Stochastic Processes -1
Dr. S. Dharmaraja
Department of Mathematics
Indian Institute of Technology – Delhi

Lecture - 05
Joint Distribution of Random Variables

This is the continuation of module 1, probability theory refresher and this is lecture 2. So in lecture 1, we have covered what is the motivation behind the stochastic process, then we have given a few examples and followed by that we have explained what are all the minimum things is necessary to study the stochastic processes. We started with random experiment and events, then the probability space and to create the probability space we need a sigma algebra.

Then after creating the probability space, then we have defined the conditional probability, then we discussed independent of events, then we have list out a few standard discrete random variables as well as standard continuous random variable, even though we have discussed only three or four discrete and continuous random variable, there are more, but whenever the problem comes.

We will discuss those standard distributions when we come across those distributions. In the lecture 2, we are going to continue whatever we have discussed in the lecture 1, basically the probability theory refresher.

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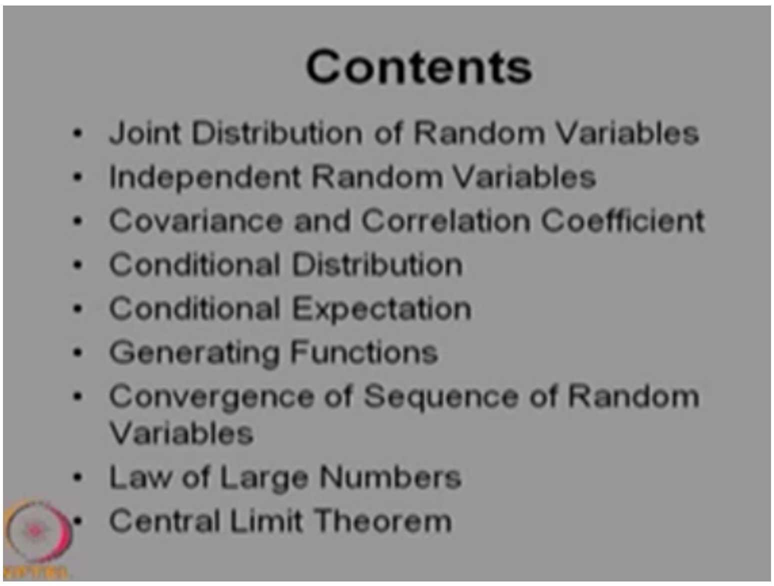

In these, we are going to give a brief about what is joint distribution and if the random variables are independent what is the behaviour of joint distribution and so on. Then we are going to discuss covariance and correlation coeffecients. After that we are going to discuss the conditional distribution, then followed conditional expectation also. And we are going to list out a few generating functions, probability generating function, (()) (02:13) generating function and also the characteristic function.

Then, at the end of the probability theory part, we are going to discuss how the sequence of random variable converges to some random variable and for that we are going to discuss a low of large numbers also and at the end of the lecture 2, we are going to complete with the central limit theorem. So let me start with the joint distribution of random variable.

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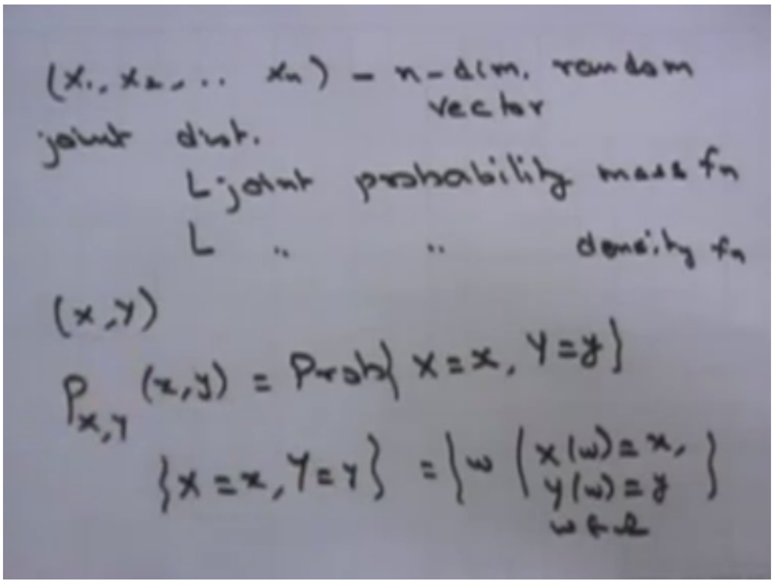

