Stochastic Processes - 1
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Lecture - 37
Example of Classification of state (Contd.)

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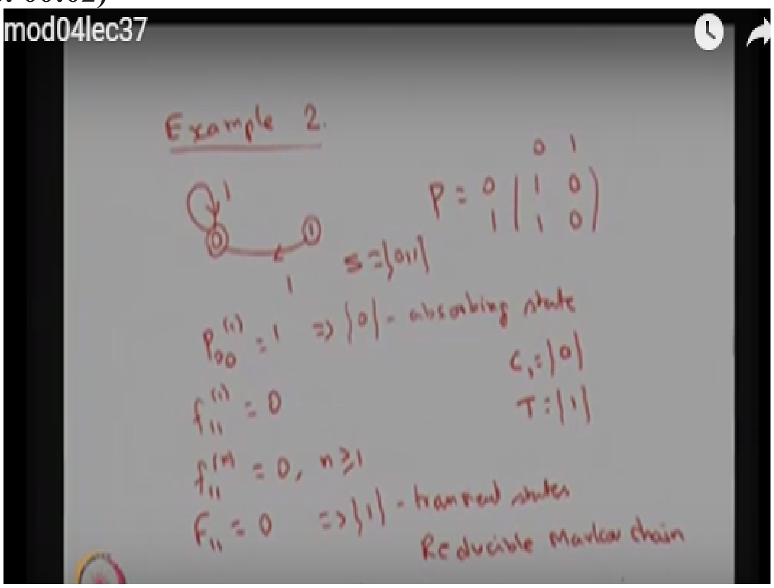

Example 2, here I am going to discuss Irreducible Markov Chain. Here also we have an only states. The Probability of system is moving from state 0 to 0 in the next step, the probability is 1 and the system is coming from the state 1 to 0 and in 1 step that probability is 1. So, this is the state transition diagram of a Time Homogenous Discrete Time Markov Chain. So, I am going to write what is the 1 State Transition Probability Matrix for this state transition diagram.

Or for this Discrete Time Markov Chain. So, 0 to 0 1 step that probability is 1, 0 to 1 is 0, 1 to 0 is 1, 1 to 1 is 0. You can verify whether this is going to be Stochastic Matrix because each element are lies between 0 to 1 and the row sum is 1. Therefore, this is a Stochastic Matrix so both are equivalent in the state transition diagram and 1 state probability matrix is 1 and the same.

Now we will try to find out what is the Classification of the states Go for the state 0. The P 0 0 of 1 that is 1, that is 1 step a transition of system is moving from state 0 to 0 that is going to 1. This implies the state 0 is a absorbing State. Now we will try to find what is a Classification of a state

1. So, if you find out f 1 1 of 1 what is the Probability that the system will come to the state 1. Given that it was in the state 1 and the first time we see to the state 1.

Exactly the first step so that is going to be not possible because if the probability is 1. It moved to the state 0 therefore this is going to be 0. And if you find out f 1 1 of 1, all the subsequent steps also that is also going to be 0 because if the system start from the state 1 in the next step itself it goes to the state 0 with the Probability 1 and it is not coming back. Therefore, now you tried to find out what is the capital F 1 1 that is nothing but the summation of all the F I's and that is going to be 0.

If you recall the way you classify the state is going to be recurrent or transient, we said f i i is going to be 1 or f i i is going to be less than 1 so that less than 1 includes f i i is equal to 0. So basically, our interest is to classify whether there is Proper Distribution, the system is coming back to the same state with the probability 1 that is f i i is equal to 1 and all other things we say that a Transient State it includes f i i equal to 0.

So here if the probability 0 the system is not coming back to the state 1, if the system starts from the state 1. This is always a Conditional Probability and this Conditional Probability f 1 1 is equal to 0 implies the State 1 is going to be a Transient State. So, whenever any state i, f i i is equal to 1 that concludes the state is going to be recurrent state and whenever the f i i is lies between including 0, excluding 1 that is less than 1, then that state is going to be called it as a Transient State.

