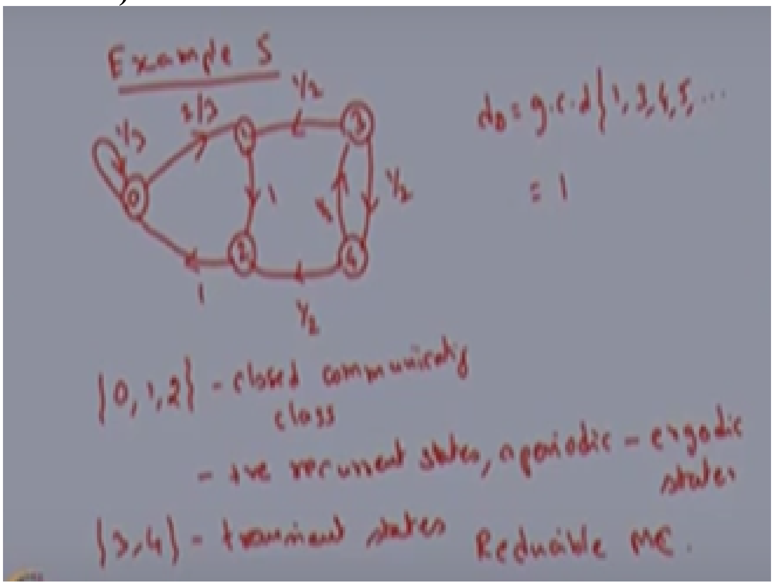
Stochastic Processes - 1
Dr. S. Dharmaraja
Department of Mathematics
Indian Institute of Technology - Delhi

Lecture - 39
Example of Classification of States (Contd.)

(Refer Slide Time: 00:02)



I am going to give the next example that is example 5, which has the finite states states 0 1 2 3 4 and this one step transition probabilities ,that is one-third, two-third and for the state 1 with the probability 1 it moves to the state 2, and for the state 2, with the probability 1 it moves to the state 0, for the state 3 with the probability of, it goes to the state 4, with the probability of it goes to the state 1, for the state 4 it is with the probability of it goes to the state of state 3, with the probability of it goes to the state 2.

The way I have drawn the state transition diagram by taking care the row sum is going to be 1 so you can equivalent lee have a one-step transition probability matrix also. So here I have only the state transition diagram for this DTM. From this diagram either by calculating I of I I and the capital FI, you can conclude it is going to be a recurrent state or transient state. Then you can conclude whether it is going to be a positive recurrent or null recurrent.

But whenever the Markov chain is going to be finite without doing the calculation, from the diagram you can conclude these states are going to be a positive recurrent and these states are going to be a transient state. So that I am going to do but at the same exercise you can do it and get the result also. The way the arcs are here, if you see the state at 3 and 4 states 3 and 4, it has the only outgoing arc to the state 1 and 2.

Whereas a 0 1 and 2 has a loop form, and the state 0 has a self-loop, with the probability one third. Sometimes if the outgoing arcs the probabilities are not going to be one that summation, that means you can make out the self-loop as the probability ,1 minus of all the outgoing arcs. But that is a default scenario but always we should draw the correct state transition diagram. If it has some positive probability with the self-loop.

We should always draw the self-loop with the positive probability. That is a correct way of drawing the state transition diagram. So now you can make out the state 0 1 and 2 are forming some sort of loop, that means if the system start from the state 0 or 1 or 2 it will be only within these 3 states over the number of steps. Even for a longer run, the system will be any one of these three states only, so these three states will be communicating each other.

Not communicating with the states 3 and 4. Whereas there is accessible from the state 3 to 1, but there is no accessible from 1 to 3, therefore 1 and 3 are not communicating states, similarly 2 and 4 or not communicating states, because one side accessible is here, not the other side accessible. Therefore, you can make a set 0 1 and 2 you cannot include any more states to form a set and this set satisfying the property closed as well as the communicating.

So, this set is called a closed communicating class. And all these three states are communicating each other and if you find out F00, F11, F22 so on, you come to the conclusion that value is going to be 1, and all these three sets are going to be a positive recurrent states. Whereas the states 3 and 4, if the system start from the state 3 or 4, it has the loop structure with the probability of, but with the probability of.

