Stochastic Processes-1
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Lecture - 34
First Passage Time and Mean Recurrence Time

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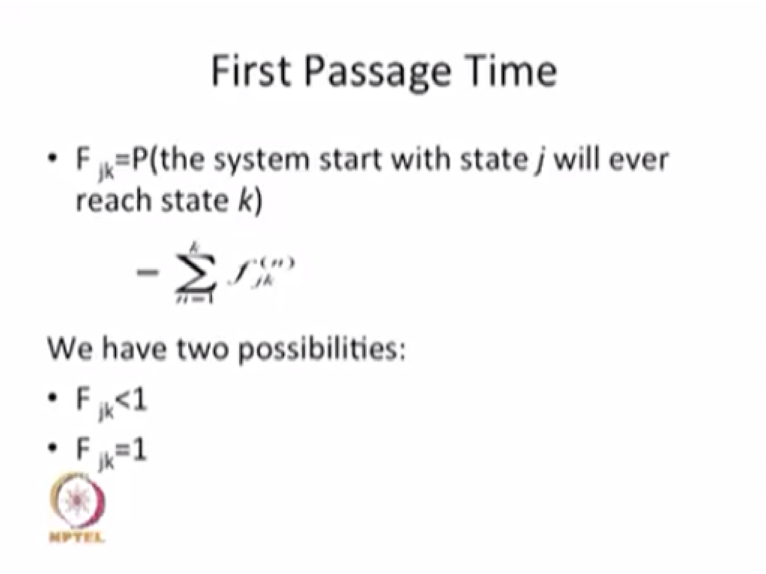

Now I am going to give the next concept called a First Passage Time Distribution. So that is written in the F suffix JK that is nothing but what is the probability that the system start with the state J will ever reach state K. So this probability I am writing as a F suffix J, K therefore this same as there is a possibility it would have gone to the state K in N steps first time and all the possible steps for the first time that union will give FJ, K.

What is the conditional probability that the system is starting from the state J and ever entering into the state K that is all the possible of first time to reaching the state N and all possible N that will give the probability F ever visiting the state K starting with the state J. Now we have two issues or two cases. One what is F, J, K which is less than 1, what is the situation corresponding to this probability is going to be less than 1.

The other case of interest is when F, J, K is equal to 1 that means with the probability 1 you will be ever visiting the state K by starting from the state J with the probability 1 or whether this probability is going to be less than 1. If it is less than 1 then it is not the correct one that means with the 1 minus of this probability, there is a possibility you would not ever visit the state K if you start from the state J the first case.

The second case that says with the probability 1 you will always reach the state K whatever be the number of steps starting from the state J. So our interest is both less than 1 as well as equal to 1. So the F, J, K equal to 1 that will give the probability distribution and that distribution is called a First Passage Time Distribution. So this case is our interest and this will give the First Passage Time Distribution.

Because whenever the system is starting from the state J whatever be the number of steps if you are reaching the state K the probability 1 that means you have the whole mass is 1. And this is going to be the distribution of the First Passage Time.

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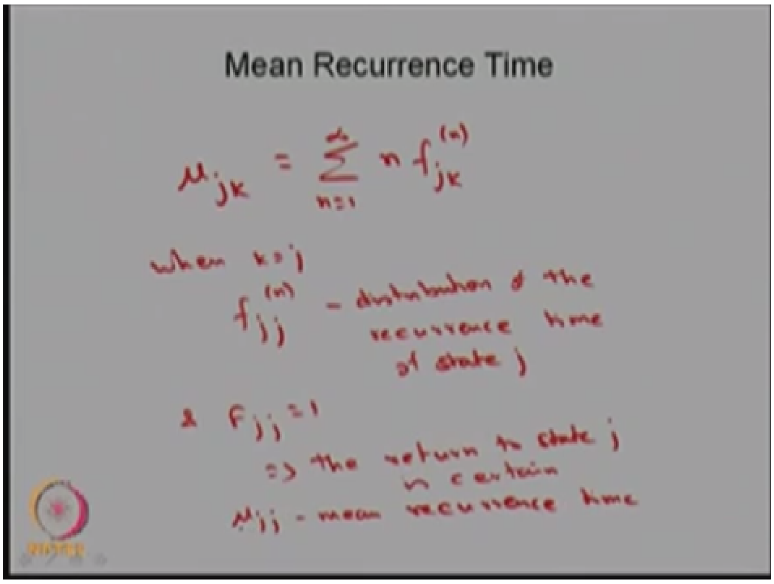

So using this I am going to give the next concept called Mean First Passage Time or Mean Recurrence Time. Mean First Passage Time is same as the Mean Recurrence Time that is defined as M suffix J, K that is nothing but what is the average First Passage Time or average recurrence time whenever the system start from the state J to the state K that is how many steps you have taken.

