Stochastic Processes -1
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Lecture - 38
Example of Classification of states(Contd.)

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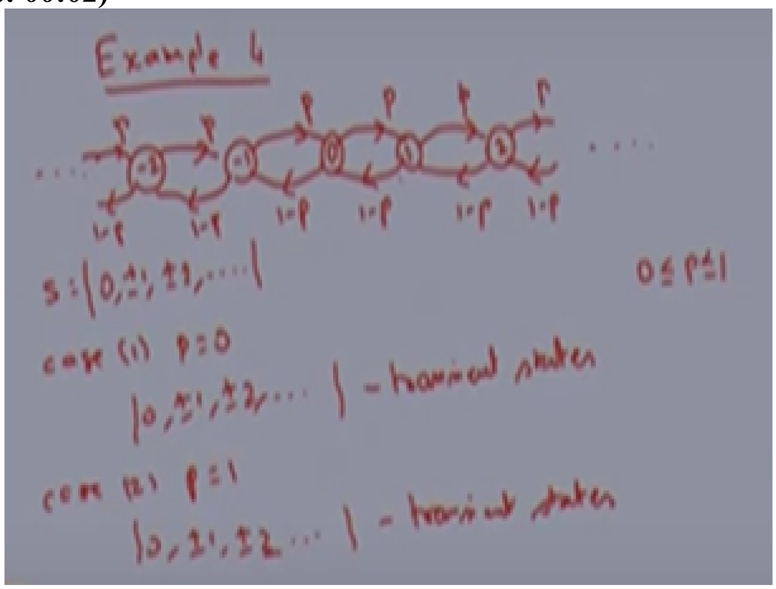

Now we are moving into the fourth example. This has the infinite number of states. Suppose the state space, let me draw the state transition diagram, state 0, 1, 2 and so on. The left-hand side it has the states minus 1, minus 2 and so on. So, the state space of this Markov chain has a countably infinite number of elements with the states 0, plus or minus 1, plus or minus 2 and so on. Let me draw, give the transition rates, transition probabilities.

So, the system is moving from state 0 to 1 with the probability P and system is moving from state 0 to minus 1 with the probability 1 minus P. Therefore, if you see the state transition diagram the state one state transition probability matrix the whole sum is going to be 1. So, you keep P is lies between P can lies between 0 to 1. Similarly, you go for the all other states. The system is moving from the state to the forward one state that is with the probability P backward state with the probability 1 minus P.

With the forward is P and coming back to the one step less, one state less that is 1 minus P so this is the way it goes for all the states. 1 minus P and this is one P and you have a countably infinite number of states. Fine. Now let me go for the case 1 in which the P is going to be 0. Suppose P takes the value 0 what happens, or how to classify the states when P is equal to 0 in this time. When P is equal to 0 there is no forward arc.

When P is equal to 0 implies the system is always go to the one state one step less, one state less with the probability 1. Because P is equal to 0. Therefore, you should able to visualize what is the state transition diagram corresponding to P is equal to 0, there is no forward arc arrows. That means whenever the system starts from some state it will keep on going to the one state less in every step and you can visualize for a longer run where the system will be.

Whether it will be in the positive side or in the negative side. You can visualize whenever the system starts from any finite state over the period it may be in some state with some positive probability for the finite number of steps or for an infinite number of steps or for a longer run the system will be in the negative side, for a longer run. So that is the limiting distribution. But here we are discussing the classification of the states.

Therefore, with the probability 0 it will not be back at all, if the system starts from any state, it will not be back to the same state with the probability 0, Therefore all the states are going to be, all the states are going to be the transient states. If you calculate a F for you take any finite states one or something then F 11 of 1, F 11 of 2 and so on if you calculate then you may land up a F 1, 1 capital one that is going to be always less than one.

Therefore, if you start with one state you can conclude it is a transient state and all other states also of the same way, therefore all the states are going to be same. Suppose we discuss the case 2 with the P equal to 1 what happen. If P equal to 1 then you have all the forward arcs not the backward arcs, that means whenever the system starts from any state then the system will go to the forward all the states in subsequent steps in the probability 1.

In a longer run the system will be in the positive side, positive infinite side in a longer run therefore with the probability 0 it will be in any one of the finite states in a longer run. whereas for the any finite steps the system will be in some of the states and it will be keep moving forward state over the number of steps. Therefore, here also you land up all the states are going to be transient states. Both the two cases the situation for the limiting distribution may change.

