Stochastic Processes - 1
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Lecture – 46
Stationary Distributions and Types of Reducible Markov chains

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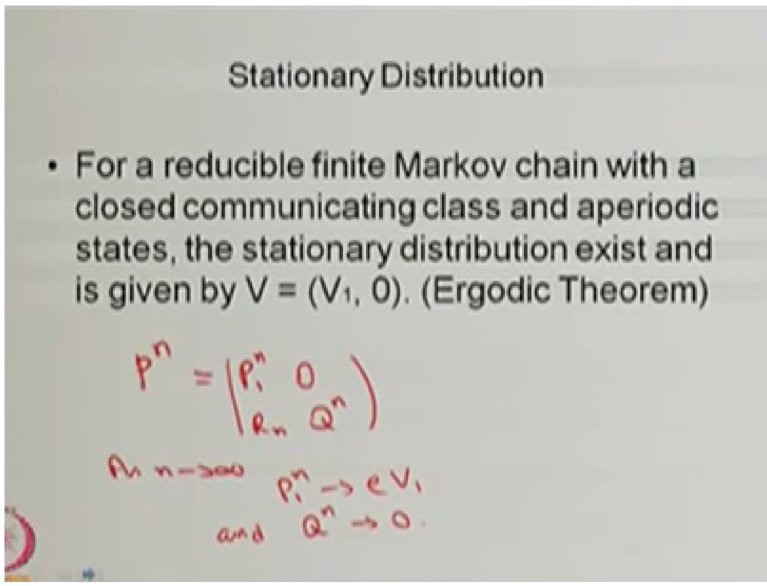

How to study the Stationary Distribution for a reducible Markov chain along with the assumptions one is aperiodic and the finite state. Here I am making one more, here I am giving the stationary distribution. So I am giving the result for a reducible finite Markov chain, Markov chain is as in the finite chain and it is reducible one. With the closed communicating class has aperiodic states.

There is a mistake the closed communicating class of state has aperiodic states, aperiodic the closed communicating class of states aperiodic then the stationary distribution exists that is going to be unique also and that is given by the vector mean its consist of two sub vectors V1, 0 vector that you can find out. And this is nothing but the Ergodic Theorem at the reducible Markov chain with the assumption finite state space and the states of closed communicating class has aperiodic states.

In that case you get the unique stationary distribution and that unique stationary distribution has a two that sub one that vectors are V1 and vectors of 0 aims. Before you can get the stationary distribution you can find out what is the N-step transition probability for the same reducible Markov chain model. So the P1 is going to be, you have a sub matrix Stochastic sub matrix P1 therefore that is going to be P1 power n whereas for every N this is going to be a function of n.

R is the sub matrix which is the one step going from the transient state to the closed communicating class. Now the Rn is nothing but a function of N that element that sub matrix is corresponding to the transient state to the communicating class. Whereas the transient to transient that is going to be a power. It is a Q matrix Q matrix is the sub matrix for one step T to T whereas the Q power n is the element corresponding to the N step transition probability matrix.

So as an to infinite the Stochastic sub matrix that power n that will the vector of V1, V1 is the sub few elements that is corresponding to the stationary state probability for the state corresponding to the closed communicating class of states. So e is the vector of the entries one, one, one and so on multiplied by the V1. And the transient to transient N-Step transition probability as an entrance to infinity this will tends to 0 this is obvious.

Because the states are transient states for a finite N you have a probability Q power n whereas as an entrance to infinite the system will not be in the transient state. Therefore, the long run proportion of the time the system being in the transient states that is 0 as entrance to infinite. Therefore, Qn is entrance to 0 or this will tens the to the stationary state probabilities. Therefore, this stationary distribution vector V consist of few elements of zeros that is corresponding to a transient states transient state probabilities in a long run.

And V1 is the study state probabilities in a longer run it is not study state its stationary distribution probability in a longer run for the closed communicating class of states. So this one can solved by using equation pi P is equal to pi you can get this pi e that pi e is in the notation and here it is Vi P1. So now I am making a further assumption the states are going to be the positive recurrent.

So already I made a aperiodic state now I am making one more assumption it is a positive recurrent. Once it is a positive recurrent then the limiting probability is limit entrance to infinite that probability is going to be Vj for the positive recurrent states and for all the transient states the probabilities are going to be zero. And since we have a reducible Markov chain with one closed communicating class.

And all other states are transient states this stationary distribution stationary state probability these probabilities are independent of the initial state i. That means either the system you can start a time zero in the one of the states in closed communicating class or transient states. In a longer run, ultimately the system will be in one of the states in the closed communicating class whether it is started initially from the closed communicating class or transient state.

Therefore, this stationary distribution is independent of initial state i and for transient state you can conclude immediately these probabilities are zeros and for a positive recurrent states you can make it Vj and you can compute this Vj.

