Stochastic Processes-1
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Lecture - 04
Discrete Uniform Distribution, Binomial Distribution, Geometric Distribution, Continuous Uniform Distribution, Exponential Distribution, Normal Distribution and Poisson Distribution

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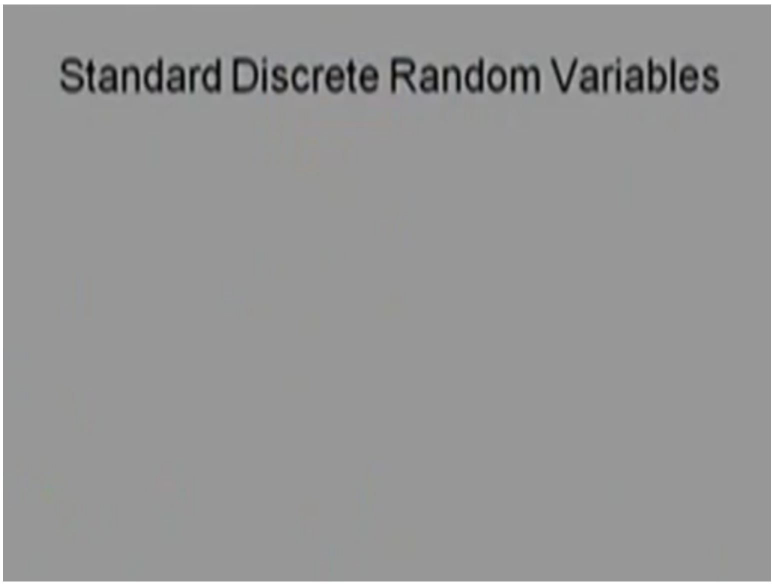

So here I am going to list out few standard discrete and continuous random variable. So these are all the standard one, we are going to use it in our course.

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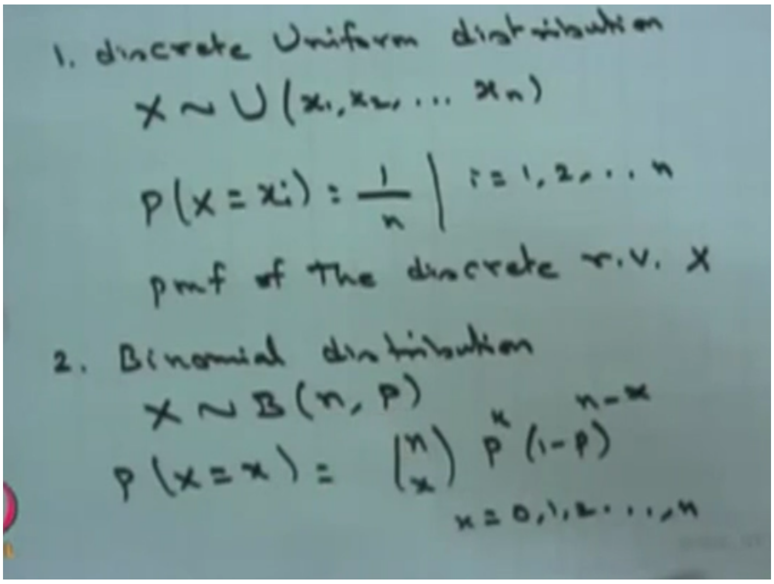

So the first one is discrete uniform distribution or the random variable is discrete uniform distributed random variable. Suppose I make the random variable x is uniformly, discrete uniformly distributed with the discrete point x m. That means that random variable takes the possible values x1 to x m and it has the masses at the xi of equal mass for i is varying from 1 to n and all otherwise it is going to be zero. Then in that case we say the random variable is going to be called it as a discrete uniform distribution.

That means it is going to satisfy the property the summation of all the excise as going to be one and the probability of x equal to xi is going to be greater than or equal to zero that means for this xi it is going to be greater than zero and all other point it is going to be zero. Therefore, it is satisfying the probability mass function of the discrete random variable, therefore this is the probability mass function of the random variable of the discrete random variable x.

So the probability of x equal to xi is going to be the probability mass function of the discrete random variable x. The second one, the discrete case, that is binomial distribution. When we say the random variable x is going to be call it as binomially distributed with the parameters n and p. Then the probability mass function for the random variable is going to be n c x p power x one minus p power n minus x., where x takes the value from 0, 1, 2 and so on.

