Stochastic Processes - 1
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Lecture - 36
Introduction and example of Classification of states

Good morning. This is model 4 of a stochastic process video course. In this we are going to discuss the limiting distribution and stationary distribution in the lecture 4. In the last 3 lectures, we have discussed the time homogeneous discrete Markov chain and in the last lecture, that is on lecture 3 we have discussed classification of states concepts and definitions, but we have not discussed the simple examples for that.

So, in this lecture, I am planning to explain, I am planning to give few examples of classification of the states, then I am going to give the definition of limiting distributions then followed by stationary distributions, then the same examples I am going to explain how to get the stationary distribution if it exists. So, if you recall our earlier lecture, that is our lecture 3, we have given the lot of concepts through those concepts you can classify the states.

The state has a transition state or a current state then the recurrent state can be classified into the positive recurrent state and then null recurrent state and you can find out the periodicity of the states and if the period is going to be 1, then we say that state is going to be the a periodic state. And if any state is going to be a positive recurrent and a periodic then we say that state is the ergodic state.

If one step transition probability, if Pi a is equal to 1, then that state is going to be called as a absorbing state. And, we have discussed irreducible Markov chain that means the whole state space is not able to partition into more than one closed communicating classes, then that is going to be closed. That is going to be called as a irreducible Markov chain, otherwise it is a reducible Markov chain.

Now I am going to give simple example through that we are going to explain the classification of the states.

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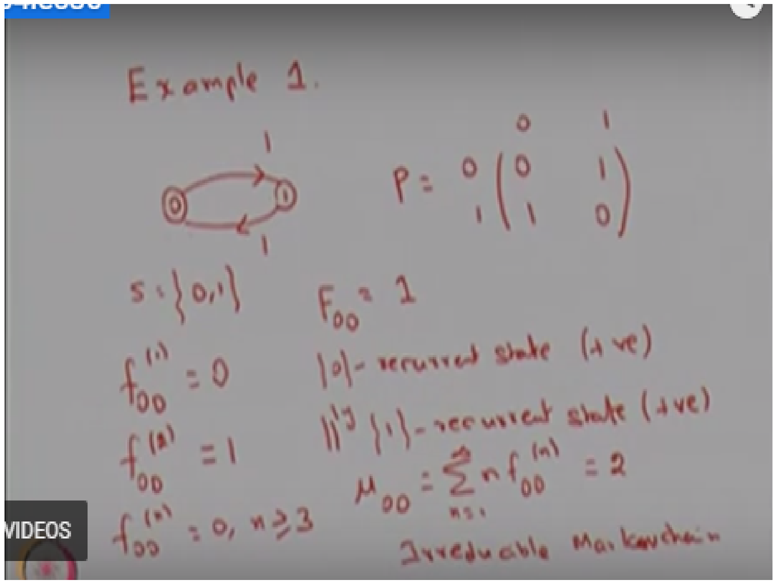

The first example the simplest one. In this first simple example we have a only two states, so the states space contains only two elements 0 and 1. The transition, the one step transition probability, from the system is moving from straight 0 to 1. that probability is 1. And the system is moving from the state 1 to 0 that probability is also 1. So, the one step transition probability matrix can be obtained from the state transition diagram, both are one and the same.

So, this is the one step transition probability matrix and this the state transition diagram, both are one and the same. So, 0 to 0 that probability 0, 0 to 1 that probability is 1, 1 to 0 that probability is 1, and 1 to 1 is 0. Now we can find out whether these states are going to be a recurrent state or transient state. If you recall to find out the recurrent state or transient state, you have to find out what is the FII.

So, we start with the state 0, so if you try to find out f00 of 1, what is the probability that, if the system starts from the state 0 and reaching the state 0 in exactly first step, for the first time. Then that probability is not possible. That is equal to 0. If you try to find out f00 of 2, first visit to the state 0. Given that started in the state 0 exactly in the second step it reaches the state 0, that is a possible because by seeing the state transition diagram you can make out that the first time.