So suppose you have random variables x1, x2, xn. We say this random variable is n dimensional random vector. Each random variable x1, x2, xn are the random variables, either it could be a discrete random variable or continuous random variable and we are going to make it as together in the vector form and each one is going to be a random variable, then it is called n dimensional random vector.

Once you have together as a vector form, then we can go for giving the joint distribution. So the joint distribution, we can discuss in two ways, either it is joint probability mass function or we can define as a joint probability density function. Suppose you take example of two dimensional random variable x, y, if both the random variables are discrete.

Random variable x is discrete as well as the random variable y is discrete, then you can define what is the joint probability mass function of this two-dimensional discrete random variable as probability of x, y. Here the x, y are the variables and this x, y denotes the two dimensional random variable. This is nothing but what is the probability that X takes the value, random variable X takes a value x and the random variable Y takes a value y.

And based on the possible values of x and y, you have the probability of this. That means you land up creating what is the event, which is corresponding to X = x and Y = y, that means if you are not getting any possible outcomes, that gives some possible values of x, y, then it may be the empty set, otherwise you land up with a different possible, that means X is equal to x and Y is equal to y for all possible values of x, y you may relate with.

What is the event in which x of w give a value X as well as y of w gives a value Y, where w is belonging to omega. That means you collected a possible outcome w such that it satisfies both the conditions where we is belonging to omega that means this is going to be the event. Therefore, this is the probability of event and you know by using the axiomatic definition, the probability of event is always greater than or equal to zero.

And the probability of omega is equal to 1 and if you take mutual exclusive events, then the probability of union of events is going to be the summation of probability. Therefore, this is the way you can define when the random variables x and y both are discrete type, then you can give the joint probability mass functions and therefore, here this is the joint probability mass function and this satisfies all the values are always greater than or equal to zero for any x and y.

And if you make the summation over x as well as summation over y, then that is going to be 1. Therefore, it satisfies the property of always greater than or equal to zero for any x and y and double summation over x and y is equal to 1. Therefore, this is going to be the joint probability mass function corresponding to the discrete type random variable.

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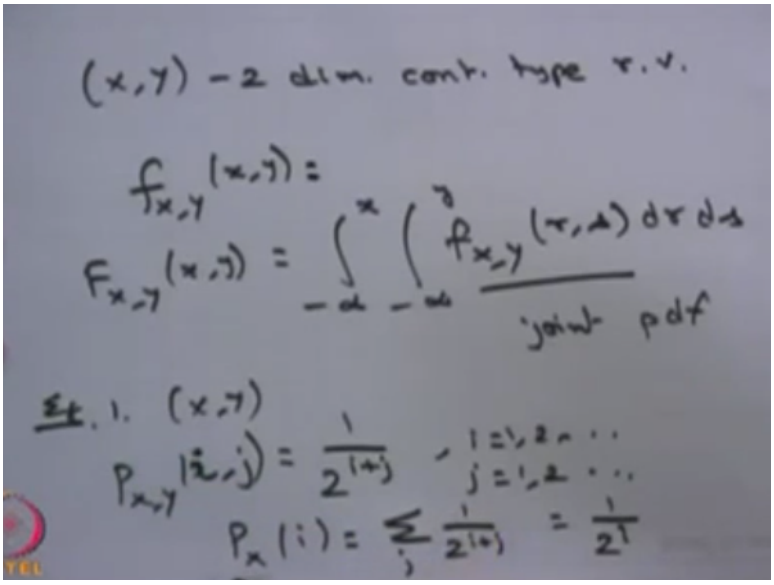

Suppose the random variable x and y are, each random variable is a continuous type random variable, therefore, you will land up with the two dimensional continuous type random variable or random vector, in that case, you can define what is the joint probability density function is of the form f of x, y that is going to be, so you can have a joint probability density function and you can relate with this joint probability density function with the CDF.

