Stochastic Processes - 1

Dr. S. Dharmaraja
Department of Mathematics
Indian Institute of Technology – Delhi

Lecture - 06

Independent Random Variables, Covariance and Correlation Coefficient and Conditional Distribution

Now I am going to discuss what is the meaning of independent random variable. Suppose you have a two random variables x and y.

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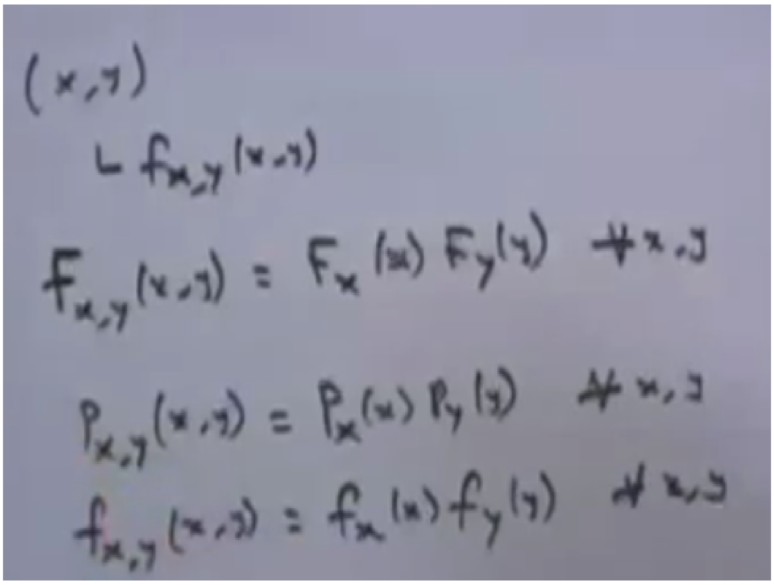

And you know what is a joint probability density function or joint probability mass function based on the random variable both are discrete or continuous then, if both the random variables are independent, then the CDF of this random variable, random vector is same as the product of the CDFs of individual random variable, whether it is a discrete random variable or continuous random variable. And this is valid for all x, y.

That means if you have a two random variables and this satisfy for all x, y, that means the joint CDF is same as the product of CDF. This is basically a if and only if condition, if this condition is satisfied, then both the random variables are called it as a independent random variables. So this, suppose these random variables are both are discrete, then you can come down from the CDF into the joint probability mass function.

In the joint probability mass function, you can write it as the product of individual probability mass function for all x, y. If both the random variables are continuous, then you have a joint probability density function. The same joint probability density function will be the product of individual probability density function. That means based on the random variables are discrete or continuous you can cross check whether this property is satisfied.

So, if this property is satisfied, then you can conclude the random variables are independent. Similarly, if the random variables are independent, then this property is going to be satisfied. So whether it is a discrete or continuous you can always check in the CDF level also. If the CDF, joint CDF or individual CDF satisfies this property, then you can conclude both the random variables are going to be independent random variable.

And this logic can be extended for the any n random variables. So instead of two random variables you can go for having a n random variables, then finding out what is the joint CDF. If the joint CDF of n dimensional random variable is going to be the product of individual random variable, then you can conclude both, all n random variables are mutually independent random variables.

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Now we are moving into the next concept, there are some moments we can find out from the random variables.

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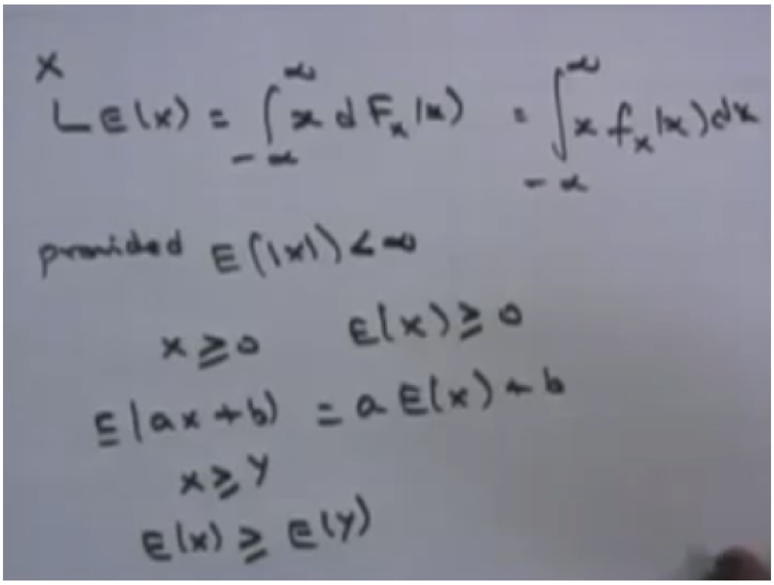

The way you are computing, suppose you have a random variable x you can able to find out the expectation of x if it exists. That means if the random variable x is there, you can always write expectation of x is from minus infinity to infinity x times d of F of x, where F is the CDF of random variable, whether the random variable is a discrete or continuous or mixed type.

