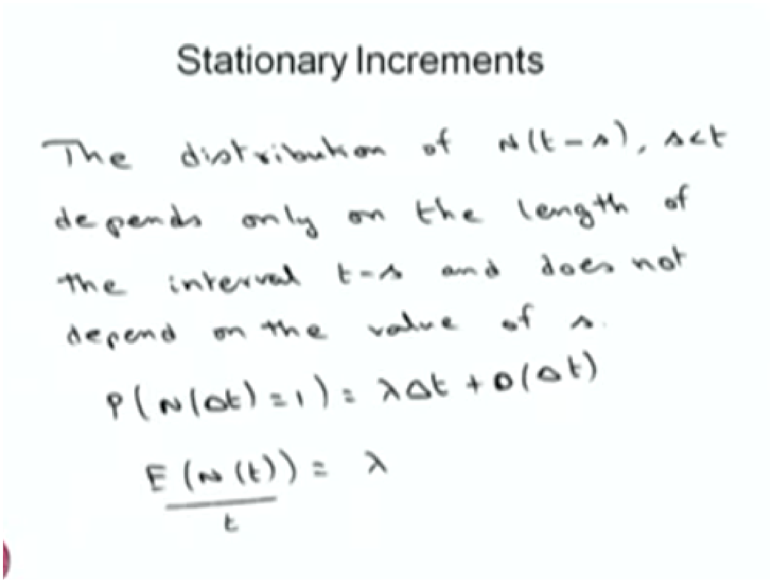
Stochastic Processes - 1
Dr. S. Dharmaraja
Department of Mathematics
Indian Institute of Technology – Delhi

Lecture – 59
Superposition and Deposition of Poisson Process

(Refer Slide Time: 00:00)

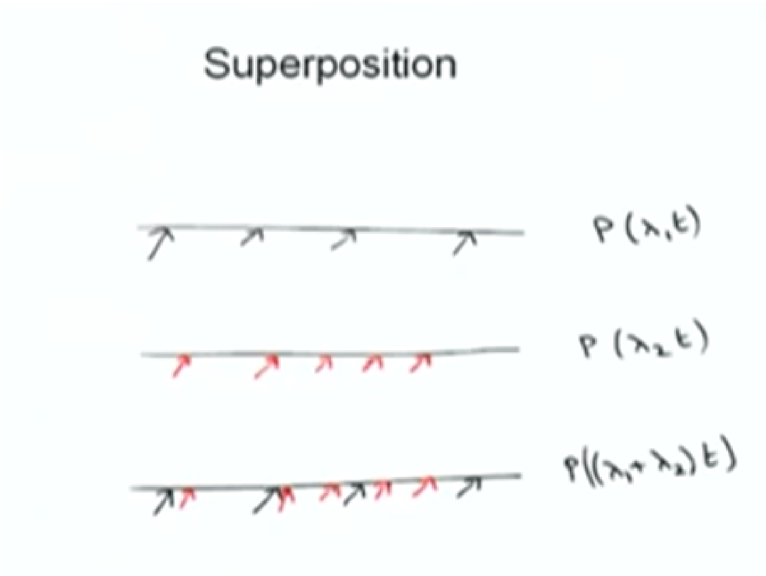


Now I am going for the stationary increment. The distribution of n of t minus s depends only on the length of the interval t minus s and does not depends on the value of s. That means, during the interval delta t, the one arrival is going to lambda times delta t, order of delta t that will tends to zero as n tends to as delta t tends to zero.

That means, the stationary increment means if you find out the rate that means you find out the average per unit of time, then that is going to be constant. So this is the assumption we have taken it in the car insurance problem, the average rate per unit day, that is going to be constant and that is an assumption we have taken at going to be a constant throughout the year and also the different times of a day.

So here also we will get whenever we have a Poisson process, then the average rate is going to be a constant because of the stationary increment.