Since we have only 2 states that the state space is 0 and 1 and you land up having a 1 Observing State and 1 Transient State. Therefore, the state space is partition into 1 closed communicating class which has only 1 element and the Transient state is 1. Therefore, I can say the state space S is a partition into closed communicating class C 1 which consist of only 1 element and the collection of all the Transient States that is only 1 element.

So, this a notation for Capital T collecting all the Transient States in the States space in the D T M C and C 1 is the first closed communicating class and which has only 1 element. If any closed communicating class have only 1 element then it is going to be call it as Observing State. Therefore 0 is a Observing State and 1 is a Transient State. Since you have a C 1 Union T become state Space S. Therefore, this Markov Chain is not an Irreducible Markov Chain.

Therefore, this is called a reducible Markov Chain. Whereas the previous example is a Irreducible Markov Chain. There we have 2 elements and both the elements form a only 1 Closed Communicating Class. Whereas here you have 1 closed communicating class with 1 element and the Transient State is 1. Therefore, it is going to be a Reducible Markov Chain. There can be more than 1 Transient State.

Now I am moving into the third example so that I am explaining some more concepts through the examples.

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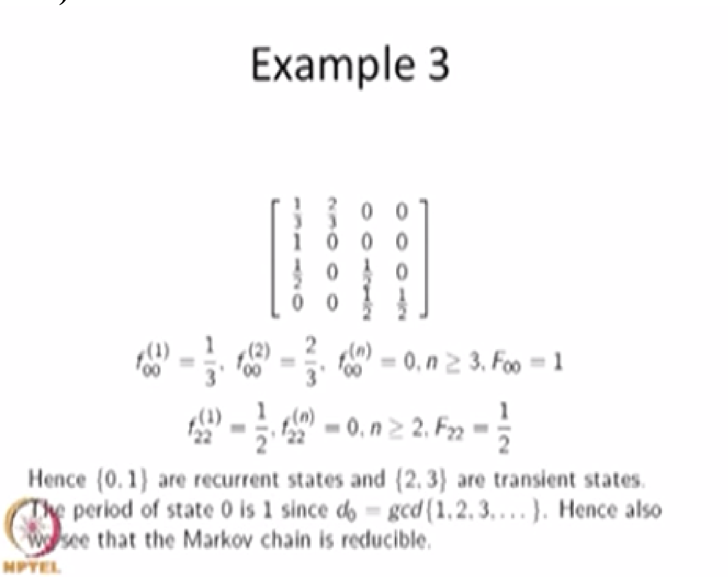

Example 3, Here I go for 4 states which consists of states 0,1,2 and 3. It is easy to explain through the State Transient diagram than the 1 State Transient Probability Matrix. So, I am just drawing the State Transient diagram for this DTMC. So, 0 to 0 1 step that probability is 1 third and 0 to 1 is 2 third. Therefore, row sum is taken care, that summation of probability is 1. Now I am going for the state 1. State 1 to 0 that probability is 1 therefore that row is taken care.

Now I am moving to the State 2, state 2 the self-loop has the probability of and going from the state 2 to 0 that probability is half. Therefore, this is row is also taken care. Now I am moving to the state 3. State 3, it has the Self-loop with the probability half and it has the moving from the state 3 to 2 that probability is half. My interest is to classify the states for this Markov Chain. Markov Chain has 4 states 0, 1, 2 and 3 that the state Space Capital S.

Now we will start with the State 0. So, you find out what f 0 0 of 1 in 1 step has a first visit. The system has to come back to the same state 0. So that probability is 1 third. you find out f 0 0 of 2, f 0 0 of 2 exactly 2 steps as a first visit you have to come to the state 0. That means, you can go to the state 1 while starting from state 0 and come back to the state 0 in the next step. Therefore, it is 2 third into 1. Therefore, it is going to be 2 third.