If this integration is going to be exist, then you are able to give expectation is equal to this much. if the integration does not converges, that means the integration is diverges, then you cannot go for writing expectation of x. Suppose the random variable is a continuous random variable, then the CDF is going to be a continuous function, therefore this is same as minus infinity to infinity x times f of x dx, if the random variable is a continuous random variable.

In that case also we have to cross check whether this integration is going to be, see provided, it says expectation of absolute x is convergence. This is because absolute convergence implies convergence, that means whenever you replace x by absolute of x and you find out if this provided is satisfied, then without absolute whatever quantity you are going to get in the, if it is in the continuous random variable, the integration is, whatever the value you are going to get, that is going to be the expectation of the random variable.

So the expectation of the random variable has, expectation has a few property this is going to be always a constant. This is not a random variable, and the expectation of x if the random variable is greater than or equal to zero, then the expectation of x is always greater than or equal to zero. And the expectation of x has a linear property, if you have a two random variables, then the expectation of x is greater than or equal to the expectation of y.

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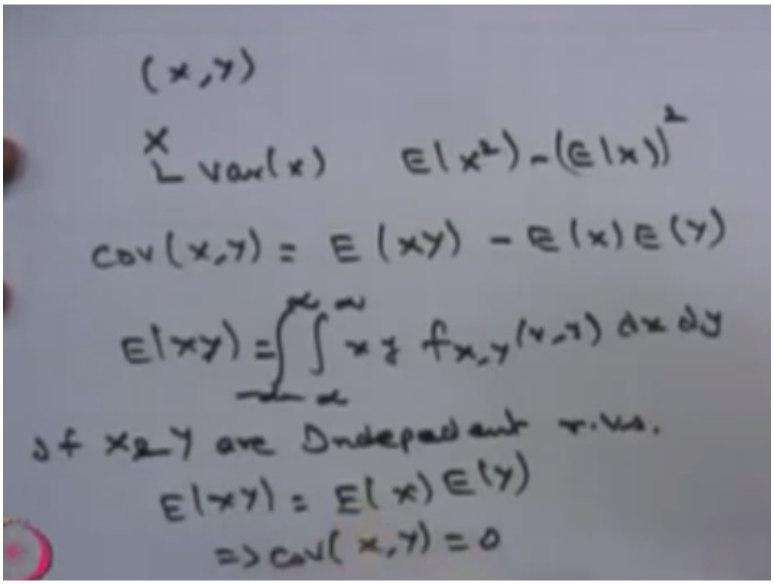

So now we are going to discuss, since we have more random variables we are going to discuss what is other than expectation we can go for finding out the variance of the random variables also. Variance is nothing but the second order moment, that is E of x square minus E of x whole square. So here also as long as E of x square, that means expectation in absolute x if that is convergence, then you can able to get expectation of E of x square.

And once you have a second order moment is exist, obviously the all the previous order moment exist. But that does not imply the further moment exist. So now I am going to define what is the covariance of x, y. So covariance of x, y is nothing but expectation of x into y minus expectation of x into expectation of x, provided the expectation exist.

So here it is expectation of x into y that means you have to find out what is the expectation of x into y, based on the random variable is a discrete or continuous, you can able to use functions of random variable method and getting the expectation. And note that even you do not know the distribution of the x into y, you can always find out the expectation of x into y.

Let me give a one situation, if both the random variables are continuous, then the expectation of x into y is going to be x into y. And the joint probability density function of f of x, f of y, that means this is going to be the value of xy, and what is the joint distribution of this. That means you are not finding out, what is the distribution of xy. But still you can find out the expectation of xy by possible values and corresponding joint distribution you can get the expectation.

And here also provided the absolute sense exists, then without absolute sense it is going to be E of x into y. Suppose x and y are independent random variables, then the expectation of x into y, the way I have given the situation with both the random variables are continuous this integration will be splitted into the two parts, such a way that this F of x, y, this is going to be, sorry this is minus infinity to infinity.