That means this is the probability mass function of a binomial distribution, it takes a value 0 to n, that means it has the jump points n plus one jump points and this we call it as a binomial distribution, if we put n equals to one, then that is going to be the Bernoulli distribution random variable and here the p is nothing but the probability of success in each trials and you can create the binomial trials by having a n independent Bernoulli trials and each trial the probability of success is going to be p.

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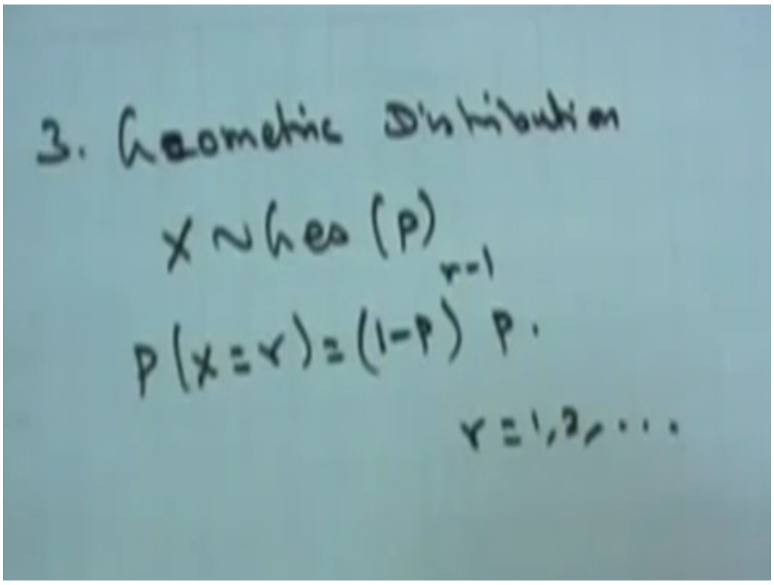

The third discrete random variable which we are going to use that is geometric distribution. When we see the random variable x is geometrically distributed with the parameter p, then the probability mass function of this random variable is going to be one minus p power r minus one multiplied by p, where r can take the value from 1 2 and so on.

That means if you have any discrete random variable and that random variable probability mass function is going to be of this form then we say that random variable is geometrically distributed with the parameter p and here the p can be treated as the probability of success in each trial and you can say what is the probability that, the r the trial getting the first success. That is same as all the trials are independent.

Therefore, you have a r minus one trials, you have the success subsequently, failure subsequently and you get the success first time in the rth trial therefore you will end up one minus p power r minus one for all such failure or all such non success, r minus one trials and first success in the rth trial.

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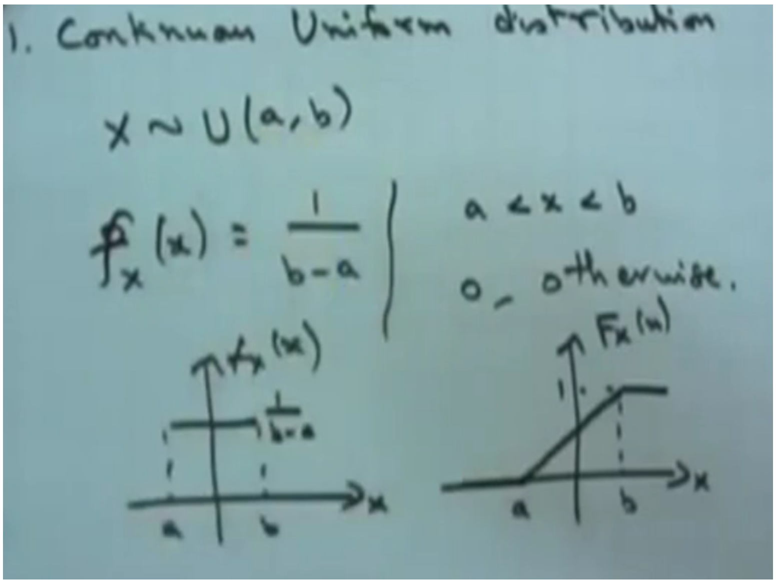

Next we are moving into the discrete continuous random variables. The first one is continuous uniform distribution, when we say the random variable x is continuous uniform distribution between the interval a to b then the probability density function for the random variable x is going to be of the form 1 divided by b minus a between the interval a to b and all other it is going to be zero.

That means the probability density function for this random variable is they have the height a and if you treated this as b and this height is one divided by b minus a, that means if you find out the integration between the range a to b of height one third b minus a then that is going to be one and this is going to be greater than or equal to zero always.

