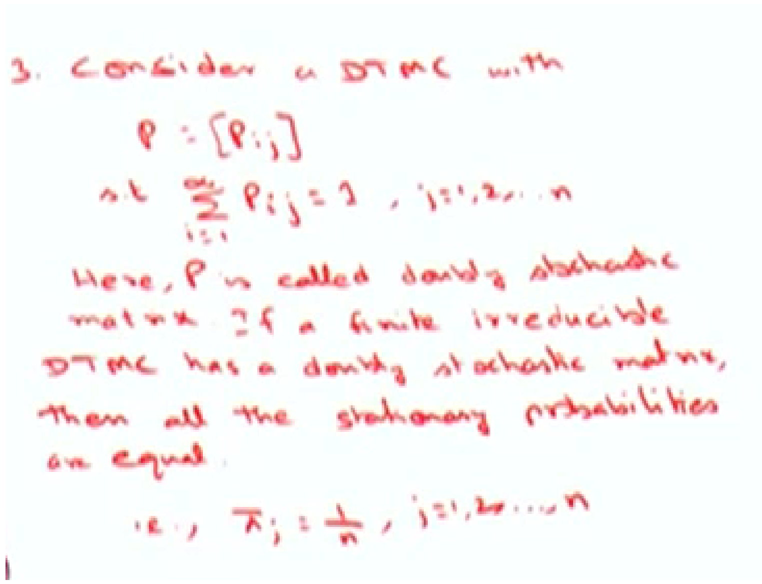
Stochastic Processes - 1
Dr. S. Dharmaraja
Department of Mathematics
Indian Institute of Technology - Delhi

Lecture - 43
Examples of Stationary Distributions

(Refer Slide Time: 00:00)



The third example I am considering a discrete time Markov chain obviously it is a time homogeneous discrete time Markov chain with the one step transition probability matrix satisfies the additional condition that is the - the column sum that is also going to be 1, obviously the stochastic matrix means the row sums are going to be 1 and here I am making additional condition along with the row sum the column sum is also going to be 1 for a finite Markov chain.

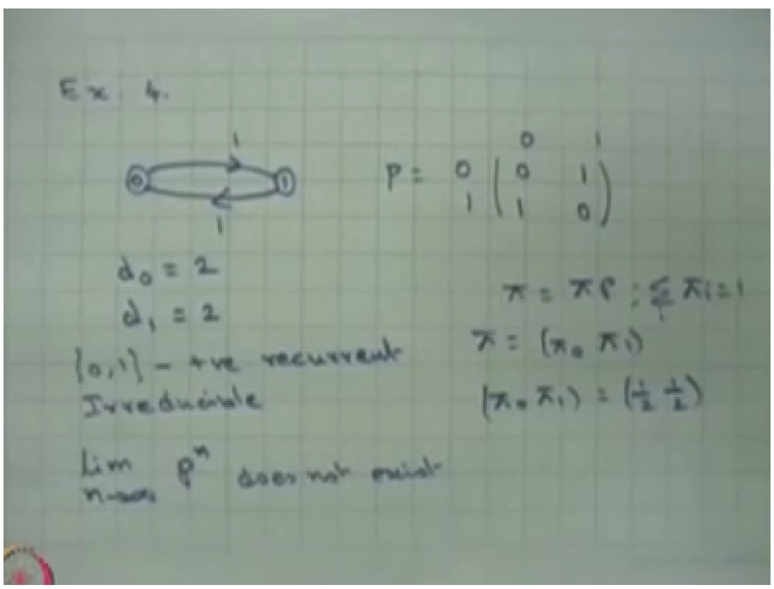
In this model, in this situation this stochastic matrix is going to be call it as a doubly stochastic matrix, that means it is a stochastic matrix that means each entities are lies between 0 to 1, and row sum is going to be 1, along with row sum, the column sum is also going to be 1, then that matrix is going to be call it as a doubly stochastic matrix.

If you have a discrete time Markov chain with the finite and doubly stochastic matrix and also it is irreducible I am making additional condition if it is a finite irreducible with the probability one step transition probability matrix is a doubly stochastic matrix then the stationary probability exist as well as that stationery probabilities are going to be a uniformly distributed that is that values are 1 divided by n, where n is the number of states of the discrete time Markov chain.

To get this result you can use all the previous results also, it is a irreducible Markov chain therefore and also it is a finite, so for a finite irreducible Markov chain all the states are going to be a positive recurrent, you can use the previous result only the periodicity is missing, but since it is a double stochastic matrix that a periodicity is taken care therefore the stationary probabilities exist.

Now if you compute the stationary probabilities for a doubly stochastic matrix situation, then the pi is equal to pi P, if you solve the summation of pi i is equal to 1, since the matrix is going to be a doubly stochastic that means it is a column sums are going to be 1, therefore it is going to be boils down or the simplification is boils down to the state probabilities are going to be 1 divided by n. I'm not going to give the derivation for that - that can be worked out.

