Stochastic Processes-1
Dr. S. Dharmaraja
Department of Mathematics
Indian Institute of technology – Delhi

Lecture - 35
Recurrent State and Transient State

Now we are going for the actual classification of a state using the concept of accessible communicate closed set then communicating class then we have defined a first visit then we have defined a Mean Passage Time or Mean Recurrence Time or Mean First Passages Time. So using these concepts we are going to going to classify the states.

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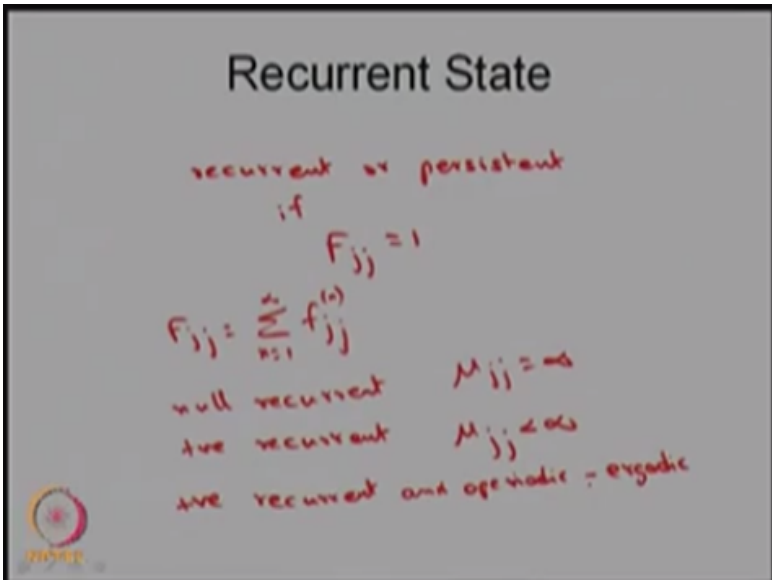

The first definition is a Recurrent State. A state J is said to be recurrent or the other word called a persistent if the F suffix JJ equal to 1. If you recall what is FJJ. FJJ is the probability of ever enter to the state J given that it was in the state J. So the FJJ I have given in the summation form of a small Fjj of n using the first visit. So, if you recall the FJJ is nothing but what are all the possible ways the system can reach the state J as a first visit.

You add all the combination, all the probabilities that is going to be the capital FJJ. So if capital FJJ that means the probability of returning to the same state J if that probability is certain that means if the probability is 1 then that state is going to be the recurrent state. We can classify the recurrent state into two form. One is called the null recurrent. And the other one is called a positive recurrent based on the mean passage time value.

So based on the capital FIJJ that is the probability we classify the state is going to be a recurrent state now based on the first passage time distribution the mean first passage time we are going to classify that recurrent state is going to be a null recurrent or positive recurrent accordingly Mu JJ if it is a finite value then we say that recurrent state is going to be positive recurrent state.

If a Mu suffix JJ is going to be an infinite value that means on average the first passage time is going to be infinite, then that corresponding recurrent state is going to be call as a null recurrent state. So whenever any state is going to be call it as a recurrent state if the probability of ever entering into the state J starting from the state J it is certain or the probability is 1 then that is the recurrent state.

And the recurrent state is going to be called it as a null recurrent if the mean first passage time or mean recurrence time or mean return time is infinity. If that is going to be a finite quantity, then the recurrent state is going to be call it as a positive recurrent state. If any state is going to be a positive recurrent as well as a periodic then that state is going to be call it as a Ergodic state.

Any state is going to be call it as a Ergodic whenever that state is a positive recurrent as well a periodic. The periodic means the periodicity of that recurrent state is 1 that means that the greatest common divisor of all positive steps in each of the system coming to the same state that value is 1. If the period is 1 and as well as the positive recurrent it should be recurrent as well as positive recurrent that means the mean recurrence time is going to be a finite quantity, then it is going to be call it as a Ergodic state.

In a Markov Chain if all the states are going to be a Ergodic one that means all the states are going to be as a positive recurrent as well as a periodic then we call that Markov Chain itself Ergodic Markov Chain. That means there is a possibility the Markov Chain maybe a irreducible that means you will end up with one class in which all the states are going to form a one closed communicating class.