The system is moving from 0 to 1 and 1 to 0 it is possible coming back to the same state, taking exactly two steps, for the first time. Therefore, F 00 of 2 that probability is 1 and by seeing the state transition diagram, you can visualize, since it comes to the same state, exactly second step. Therefore, all the further steps, for the first time that is not possible. Therefore, all the f00 of n, that is going to be 0, for n is greater than or equal to 3.

For n is greater than or equal to 3, the f00 of n, is equal to 0. Now if you try to find out what is capital F 00 that is a probability of ever visiting to state 0, starting from the state 0, that is going to be the summation of f 00, super script we can bracket n, for all n vary from 1 to infinity if you sum it up, then that is going to be 1. Since F 00 is equal to 1, you can conclude the state 0 is the recurrent state. You can conclude this state 0 is the recurrent state.

Similarly, if you do the same exercise for the state 1, by starting with f11 of step 1 what is a probability, f11 of step 2 what is a probability and f11 of all the ends and find out the summation, so you land up f11 is also going to be 1. We can conclude similarly the state 1 that is also recurrent state. Here after finding the recurrent state, now we can find whether this a positive or null recurrent state.

For that u have to find out what is the mean recurrent state or mean pass each time. So, find out what is Mu 00, that is summation N, F i i of n, n varies from 1 to infinity. Here the i is nothing but 00, n times i of 00 of n, because this takes the value of n 00 of 2. Therefore, you will get two times 1 and all other quantities are 0, therefore this is going to be 2. And this is going to be a finite quantity.

Therefore, u can conclude state 0 is the positive recurrent state. The same exercise you can do it for Mu 11, that is also you may land up getting 2. Therefore, you can come to the conclusion state 1, that is also positive recurrent state. So, in this finite discrete time Markov chain you have 2 states. And both are positive recurrent state. And both are communicating states, therefore you have a class that has the two states and the state space is also 0 and 1.

And the closed communicating class is also 0 and 1. Therefore you are not able to partition the state space, into more than one communicating class and so on therefore, we land up this Markov chain is going to be, this Markov chain is going to be the irreducible Markov chain. This Markov chain is irreducible Markov chain, because the state space has only 2 elements. Both the elements are, both the states are communicating each other.

Ad we land up only one closed communicating class, therefore this is going to be irreducible Markov chain. We can find out what is the periodicity of these states also. You can find out the periodicity for the state 0, by evaluating the 0, that is nothing but, what is the greatest common divisor of all possible steps, in which the system is coming back to the same state. So, if we find out the system can come to the same state.

If you see the state transition diagram if the system starts from the state 0 coming back to the same state either by 2 steps or 4 steps or 6 steps and so on. You should remember that when you are trying, when you are finding the periodicity, you are finding the number of steps, coming back to the same state, not necessarily the first visit. Whereas the f00 f00 of n, to conclude it is a recurrent state you are fine using the first time teaching that state.

In the exactly and it is tough, so there is a difference. So, the gcd of all the possible steps, in which the system is coming back to the same state. So, it can come back to the same state 0 in 2 steps or 4 steps or 6 steps and so on. so, the gcd is going to be 2, that means the period for the state 2, sorry the state 1, the state 0, period for the state 0 is 2. Similarly, you can find out what is the period for the state 1 also, if you do the same exercise.

But seeing this diagram, you can make out the state 1 also going to have the gcd of 2,4,6,8 and so on. Therefore, the period for the state 1 also going to be 2. Otherwise also we can conclude, both are communicating states since the period for the state 0 is 2 and since the state 1 communicating with states 0, that means it is accessible in both ways. Therefore, the state 1 is also having the same state, same period.

In conclusion you can make out, if you have a 1 class, with more than 1 states, then all the states are going to have the same period.