It can go to the state 2 or it can go to the state 1, via state 3 or via state 4 accordingly. Then land up the system is not coming back to the state 3 or 4. Once it is going away from the state 3 and 4 starting from these states it is not coming back. Therefore, these states 3 and 3 will form a transient state. Even though the state 3 and 4 are communicating each other, even though the state 3 and 4 are communicating each other this state will form a transient state.

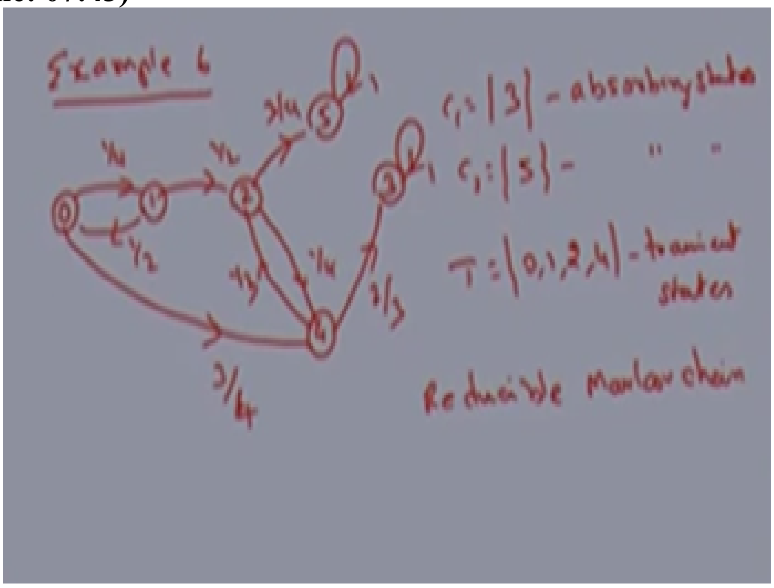
Because f33 and f44, it is going to be less than 1. And if you try to find out the periodicity of this state, you can find out the periodicity of any one state. Then that is going to be the period for all other states in the same class. Therefore, if you find out the periodicity of the state 0, that is d 0, that is the greatest common divisor of what are all the steps, the system will be come back to the same state.

So, either it can take 1 step or 1 2 3 4 or it can take only 3 steps, not making a self-loop 1 2 and 3. So it can make 1 or 3 steps or 4 steps or 5 steps. 5 steps means, it makes a 2 steps self-loop, then third step going from 0 to 1 and 1 to 2, then 2 to 0. Therefore, it is 5 steps and so on. Therefore, the greatest common divisor is going to be 1. So, since the period for the state 0 is going to be one.

This is going to be a periodic state. and all the states are going to be a periodic states. Since these states are the positive recurrent aperiodic, these states are also going to be call it as a ergodic states. These states are going to be ergodic states, since the state space s has the union of the closed communicating class and this a transient state, therefore this is going to be a reducible Markov chain.

So, in this example, we come to the conclusion, we have five states and this is going to be the reducible Markov chain, because of the closed communicating class is consist of element 0 1 and 2 and the transient states are 3 and 4.

(Refer Slide Time: 07:43)



Moving to the next example example 6. In this example, I have 6 states and the one step transition probability values are, one fourth and three fourth it goes to the state 4, and for the state 1 it is off and this is also off. For the state 2 with the probability three fourth, it goes to the state 5 and with the probability one fourth, it goes to the state 4. Whereas for the state 3 there is nothing.

For the state for with the probability one-third it goes to the state 2 and two third, it goes to the state. Since there is no outgoing arc in the state of five and three you can make out the self-loop has the probability 1. Or you can draw also with the probability 1. So now you can go for classifying the states, because of the self-loop with the probability 1 for the state 3 and 5, you can directly make out, the state 3 is going to be absorbing state.

And this is going to form a one class c-1, this close communicating class has only one element, which is state 3. Similarly, I can go for the second class, which has the only one element that is state 5. That is also absorbing state that is also absorbing state now I can go for classifying the state 0 1 2 and 4, because it is a finite state discrete time Markov chain, if the system start from the state 3 or 5, it will be in the state 3 or 5 forever, because both are observing state.