What is the CDF of the random variable? That is nothing but what is the integration from minus infinity to x and what is the integration from minus infinity to y, the integrant is going to be r, s where is r is with respect to x that is dr and this is ds. That means since you have a continuous type random variable, therefore you will land up with a continuous function with the two variables x, y by using the fundamental theorem of algebra.

You can always land up a unique integrant and that is going to be the density function for this two-dimensional continuous random variable and you can able to write the left hand side continuous function in x and y can be written in the form of integration from minus infinity to x and minus infinity to y of this integrant and the s, whereas this function is going to be called as joint probability density function for the random variable x, y.

So there is a relation between joint probability density function with the CDF, I can give a few examples. Example 1, suppose you have both random variables x, y is a discrete type, then I can give one simplest example of i, j that is going to be 1 divided by 2 power i plus j, for i is belonging to 1, 2, and so on and j can take the value 1, 2, and so on and this is going to be the joint probability mass function for two dimensional continuous type random variable.

If you made a summation over x and summation over i and summation over j, then that is going to be 1 and you can get by summing at over only j, you will get the probability mass function for the random variable x. Similarly, if you make the summation over i with the probability mass function, then you may get the probability mass function for the random variable y.

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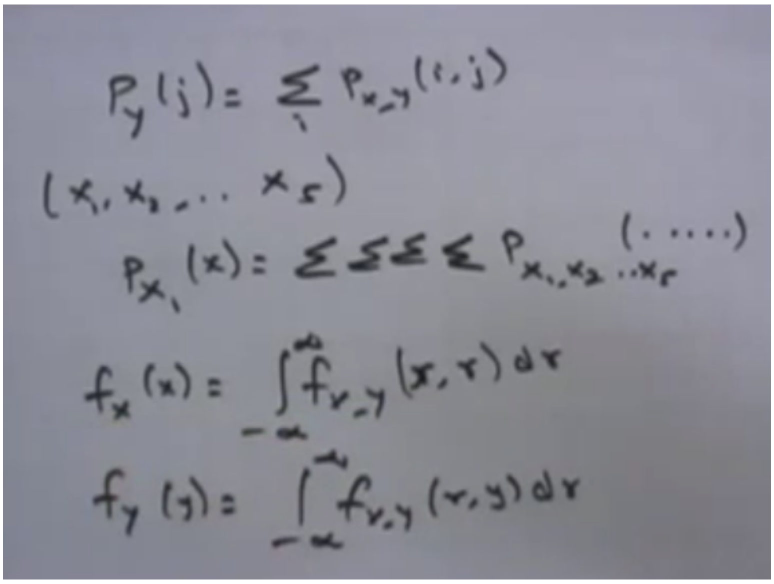

That means from the joint probability mass function by summation over the other random variable, you can get the distribution of the single random variable. Suppose, if you have a n dimensional random variable, for n is equal to 5, you have a five dimensional random variable, suppose all the random variables are of the discrete type, then you may have the joint probability mass function of this five dimensional discrete random variable.

And by summing over each random variable and if you want to find out what is the probability mass function for the one random variable, you can always make the summation over other random variables with respect to the other random variables, you can get what is the probability mass function for. By summing it over the other random variables, you will get marginal distribution of the random variable x1.

Similarly, you can get the marginal distribution of x2 or any other random variable. Similarly, suppose you have a joint probability density function, from that you can get the marginal distribution of one random variable by integrating with the other random variable. By integrating with the other random variable, so this is the joint probability density function.

From the joint probability density function by integrating with other random variable, you will get the probability density function for this random variable y.

Similarly, you can get the probability density function for y. So by integrating with respect to r, so that means you are just finding out the marginal distribution of the random variable y and here you are finding the marginal distribution of x.