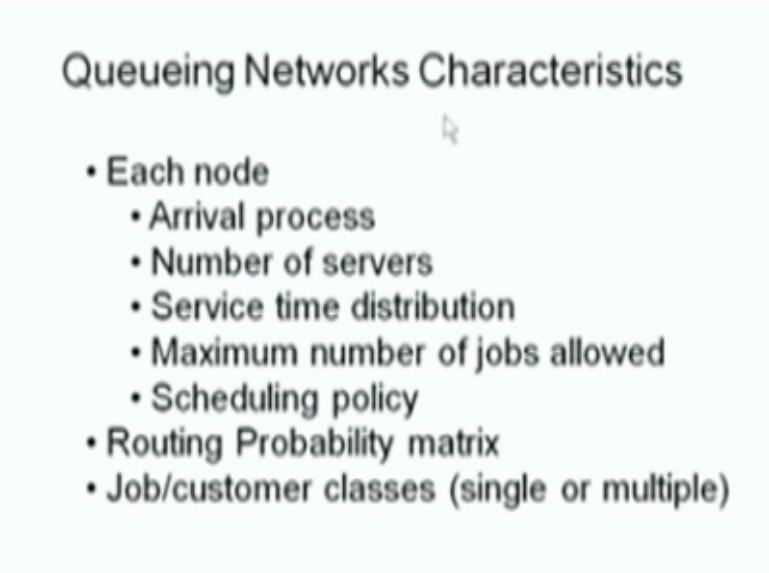

But before I move into the types of queueing networks, let me give the characteristics of queueing networks. So what are all the minimum information is needed or is provided to study the queueing networks it consists of nodes and interconnected. So the interconnected are given by the information routing probability matrix. Suppose in this example if the node 2 after the service is over, the packets are the customers or the jobs are moving into the node 3.

Then the routing probability is P to 3 so p is the matrix routing probability matrix and second row third column element is corresponding to the routing probability of customers or jobs or packets moving from node 2 to node 3 after the service is completed. If that particular probability is zero, that means the routing probability there is no path from node 2 to node 3. So we need the routing probability matrix that matrix order is the number of nodes in the queueing networks and the entries are the probability values.

Therefore, it will be greater than or equal to zero and lies between zero to one. Obviously this routing probability matrix will be a stochastic matrix, the row sum is one all the entities lies between zero to one. Now what are all the information is needed in the each node or in the each queueing system inside the queueing networks. We have to provide the arrival process or arrival pattern.

If the inter-arrival time is a random variable, then what is the distribution of that. Then, we have to provide the number of servers in each node whether it is only one server or multi server or infinite servers, then what is the time taken for the service whether it is deterministic or probabilistic. If it is probabilistic, then what is the distribution of service time?

For each node, we have to provide if it is one server then what is the service time distribution? If it is more than one servers, then whether they are the identical servers or non identical and what is the distribution of each server's service time? What is the capacity of the queueing systems in each queues? In the whole queueing networks, what is the maximum number of jobs or customers are allowed in the waiting space.

Whether infinite capacity system or finite capacity system. If it finite capacity system, then if the customer are coming from the previous some other node and if the system is full then it will be blocked. Therefore, we should consider the capacity of each queues also. What is the scheduling policy? In what policy the customers are getting served in each que? So scheduling policy whether first come first served or last come first served or priority based or random order.

So we should know what is the service or what is the scheduling policy for the each node. If we provide this file information for each node and the routing probability matrix as well as what type of customers are coming into the whole queueing networks whether only one type of customers are moving inside the different queues or more than one type of customers are in the systems. Our interest in this course how the CTMC is used in the queueing network.

Therefore, I am going to discuss a very simple queueing network in which you can able to map the underlying stochastic process in the CTMC. Therefore, we can get some of the performance measures through the knowledge of a CTMC.

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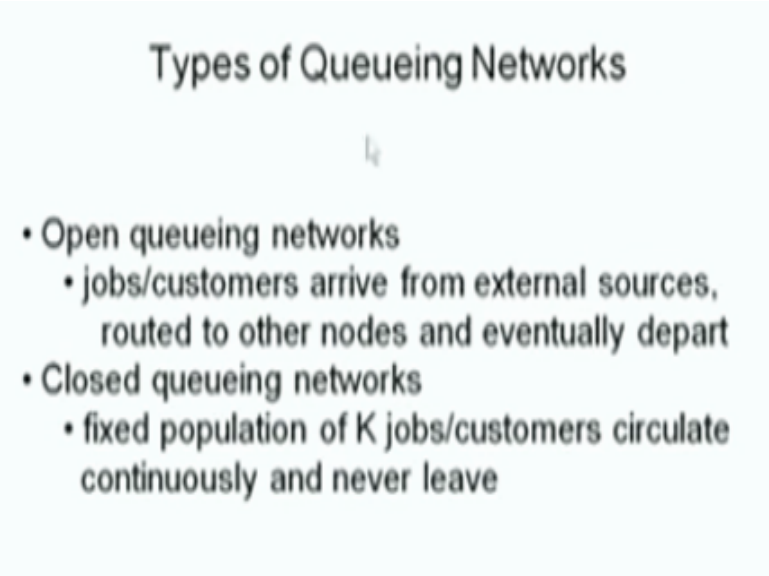

Now I am discussing the type of queueing networks, even though I have explained the type of queueing networks through these three examples. Like that we can frame many more queueing networks, but in this course I am going to cover only these three types of queueing networks. Therefore, I have given these three examples, so through these three examples I am going to classify the queueing networks as majorly in two parts.

That is the open queueing network and closed queueing network, what is the meaning of open queueing networks? The jobs or customers arrive from external sources routed to other nodes and eventually depart. Therefore, there is a source and there is a sink also. The external source and it has eventually departed from queueing networks also. Therefore, that type of queueing network is called open queueing networks.

The closed queueing networks has fixed number of population, some constant K number of jobs or customers circulate continuously and never leave. So that type of queueing network is called as closed queueing networks. That means always if you count how many customers in the system means, how many customers in each node, if we sum it up, then that total number is going to be always K that is the population size.

Therefore, no one will leave system and no one depart from the system also. So it is a constant number of customers always there in the system at the different queues. You can always convert the closed queueing network into the open queueing network by including one source and sink. So in general, we can classify the queueing network as the open queueing network and closed queueing network.