(Refer Slide Time: 02:57)



Example that is example 4, you consider a two state model the system is going from the state 0 to 1 in one step that probability is 1, and the system is going from the state 1 to 0 that probability is also 1, therefore the P matrix one step transition probability matrix 0 to 0 is 0, 0 to1 that is 1, 1 to 0 that probability is 1 and 1 to 1 that is 0, so this is a one step transition probability matrix.

And if you see that this is a finite state model irreducible it is not aperiodic because there is no self-loop so if you find out the periodicity for the state 0, the greatest common divisor of system starting from the state 0 to coming back to 0, in how many steps you find out the greatest common divisor of that and since it can come back in two steps or four steps and so on, therefore the greatest common divisor is 2.

Similarly, since it is a finite state model, if a one model is one state is of periodicity then all other states also going to be a same periodicity as long as it is a irreducible, therefore the periodicity for the state 1 that is also going to be 2 or you can compute separately coming back to the state one starting from the state one that is going to be either two state or two steps or four steps or six steps and so on, that the therefore the greatest common divisor is two.

Since it is a irreducible model, all the states are going to be of the same type, since it is a finite one is going to be a positive recurrent therefore both the states are going to be a positive recurrent and aperiodic sorry periodicity to and irreducible note that the one example which I have formulated the column sum is also 1, therefore it is a doubly stochastic matrix therefore you can use the previous result the example which I have given finite irreducible doubly stochastic.

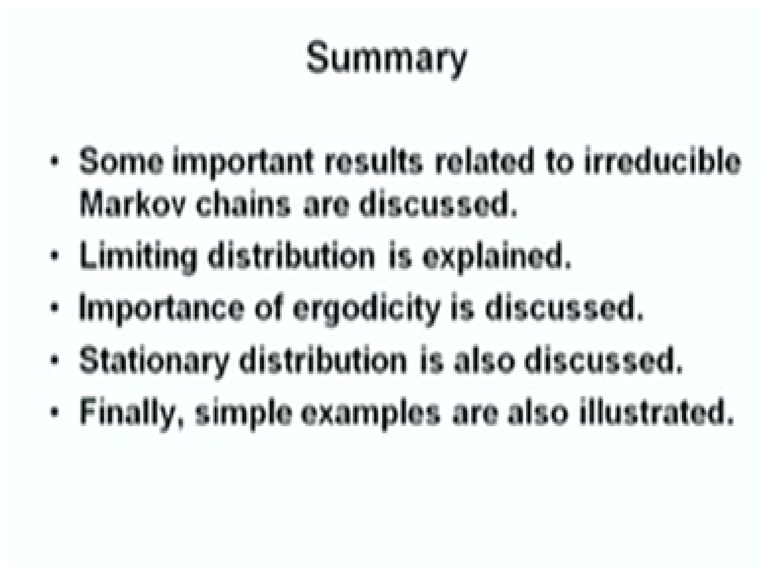
Therefore the stationary distribution exists, so if you solve pi is equal to pi P with the summation pi i is going to be 1, where pi is nothing but pi naught, pi 1 vector, so if you solve pi is equal to pi P with a summation of pi i, is equal to 1 you will get pi naught, pi 1, that is same as 1 by 2, 1 by 2, so this is the stationary distribution that exists and that value is a state probability stationary state probabilities are going to be 1 by 2, 1 by 2.

That means in a longer run the system will be in state 0 or 1 with the probability half, whereas if you try to find out the limiting state probabilities sorry, limiting distribution that means the limit n tends to infinity P of n, that means find out the n step transition probability matrix, then you make n tends to infinity this does not exist for this model. If you see the result which I have given the - the limiting distribution it is going to be exist and unique and so on.

There I have made, there I have not discussed the periodicity, there I have made it aperiodic, so here it is a periods period to model, so whenever you have an irreducible positive recurrent state if the periodicity is not a one that means it is not aperiodic model there is possibility the limiting distribution won't exist but still the stationary distribution exists. So this is an example in which and the limiting distribution does not exist whereas the stationary distribution exists.

But if the model is irreducible aperiodic positive recurrent then the limiting distribution exist as well as the sorry, stationary distribution exist as well as the limiting distribution exists and both are going to be same. Now I am going to give the conclusion.

(Refer Slide Time: 08:00)



So in this talk we have discussed some important results for the irreducible Markov chain, then I have discussed what is the meaning of a limiting distribution and I have given one example of how to compute the limiting state probabilities, then I discussed the ergodicity. Then I have discussed the stationary distribution, and how to compute the stationary distributions for irreducible aperiodic a positive recurrent whether it is a finite state or infinite state Markov chain.

I have given few examples and I have given example in which the stationary distribution exists whereas the limiting distribution does not exist and I have given some examples also with this I completed today's lecture thanks.