Then you go for what is the possibility I will take 3 steps exactly 3 steps coming to the state 0. As a first visit it is not possible whereas the p 0 0 of 3 is possible. f 0 0 of 3 is not possible because in 3 steps you cannot make a first visit. Therefore, that is going to be 0 not only f 0 0 of 3 and for all other things also it is going to be 0. f 0 0 of n is equal to 0 for n is greater than or equal to 3. So now I can find out what is the capital F 0 0.

If I find out capital F 0 0, I have to add all the values so it is 1 third plus 2 third plus all the further terms are 0. Therefore, it is going to be 1. Since f 0 0 is equal to 1. You can conclude the state space 0 is the state 0 is going to be the Recurrent State. The similar exercise you can do it for the state 1, the same way you can conclude f 1 1 is also going to be 1. The other way, since the state 1 is communicating with the state 0. Therefore, this is also going to be of the same type.

Therefore, the state 1 is also going to be the Recurrent State. Now we can go to the State 2. So, the state 0, 1 that is going to be Recurrent State. Now I will move it to the State 2. So whereas the state 2 if you find out f 0, f 2 2 of 1 in 1 step coming back to the same state that is going to be half, f 2 2 of 2 steps exactly 2 steps that is not possible that is going to be 0 and so on. Not only 2 and all the further steps also going to be 0.

Because with the probability half, I takes only 1 step come back and all the further steps it takes with the probability half, it is not coming back at all. Therefore, this is going to be for greater than equal to 2, it is going to be 0. Therefore, if you compute capital F 2 2 then that it is going to be half plus 0 plus so on. Therefore, you land up half which is less than 1. Therefore, you can conclude the state 2 is going to be Transient State.

Not only the state 2, if you do the similar exercise for the state 3. The same thing, you may land up f 3 3 is also going to be less than 1 whatever be the number. You can conclude the state 3 that is also going to be the Transient State. You can find out Periodicity for the Recurrent State only not for the Transient State. Therefore, now you can try to find out what is the Periodicity for the state is 0 and 1.

Before that we will try to find out what is the type of Recurrent State whether it is going to be Positive Recurrent or Null Recurrent If you out Mu 0 0 that is nothing but 1 times 1 third, 2 times 2 third, 3 times 0, 4 Time 0 and so on. So, if sum it up everything you may land up 1 times 1 third plus 2 times 2 third that is going to be 1 third plus 2 times 2 third so that is going to be 3, 1 plus 4 that is 5 so which is a finite quantity.

You can conclude the state 0 is going to be Positive Recurrent. Similarly, if you calculate f Mu 1 1 also you may land up with the finite quantity. So, you can conclude both the states are going to be a Positive Recurrent States. Here, the states space is classified into 2 Positive Recurrent State and 2 Transient State. Therefore, this Markov Chain is going to be a Reducible Markov Chain. In short form M C.

It is a Reducible Markov Chain because the whole state space capital S is partition into 1 close communicating class which consist of the state 0 and 1. And the Transient States 2 and 3. Therefore this is going to be a Reducible Markov Chain. You can find out the Periodicity of these 2 Recurrent State also. So, if you find out d 0 that is going to be greatest common divisor of, what are all the steps in which the system will be come back if the system starts from the state 0.

So, either it can come back with the 1 step or either it can come back with the 2 steps or it can make a 1 loop here then 1 loop then here. Therefore, it can come back from the 3 steps and 4 steps and so on. Its need not be the first visit. Therefore, the gcd of 1 step or 2 steps and 3 steps and so on. Therefore, this is going to be 1. That means it is a Periodic State. Therefore, whatever we have done it for state 0. You can do it for the state 1 also so that also going to be 1.

A period is going to be 1. Therefore, both the states 0 and 1 are the Positive Recurrent and a Periodic States and other 2 are going to be Transient States. Since this state 0 and 1 are going to be Positive Recurrent as well as Periodic. And these 2 states are Ergodic States. Later we are going to explain Ergodicity the Property.

For that property, you need to understand what is Ergodic State so whenever the Markov chain has few states going to be a Positive Recurrent and a Periodic then those states going to be called it as a Ergodic States. Later I am going to give the definition of Ergodicity and so on.