Suppose if one state is going to be a positive recurrent and a periodic then all other states are also going to be of the same type and the same period therefore all the states are going to be Ergodic states then that Markov Chain is going to be call it as a Ergodic Markov Chain.

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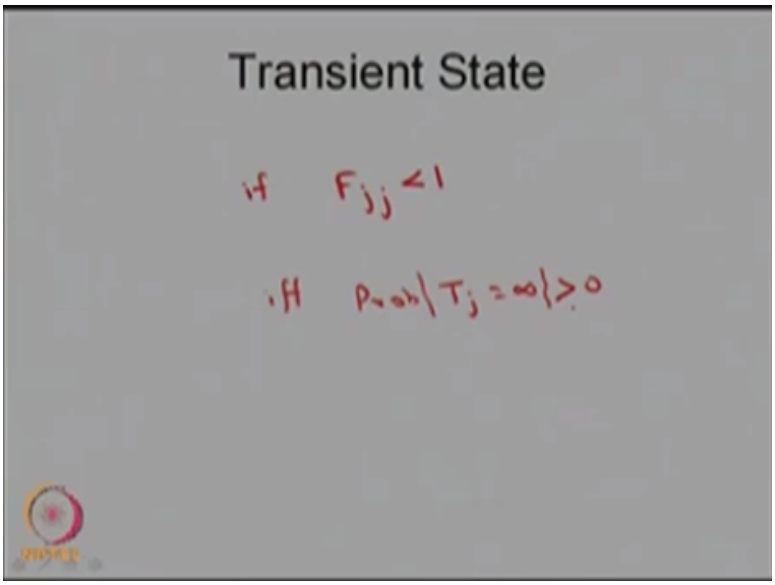

Now I am going to classify the state as a transient state whenever the FJJ value is less than 1. If we recall, we have considered only two cases whether the FJJ is less than 1 or FJJ is equal to 1. Equal to 1 land up recurrent state and FJJ is less than 1 that gives a transient state. That means the probability of returning to the state J starting from the state J is not certain that means one minus of this probability with that much probability.

The system may not return to the same state J if the system starts from the state J that means with some positive probability because 1 minus this values this value is less than 1 therefore 1 minus of FJJ is going to be greater than 0. So with some positive probability the system may not return to the same state if it starts from the state J then that corresponding state is going to be call it as a transient state.

By seeing the one step transition probability matrix or by seeing the state transition diagram of a Discrete Time Markov Chain you can easily come to the conclusion the state is going to be a recurrent state or transient state. Whenever it is going to be a finite number of states this is easy to come to the conclusion if it is a infinite number of states then we need some work to be needed to come to the conclusion whether it is a positive recurrent or null recurrent.

So but easily you can make out the given state is going to be a transient state that you can make out from the state transition diagram or one step transition probability matrix. The conclusion of the state is going to be the transient state that can be given by a random variable TJ also. So the state J is a transient if and only the probability of the T suffix J is

equal to infinity and that probability 0. Sorry I had a mistake.

If this probability is strictly greater than 0 the probability of the system returns to the first passage the first passage return time that is infinity if that probability is greater than 0. That means there is a certainly however the system returns to the state J with the infinite amount of time going to take if that event is going to be with the positive probability then that state is going to be the transient state.

So there are two ways you can conclude the given state is going to be the transient state if either FJJ is less than 1 or the probability of TJ is equal to infinity which is greater than 0.