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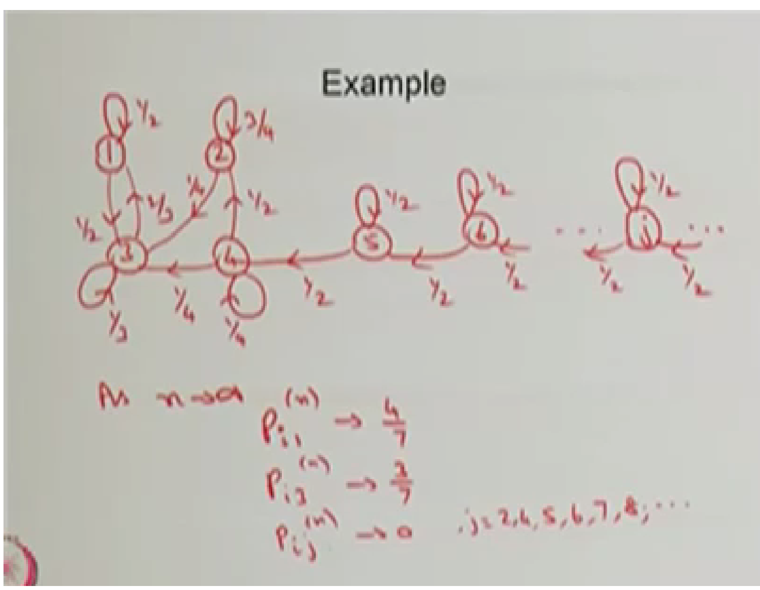

Now I am going to give one simple example in which we have a infinite state. This is going to be a reducible Markov chain because the states till five not till five including four and two the system come to the state three there is no arc from three to four or three to two therefore the states 2,4,5,6 and so on all those states are transient states whereas the state one and three are going to form a one close communicating class.

Therefore, this is the reducible Markov chain with the one closed communicating class one and three and all other states are going to be transient states therefore as an entrance to infinite this probability is are going to be zero for these states 2,4,5 and so on and these probabilities are independent of the initial state i.

So wherever the i whether i is belonging to the one of the event one of the states in the closed communicating class of states or the transient state immaterial of that this is stationary distribution zeros for the transient state for the closed communicating class of states you can find out this probability by separately making the Markov chain.

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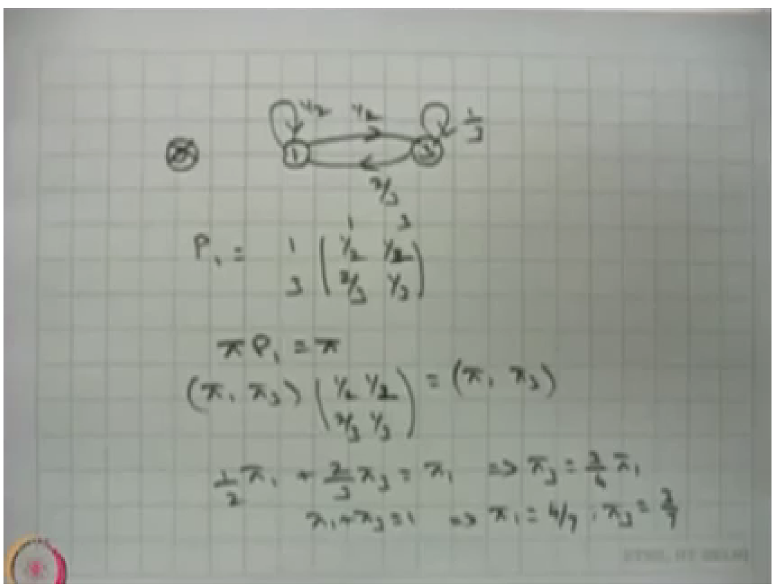

The states are one and three you can make it separately and there is a arc from one to three with the probability of there is self-loop with the probability one by two and there is a self-loop in the state three with the probability one by three and the arc from three to one is two by three. So what do you want to find out this stationary distribution for this two states. Therefore, you make as stochastic sub matrix with the state ones and three.

That is one by two, one by three, one by three and one by three this is also stochastic matrix you can verify. Now if you want to find out the stationary distribution for these two states you solve by pi P1 is equal to pi. That means pi 1 pi 3 three times P1 that is one by two, one by three oh sorry, one by two, this is one by two, one by two, one by two and this is two by three; one by three that is equal to pi 1, pie 3.

You take the first equation that is pi 1 of times pi 1one plus two third three that is equal to pi one. So from here you will get pi three is equal to three by four pi 1. Now we use a pi 1 plus pi 3 is equal to one. So using this we will get pi 1 is equal to four by seven. Once you know the pi 1 the pi 3 is going to be three by seven. So you do not to find out stationary distribution for the whole model instead of that you can find out what is the close communicating class.

And you can solve only a closed communicating class that sub matrix pi P1 is equal to pi and you will get pi 1 and pi 3 and that is going to be in a longer run that is equal to four by seven and three by seven and all other states are going to be zero.