Therefore, this is going to be the probability density function of the continuous random variable and for any continuous random variable the probability density function is going to be one divided by length of the interval in which it takes the value one divided by this much and all other it is zero, then that random variable is going to be call it as a continuous uniform distribution between the interval a to b.

And If you see the CDF of this random variable till a, it is going to be zero and after a it is going to be increasing and at the point b, it reaches one. That means you can come to the conclusion if any random variable CDF is going to be between zero to one in the interval a to b with the standing line, then you can come out what is the point in which a and b and you can come to the, find out what is the random variable in which it is going to be continuous, it is going to be a uniform distribution between the interval a to b.

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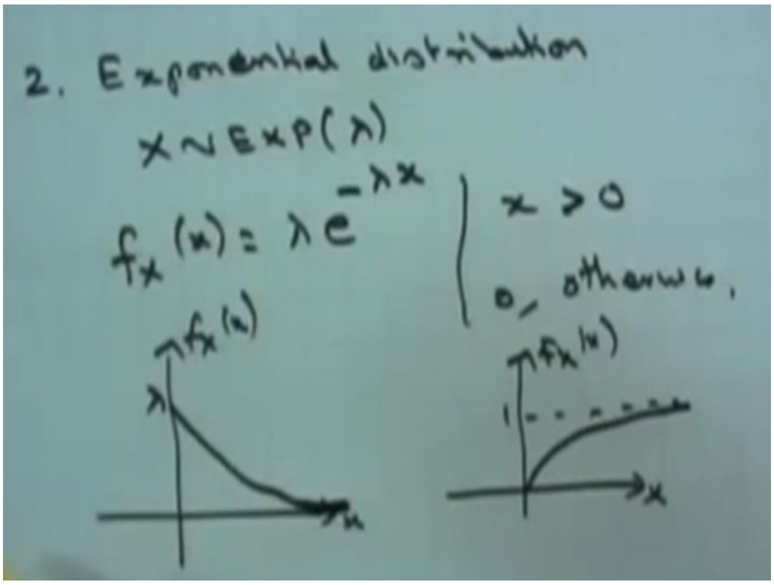

The second one is exponential distribution. When we say the continuous random variable x is going to be exponentially distributed with the parameter lambda, if the probability density function for that random variable is going to be lambda times e power minus lambda x and between the axis going to be greater than zero or it is going to be zero otherwise. That means within the range of 0 to infinity and the F of x is going to be lambda times e power minus lambda x, otherwise it is going to be zero.

So if you see the probability density function of that continuous random variable, it is going to start from lambda and asymptotically it touches zero. So this is the probability density function of the exponential distribution. And if you see the CDF of this, it reaches one at infinity. So this exponential distribution is going to be used in many of our problems later. Therefore, all the properties of the exponential distribution that I will discuss when we discuss the stochastic process in detail.

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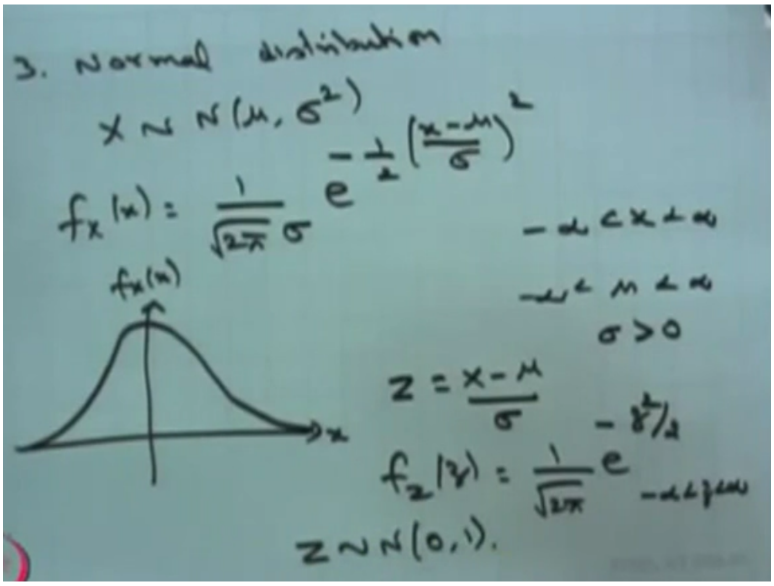

The third distribution is normal distribution or Gaussian distribution. So when we say the random variable is normally distributed with the parameters mu and sigma square, the probability density function is going to be one divided square root of 2 pi sigma e power minus half times x minus mu by sigma whole square. Here the x can lie between minus infinity to infinity and the mu also can lie between minus infinity to infinity and the sigma is a strictly positive quantity.