So this is going to be the product of individual probability mass function, therefore this is going to be integration will be splitted into two single integration minus infinity to infinity externs f of x and minus infinity to infinity y times f of y dy. Therefore, that is nothing but the expectation of x into expectation of y. That means if two random variables are independent, then implies the covariance of the two random variables going to be zero.

But the covariance of x, y = zero that does not imply the random variables are independent. So this is going to be the not if and only if, if the random variables are independent, then you will come to the conclusion the covariance of x, y equal to zero, not the converse.

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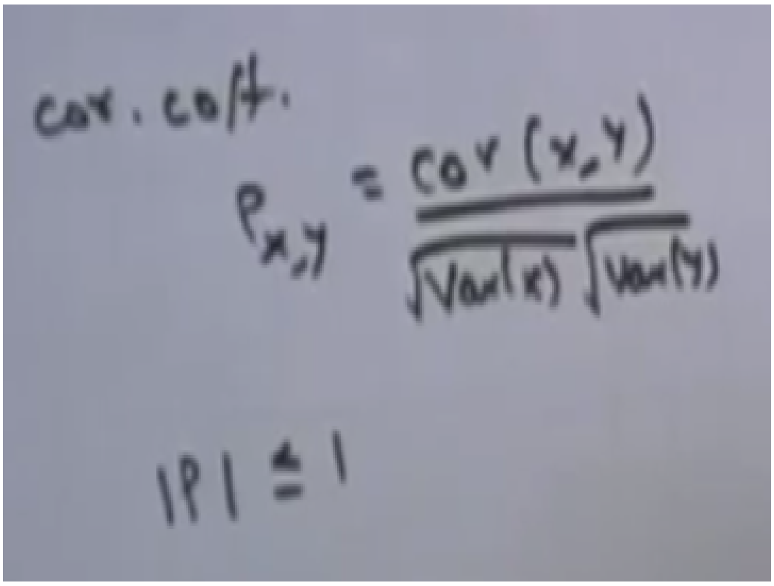

Now we are going to define the another measure that is a correlation coefficient. That is nothing but with the letter rho, rho of x, y that means I am trying to find what is the correlation coefficient between the random variables x, y. That is nothing but the covariance of x, y divided by the square root of variance of x into square root of variance of y. That means to have the existence of the covariance correlation coefficient you should have a, the random variable should have at least second order moment.

So unless otherwise the second order moment does not exist you cannot find out the correlation coefficient between these two random variables x, y, because you are using the variance as well as the covariance too. So therefore, if the random variables are independent, then obviously the rho = zero, because the numerator is going to be zero. And since you are dividing the covariance divided by square root of variance in x as well as y, this quantity in absolute is less than or equal to one or the rho lies between minus one to one.

And the way the correlation coefficient value lies between zero to one, that conclude it is a positive correlated, and the values lies between minus one to zero it gives a negatively correlated. And if the value is positive one or minus one, then you can conclude that the random variables x and y or linearly correlated, based on the value is positive side or the negative side, then you can conclude it is positively correlated or negatively correlated.

So other than the value minus one and one, you cannot conclude what is the relation between the random variable. Only if it is one and minus one, then you can conclude the random variables are correlated in the linear way.

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Now I am going to discuss conditional distribution, because these are all the concepts that are needed when you start defining some of the properties in the stochastic process. So therefore I am discussing what is conditional distribution.

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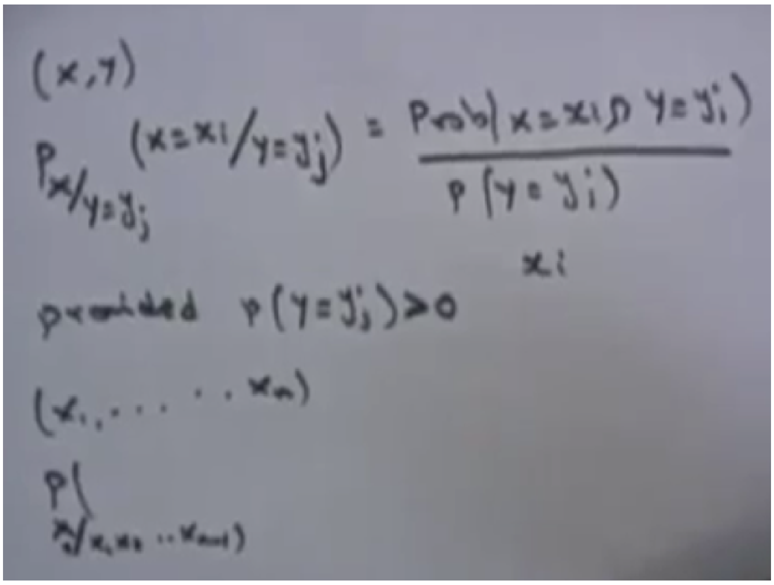