And what is the probability that starting from state J to K in the N steps and for possible values of N that summation is going to give the Mean First Passage Time or Mean Recurrence Time. When our interest will be when K equal to J return to the same state. So that means F, J, J of N that will give the Distribution of the Recurrence Time of the state J. And if F, J, J equal to 1 this corresponding F suffix J, J, N is going to be the distribution.

So correspondingly F, J, J is going to be 1 this implies the return to the state J whenever the system starts from the state J that is certain because that probability is 1 whenever F, J, J is 1 that means the probability 1 if you start from the state J you will definitely come to the state J. Therefore, that is corresponding to F, J, J is equal to 1. And the M JJ that will give what is the Mean Recurrence Time.

The M, J, J will give Mean Recurrence Time for the state J. So we are considering the second case in which F, J, J is equal to 1 so that is nothing but the return to the state J whenever the system start from the state J is certain and the small f, j, j of n will give the distribution of the recurrence time. And our interest is also for the Mean Recurrence Time that can be calculated by using MJJ.

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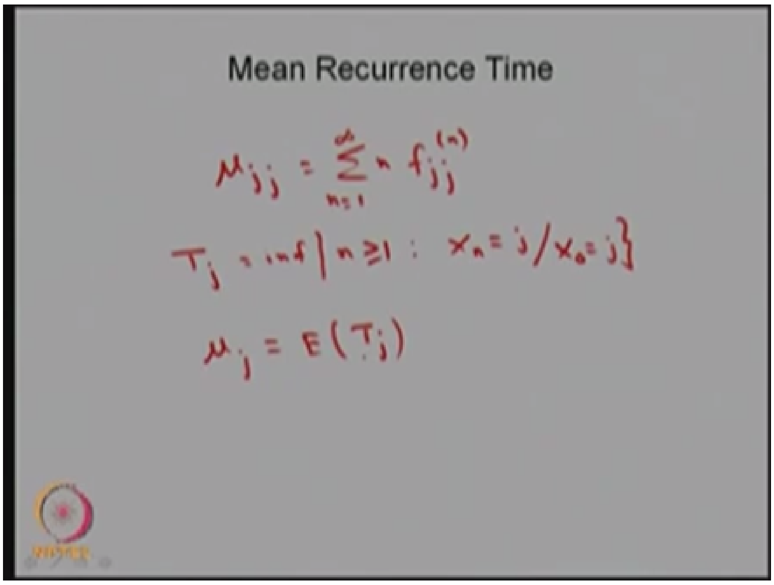

So earlier we have given M, J, J that is same as N times F, J, J of N by knowing F, J, J of N you can find out the Mean Recurrence Time for the state J. The same thing can be obtained by using another concept by introducing the random variable that is T suffix J that is nothing but inferior of N greater than or equal to 1 such that the XN is state J given that X 0 was state J.

This is a random variable denoting the first return time to the state J. The first return time, time here it is the step Nth step and you find out the first time you return to the state J starting from the state J reaching the state J. So whatever be the first number that integer and that is going to be T, J and this is going to be a random variable. So using this random variable also you can give the definition of Mean Recurrence Time.

Now I can define the Mean Recurrence Time MJ you do not want the two suffix J, J one suffix is enough. So M suffix J is nothing but what is the expected or expectation of random variable T, J. So the T, J will give the step that denotes the first return time therefore the expected first passage time that you can write it as the M suffix J. So this M suffix J and MJJ or both are one of the same.

And here you are finding the distribution and using the distribution you are getting and here you are finding the time and finding the average time using the expectation of T, J. So in both ways one can define the Mean Recurrence Time.