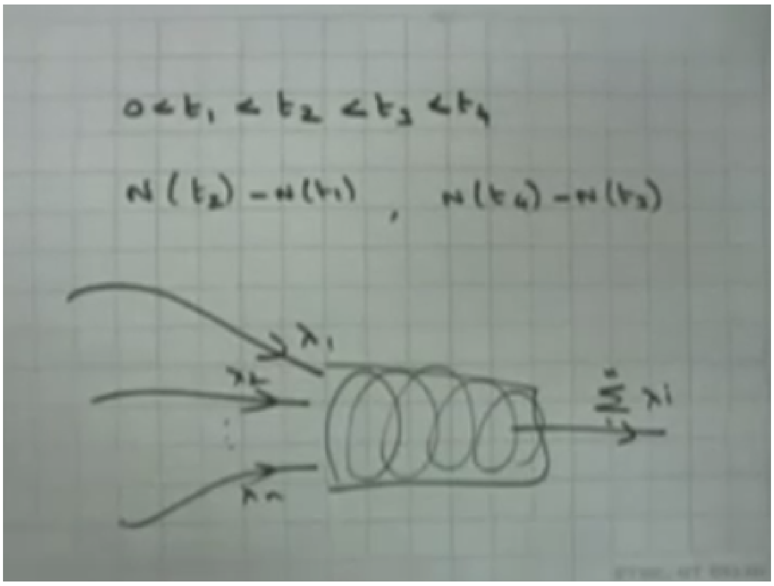
(Refer Slide Time: 01:08)



The next property, suppose you have a Poisson process of a one arrival and you have a Poisson process of the other arrival, that means one type of arrival is a Poisson process with the parameter lambda one and another type of arrival who that is also Poisson process with the parameter lambda two. As long as both are independent, the arrivals are independent, then the together superposition.

That is going to be again Poisson with the parameters lambda one plus lambda two. You can add the parameter. That means for fixed t that is going to be a Poisson distributed random variable with the parameter lambda one plus lambda two times t. Whenever you have a two independent or more than one independent Poisson process arrival, then the merging or the superposition will be again Poisson process as long as they are mutually independent with the parameter is nothing but the sum of those parameters.

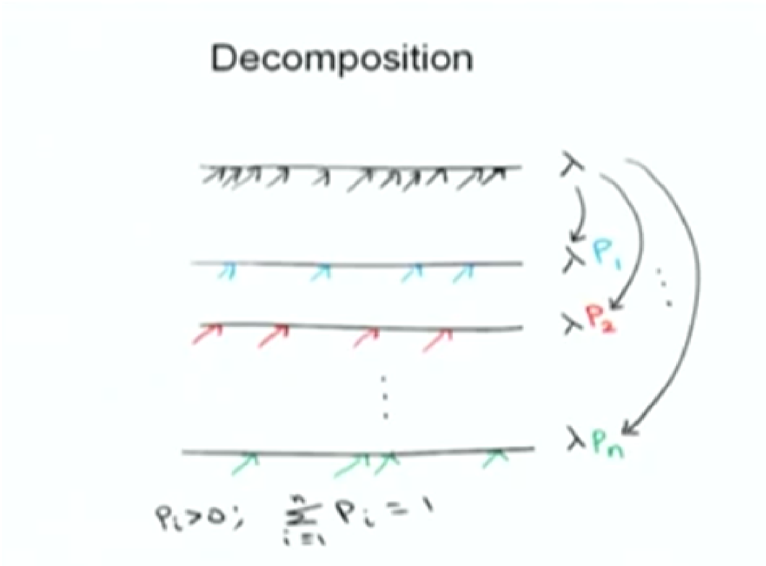
(Refer Slide Time: 02:32)



That is, you can combine many Poisson process as streams into one stream and that is going to be a Poisson stream with the parameter sum of parameters lambda 1 to lambda n. So this is possible, this is used in many telecommunication application, that means, suppose, you have a Poisson arrival of a packets from the different streams and all the streams are mutually dependent, the arrival are independent.