And the mu is nothing but the mean of normal distribution and the sigma square is the variance of normal distribution and sigma is the standard deviation. The standard deviation is always strictly greater than zero and if you see the probability density function of F of x asymptotically it starts. So I made it with mu is equal to zero and this is the probability density. So it looks like a bell shaped.

So this is going to be a normal distribution and you can always convert the normal distribution into the standard normal by using this substitution, z is equal to x minus mu by sigma. So you will end up with a standard normal, one that is one divided by square root of 2 pi e power minus z square by two where z lies between minus infinity to infinity. So this is going to be a standard normal distribution in which the mean is zero and the variance is one.

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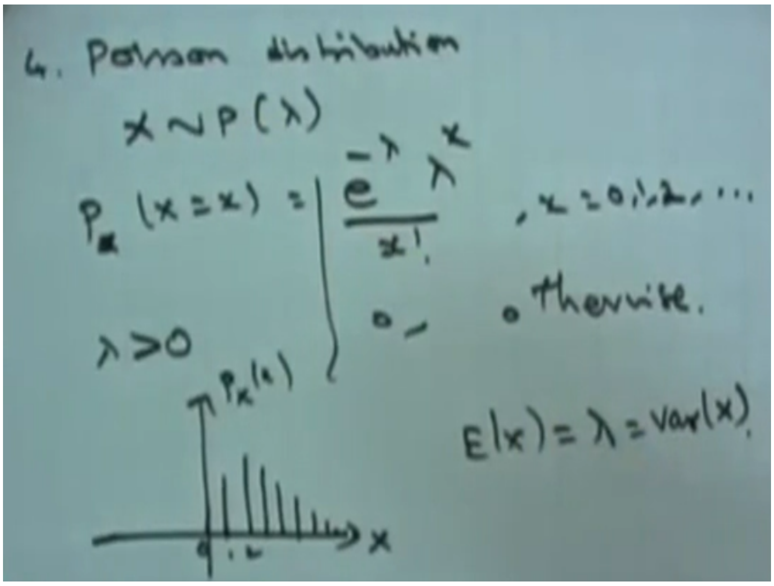

So other than the discrete standard distributions we have discussed only the discrete uniform distribution and second we discuss the binomial distribution and then we discuss geometric distribution. The fourth one that is very important that is a Poisson distribution. When we say the discrete random variable x is going to be a Poisson distribution with the parameter lambda.

If the probability mass function for the random variable x is going to be of the form e power minus lambda. Lambda power x divided by x factorial where x can take the value from 0, 1, 2 and so on. So that means this is the discrete type random variable in which it has the countably infinite masses and these are all the jump points and the masses are going to be e power minus lambda, lambda power x by x factorial.

Here the lambda is strictly greater than zero. That means if any discrete random variable has the probability mass function of this form then we can say that that random variably is Poisson distributed with the parameter lambda. And if you see the probability mass function, for the different values of x, so whatever the lambda you have chosen, so accordingly it is going to be at zero, it has some value and one it has some other value and two and so on.

So that means for fixed lambda you can just draw the probability mass function and this is going to have a countably infinite mass and if you add over the zero to infinity that is going to be one and the masses are going to be always greater than zero and all other points it is going to be zero. And this is going to be the very important distribution because using this we are going to create one stochastic process that is going to be call it as a Poisson process.

That means in the Poisson process, each random variable is going to be Poisson distributed. So for that we should know what is the probability mass function of the Poisson distribution and their properties and here the lambda is same as if you find out the mean for this Poisson distribution, the mean is going to be lambda and the variance is also going to be lambda. So this is the one particular distribution in which the mean variance is going to be same as the parameter lambda.

So in today's lecture what we have covered introduction of stochastic process by giving the motivation by giving four different examples to motivate the stochastic process. Then what we have covered is what is the probability theory, knowledge needed. In that I have covered only the probability space and the random variable and the discrete standard random variable as well as standard continuous random variables.

There are some more standard discrete random variables as well as there are some more discrete, there are some more standard continuous random variable that I have not covered here because it is a probability theory refresher and some of the distribution if it is needed then we will be covered at the time of, when we explain the stochastic process itself.

So therefore with giving few standard discrete random variable and few standard continuous random variables. I will complete today's lecture and the next lecture, I will cover some of the other probability theory concepts needed for the stochastic process that I will cover it in the next lecture. Then the third lecture onwards I will start the stochastic process. Thank you.