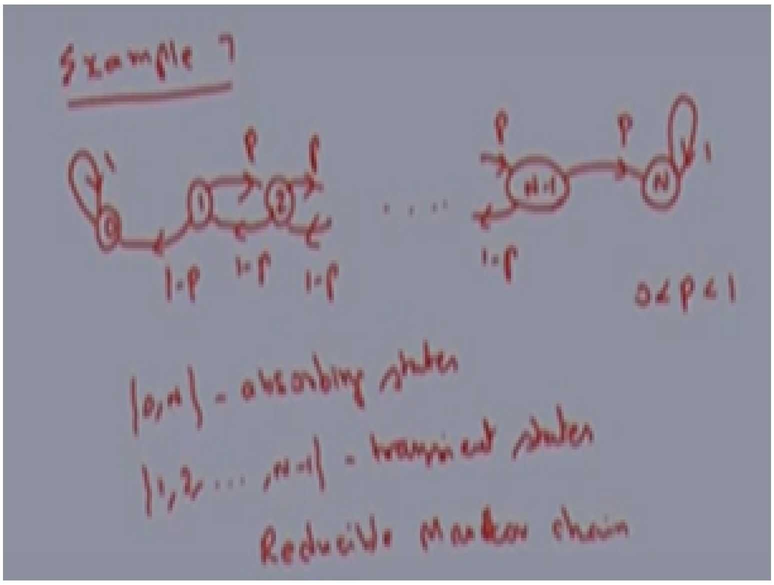
If the system starts from other than the state at 3 or 5, ultimately it comes to the state at 3 or 5, via these 2 to 5 or 4 to 3, then it wont be back. Therefore, all these states 0 1 2 and 4 will form a transient state. So, this is a collection capital T that is 0 1 2 and 4 are going to be form a transient states. I have not computed what is e of 0o f11 f 22 f 44, since it is a finite Markov chain, and these two states are going to be absorbing state.

Whenever the system starts from the state 0 or 1 or 2 or 4, either it will make a loop or ultimately land up to the state 5 or 3 with this arcs. Therefore, these 3 these 4 states are going to be a transient states and this will make a reducible Markov chain. Suppose the system start from zero or one with the arc 1 to 2 or 2 to 4, either the system can go either the system can go to the state 2, or state 4.

If the system start from the state is 0 or 1, either the system can go to the state 2 via 1 2 or state 4 via 0 4, then after that it will be keep roaming here 2 to 2 and 4. But with the positive probability three fourth and two third, it can go to the state five or state three, therefore these states are going to be absorbing state. Therefore, ultimately the system will land up the state three or five.

Therefore, this is a one type of reducible Markov chain, in which a you have a transient states and few absorbing states.

(Refer Slide Time: 12:57)



Now I am moving into next example, this is another type of reducible Markov chain, that is example 7 which has the finite states, which has the finite n plus 1 states and the transitions are like this, with the probability 1 minus P the system goes to the state 1to 0. With the probability with the probability P, it can go to the state 1 to 2. and this probability is 1 minus P and so on. So, all the forward arcs are P and the backward arcs are 1 minus P.

Whereas here this is P there is no forward or there is no forward arc therefore the state 0 and n is going to be observing States, so here the pecan lies between. Later I am going to explain this same DTM see for the gamblers ruin chain problem and here we have a n plus 1 states with the state 0 and 1 are going to be 0 and 1 are going to be absorbing states. We usually write absorbing states individually because each one will form a closed communicating class.

But here I have written both the states are observing states and all other states 1 to till n minus 1 those will form a transient States, because if the system starts from these states 1 to n minus 1, it can keep move between these states over the number of steps but with the positive probability of 1 minus P it can go to the state 0. With the positive probability of P can go to the state n, in this group of a transient states.

So, once the system come to the state 0 or n, then it will be far away, it is basically therefore we can come to the conclusion this is going to be the reducible Markov chain of the type transient States and the few absorbing states. So, with this let me stop the examples of classification of states. That means I have given the seven different examples in which it has the finite Markov chain as well as the infinite Markov chain.

And a few Markov chains are the reducible, few are irreducible, in a reducible Markov chain we have made a two three types of reducible Markov chain, that also I have explained. In the next class, we will discuss the limiting distribution and the stationary distribution. Thanks.