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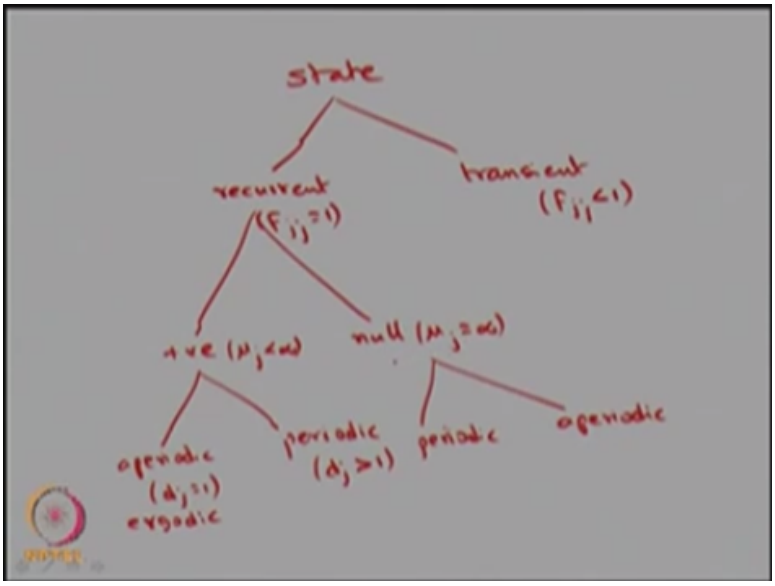

So based on this I can come to the conclusion any state could be recurrent or transient that means this is corresponding to FJJ is less than 1 and this is corresponding to FJJ is equal to 1. I can classify the recurrent sate into two form or either it could be a positive recurrent or null recurrent. Positive recurrent corresponding to the Mu J or Mu JJ both are one of the same that is going to be a finite value or null recurrent is corresponding to Mu J is equal to infinity.

That means based on the mean recurrence time you can conclude whether it is a positive recurrent or null recurrent. Again I can classify the positive recurrent into two. One is a periodic or the other one is a periodic. Periodic means that corresponding positive recurrent state that period is greater than 1. A periodic means that the TJ is 1. So the periodic positive recurrent state that is going to be call it as an Ergodic state.

Similarly, I can classify the null recurrent state into two. One is periodic and the other one is A periodic. The absorbing state is a special case of positive recurrent state where the transition probability from a state to itself is 1. So this is a way you can classify the state is a recurrent state or transient state positive recurrent state, null recurrent state again each one could be an A periodic or periodic state.

So in this lecture, we started with the few concepts of accessible then communicate then closed set then we have discussed a communicating class. Then, we have discussed what is the meaning of first visit then we have given the first passage time then we have given the mean first passage time distribution or mean recurrence time distribution. So based on those concepts we have classify the state as the recurrent state or transient state.

So this is related to the probability whereas the conclusion of the positive recurrent or null recurrent is related to the average time. So here only it involves the probability that whether in a certain probability the system will come to the same state if the probability wants whereas here there is uncertainty the system may not come to the state J. If the system starts from the state J.

If there is a uncertainty of returning that means with some positive probability the system would not be back then that state is going to be call it as a transient state. So this you can easily visualize in the state transition diagram of any Discrete Time Markov Chain you can see it whether from by seeing the state transition diagram you can come to the conclusion whether the state is going to be the transient or recurrent.

But through this diagrams you cannot come to the conclusion whether it is going to be a positive recurrent or null recurrent unless otherwise you evaluate this quantity Mu j is going to be n times e of jj of N.

So you find out that summation. So based on the summation values is going to be a finite one or infinite one accordingly that means whether the main recurrence time or mean return time or mean first passage time is going to be a finite quantity or infinite quantity accordingly you can conclude whether that recurrent state is going to be a positive recurrent or null recurrent so here you need a computation.

Whereas by seeing the state transition diagram sometime you can come to the conclusion whether it is a transient state or recurrent state. Now the issue of periodicity. The periodicity is important to conclude whether the limiting distribution exist or not whether that is going to be unique. So you need to find out the A periodic or periodic. So if the period is going to be one then that state is going to be call it as a periodic if the period is greater than 1 then it is a period with that integer.

When it is going to be null recurrent then also you can come to the conclusion whether it is a periodic or A periodic. Whenever you have a Markov Chain with the finite number of states then it is easy to find out whether it is going to be a positive recurrent or transient. So you need a quite good exercises needed whenever the Markov Chain have a infinite number of states then you need some work to be done for come to the conclusion it is a null recurrent and so on.

In today's lecture with this classification I stop here and all the simple examples and limiting distribution that I will explain in the fourth lecture. Thanks.