Then the total number of packets arriving into the particular switch or router, whatever it is. Then the multiplexed one, that is going to be always Poisson process, that arrival follows a Poisson process with the parameters are sum of, parameter is nothing but the sum of these parameters as long as they are Poisson as well as independent.

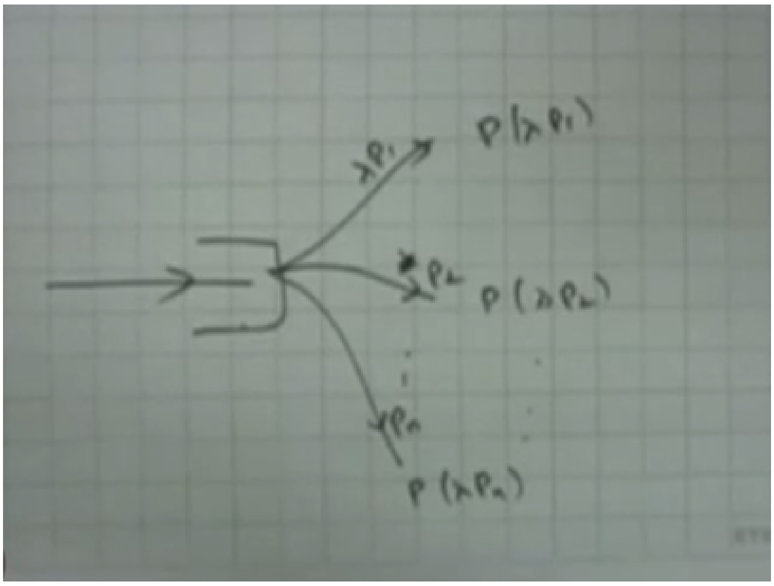
(Refer Slide Time: 03:42)



The next property, decomposition, suppose if you have a one Poisson stream, you can decompose into many Poisson streams with the sum proportion. So that proportions are the p1, p2 and pn's. So one Poisson stream can be split into n Poisson streams with the parameter lambda times p1, lambda times p2 where pi's are greater than zero. The summation of pi's has to be one.

That means these are all the probabilities. With these probabilities you can split one Poisson stream into many Poisson streams. So here I have made a n Poisson streams, that means the same arrival is with some probability p1, it lands up here with some probability p1, this put up here, with some probability p1 is put up here. So the split of one Poisson stream into n Poisson streams is allowed.

(Refer Slide Time: 04:47)



That means the same example, if you have a one router and from the router if the arrival is splitted into many streams with a probability p1, it goes to the first stream with the probability p2 it goes to the second stream and with the probability pn it goes to the last stream. Then each one is going to be a Poisson process with the parameter lambda times p1 and lambda times p2 and so on lambda times pn.

So the split is possible as well as the superposition is also possible from the Poisson process. So this also has a many more applications in the telecommunication networks. One type of packet arrival can be splitted into n proportions p1, p2, pn's and each one is going to be a Poisson process.

(Refer Slide Time: 05:55)