Suppose you have a two dimensional random variable x, y, you can define, suppose I make the one more assumption both are discrete type random variable, then I can define what is a conditional distribution of the random variable x given that y takes some value yj, and here x takes a value xi given that y takes the value yj. I can compute by finding what is the probability that x takes a value xi intersection the y takes the value yj divided by what is the probability that y takes the value yj.

And here the running index is for all xis' and this is for fixed yj, therefore the provided condition, provided the probability of y takes a value yj has to be at least be greater than zero. That means you are making a the way you made a conditional probability over the event the same way we are making, this is going to be the event y is equal to yj. So as long as the probability of the event corresponding to y is equal to yj is strictly greater than zero, that means it is not an impossible event with a probability zero.

It is an event which has a positive probability, if this happens already, then what is the probability of x takes the value xi. That means still our interest is to find out the distribution of x only, the random variable x, with the provided or given situation that at the random variable y takes a value yj.

That means from the omega you land up having a one reduce a sample space that corresponding to y is equal to yj. And from the reduced sample space you are trying to find out what is a distribution of the random variable x, for all possible values of xi. So this we call it as a conditional distribution of x given the other random variable. And this logic can be extended for more random variables.

That means if you have a n discrete random variable, then you can always define, suppose you have a x1, x2, xn, and suppose all are discrete random variable, then you can always define what is a probability distribution of xn given you know the distribution of x1, x2 till xn minus one. That means still it is a one dimensional random variable of xn given that already the random variable x1 to xn minus one takes some particular value.

Similarly, you can go for what is a joint distribution of few random variables given that all other random variables already taken some value.

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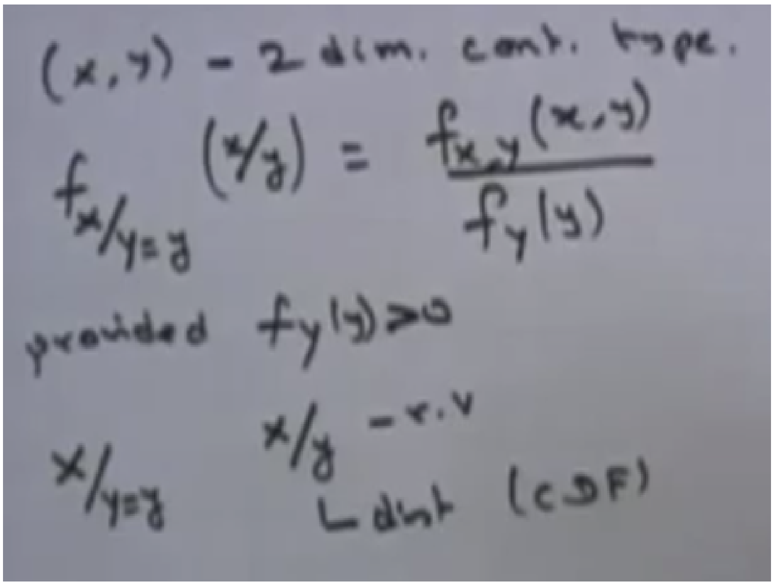

Now I can go for defining, the same way I can go for defining what is the conditional distribution of two dimensional continuous type random variable. That means you can define what is the probability density function of random variable x given that y takes a value y. That means, that is x given y, this is nothing but what is the joint probability density function of x with y and divided by what is the marginal distribution of y.

And here also the provided condition f of y is strictly greater than zero. That means where ever there is a density which is greater than zero and with that given situation you can find out the distribution of the random variable x with the given y takes the value small y. That is nothing but what is the ratio in which the joint distribution with the marginal distribution.

Once you know the conditional distribution, this is also a sort of another random variable, that means x given y takes a value y. So that I can use it as the word x, y, this is also is a random variable. Therefore, you can find out what is the distribution, therefore this distribution is called a conditional distribution. And you can find out what is a CDF of that random variable.

So the way you find out the CDF of the any discrete random variable by summing what is the mass or by integrating the probability density function till that point, you will get the CDF of this conditional distribution.