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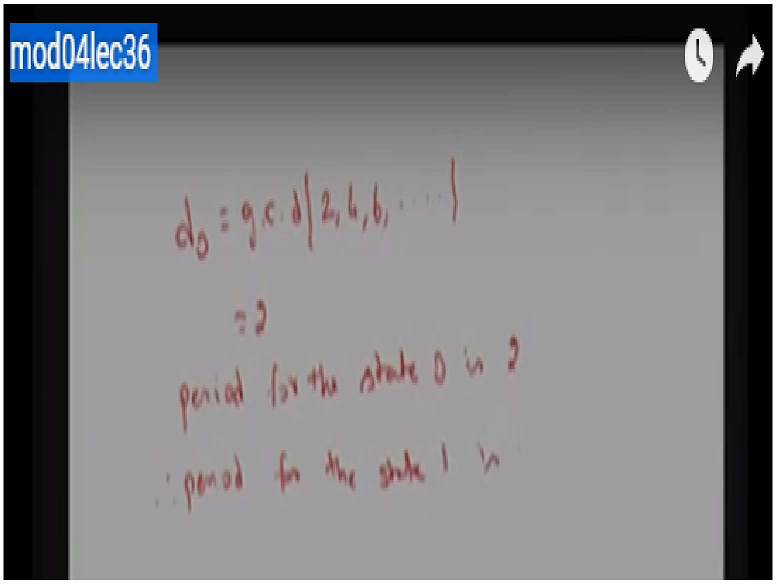

Therefore, the state 1 is also have the period for the state one, that is also.

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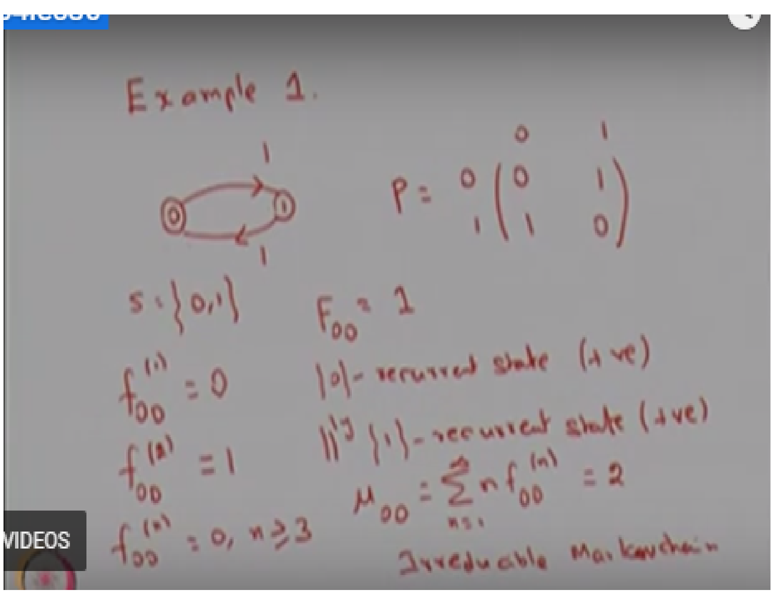

That means this example, you have only two states and this is an irreducible Markov chain and both the states are positive recurrent with the period 2. So that is the way using the classification of the states will come to the conclusion of this particular example. Later we are going to find out the limiting distribution and stationary distribution and so on, but for that we need the classification. Here also we can visualize, where the system will be for a longer run, if the system starts from the state 0 or 1.

You can visualize, because it is only two state, by seeing the state transition diagram you can make out, suppose the system start initially in the state 0, at every even number of steps, it will be come back to the stage zero, in a longer run based on the number is going to be even or odd, accordingly the system will be in any one of the states. Similarly, in a longer run, you can make out, if the system starts from the state 1 initially.

All the even number of steps it will be come back to the same state 1, and all the odd number of steps it will be in the state 0, in the longer run also it is going to be happen the same way for even n and odd n, accordingly the system will be in any one of the states. In a longer run also, the system will be any one of these two states only, because it is a irreducible Markov chain. Because these two states are communicating each other.

Therefore, in a longer run, the probability that the system will be in any one of these states, will be some value and only the system will be in any one of these two states only. Later I am going to give the definition of the limiting distribution, through that I am going to explain the same example again.