Simple Problem

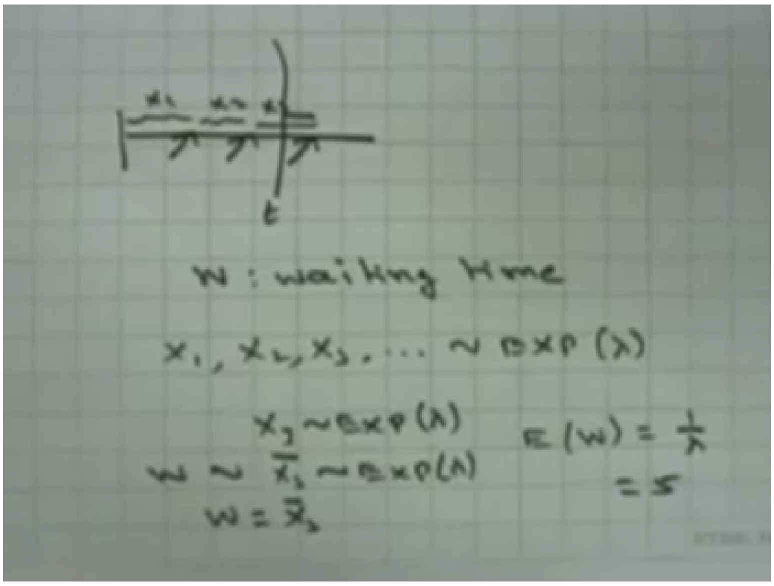
Consider the situation of waiting for a bus in a bus stand. Assume that the bus arrivals (in minutes) follow Poisson process with parameter 5. Suppose you come to the bus stand at some time. What is the average waiting time to get the bus ?

Now I am going to give the first example to illustrate the Poisson process. Consider the situation of a waiting for a bus in a bus stand. Assume that the bus arrivals in minutes follow a Poisson process with the parameter 5. With the rate, the parameter here that is nothing but the intensity or rate. Suppose you come to the bus stand at some time, what is the average waiting time to get the bus.

When you land up bus stand, there is a possibility the bus would have come before some time, the time in which the next bus is about to come, you are going to take that bus and till that time, you are going to wait in the bus stand, that is the waiting time. So the waiting time is a random variable. So that is a continuous random variable. The question is what is the average waiting time.

One can find out the distribution of the waiting time also. Here the question is what is the average waiting time. So then, what I can do, I can use the Poisson concept here.

(Refer Slide Time: 07:20)



The arrival follows the arrival of a bus follows a Poisson process. Suppose at some time you come to the bus stand and suppose the bus is going to come at this time, your waiting time is this much. So suppose, you make w is going to be your waiting time, w is going to be your waiting time, the question is what is an average waiting time. Just now I have explained the Poisson process as the property, the inter-arrival times are exponential distribution.

The inter-arrival times are exponential distribution and all the times are, all the inter-arrival times are independent also. Therefore, this x1 and this is x2 and this is x3, so x1, x2, x3 like that so many, all the inter-arrival times, that is going to follow exponential distribution with the parameter lambda. Since the waiting time is going to be the remaining time of arrival of the third bus.

So the w, the waiting time is same as a the remaining or residual time of the third bus to come into the bus stand. So x3 is exponential distribution with the parameter lambda. The residual lifetime of x3, suppose I make it as a notation x3 bar, the residual lifetime, residual time of arrival, not life time, residual arrival time of the third bus coming to the bus stand, that is also going to be exponential distribution.

This is because of the memoryless property, the residual time is also, whenever, some time is exponentially distributed, some random variable time is exponentially distributed, then the residual time is also going to be exponentially distributed using the memoryless property. Therefore, residual arrival time of bus to come to the bus stand, that is also exponential distribution with the parameter same lambda.

So this is same as the w. The waiting time w is same as a residual time. Therefore, w is always going to be exponentially distributed with the parameter lambda. That means the waiting time for the bus to come to the bus stand to catch. So the w is exponentially distributed therefore the question is what is a average waiting time. So average waiting time is nothing but one divided by the parameter.

So here it says the Poisson process with the intensity 5, that rate is lambda, that is the mean inter-arrival times between the buses is 5 minutes, that means the mean inter-arrival times between the buses is 5 minutes is nothing but it is exponentially distributed with the parameter that is average 5 minutes therefore that is the same thing. Therefore, that is equal to 5 minutes.

Because the way I have given the clue, the mean interval between the buses is 5 minutes, that means the average of x i's that is equal to 5 minutes. So that is as same as your waiting time because it is exponential distribution therefore the residual is also exponential distribution, therefore you can use the same value, therefore the average is going to be 5 minutes. So using Poisson process one can find out the different results related to the number of arrivals.