One is in the left side the other one is in the right side, whereas all the states are going to be the transient states. But our interest is for the P is lies between that is our third case our interest is P lies between 0 to 1 open interval that means if you see the previous state transition diagram you have a both the forward arcs as well as the backward arcs. Because the probability P lies between open interval 0 to 1 therefore 1 minus P is also lies between 0 to 1 in the open interval.

Therefore, whenever the system starts from any state it will come back to the same state with the even number of steps. Suppose you visualize it the state 1 it can come back to the same state 1 not in the odd number of steps but in the even number of steps, suppose if the system moves to state 0 in the first step and in the second step it can come back to the state 1. Similarly suppose the system would have move from 1 to 2 then in the second step it would have come to the state 1.

Therefore, it is two steps it can come back either via going to the state 0 or going to the state 2. Suppose you go for think of four steps it coming back to the same state that is possible need not be the first visit means it can make two times look the left side or it can make two times look in the right-hand side or it can make a one step forward and one more step forward then it can come back.

Therefore, all the possible steps if you include all the possible steps you will come to the conclusion it will take a even number of steps to come back to the same state. So, if you do the simple exercise what we have done it in the earlier case you can come to the conclusion.

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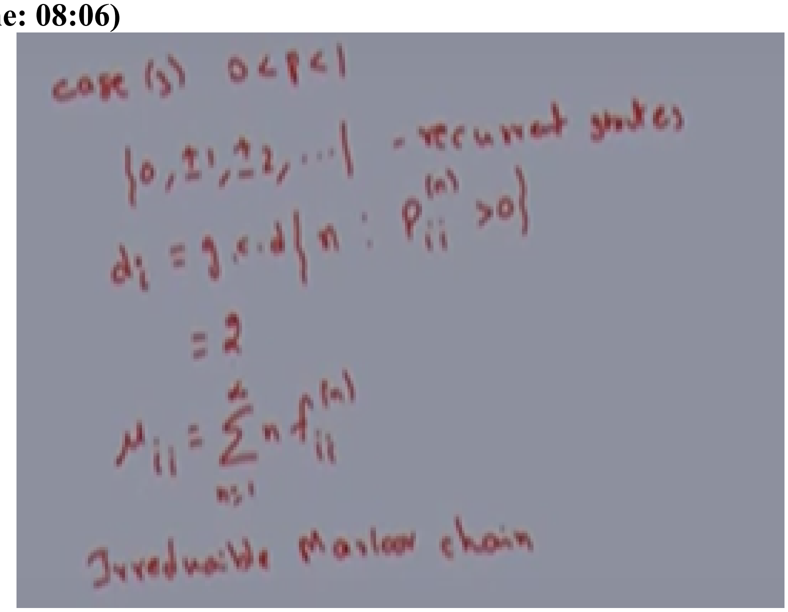

Zero plus or minus One, plus or minus Two and so on. All the states are going to be a recurrent state. Without fixing the value p you can conclude Fii capital Fii is going to be one for all the states. Therefore, now you come to the conclusion all the states are going to be the recurrent state if you try to find out the periodicity for any state.

The way I discuss a greatest common divisor of coming the greatest common divisor of n such that Pii of n which is going to be greater than 0 and this is possible for all the even number of steps therefore the system will be come back to the same state, 2 step 4 step 6 step and so on therefore the GCD is going to be 2 for this Markov chain. So, the period is going to be 2 and the recurrent state. Now our interest is whether this states are going to be a positive recurrent, or null recurrent.

But for that you need what is the value of P because without P without the value of P you cannot come to the conclusion with the mu suffix ii that is going to be the n times Fii of n you need the value but some example it is not it is possible but still by supplying the value of P or what is the range in which you can conclude whether this is going to be finite quantity or going to be infinite quantity.

Based on the range of P you can conclude these recurrent state is going to be a positive recurrent or null recurrent. Since the states space is going to be 0 plus or minus 1 plus or minus 2 and so on. And all the states are going to be a recurrent state it will form a one close communicating class both are all the states are communicating with each other therefore you land up having a only one close communicating class which is same as the state space.

Therefore, this is going to be a irreducible Markov chain. This state may be a positive recurrent or null recurrent based on the range of P but here we are just concluding this is going to be an irreducible Markov chain with all the states are going to be a recurrent with the period 2. So, since the period is 2 it will not be an ergodic states also. If you want ergodic state you need a positive recurrent as well as the periodic, since the period is 2 you can conclude this is not going to be the ergodic state.