Stochastic Processes-1
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Lecture - 33
Closed Set of States and Irreducible Markov Chain

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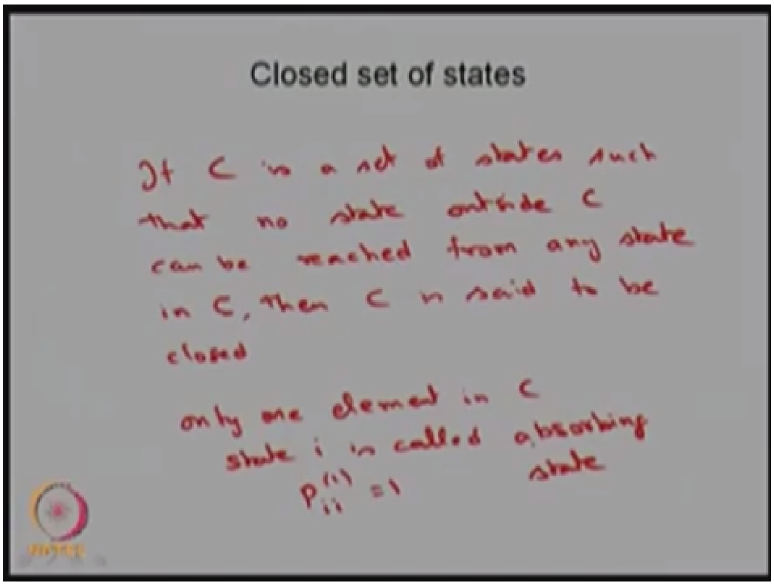

Now I am going for the next concept called a closed set of states. If C is set of states such that no state outside capital C can be reached from any state in capital C then we say the collection or the set C is said to be closed. So whenever you create a collection of states and that set we call it as a capital C. If it satisfies this property then we say that set is called closed set.

So we can combine the class property with the closed set property if both the properties are satisfied they communicates with each other as well as the closed property satisfies then you can say that a closed communicating class. So any subset in the states space S if it satisfies each element within the set is communicating each other and satisfies this property then we say that collection is going to be a closed communicating class.

There is a possibility in a set you can have a more than one elements one than one states in the collection. The class may have only one element or it maybe more than one element. If any closed set or the closed communicating class has only one element that means you cannot include one more state and to make it as the closed or communicating class then that closed set is called or that state is called a only one element in capital C then the state I is called absorbing states.

The state I is said to be absorbing state then it is going to form a closed communicating class which has only one element in that class. There is a possibility more than one element is also possible in the closed communicating class. So we can define the absorbing state through the closed communicating class or we can make it in the same absorbing state using the definition PII in step one that is going to be one.

That means if you see the one step transition probability matrix the diagonal element of that corresponding state the corresponding rho the element is going to be one that means the system starting from the state I and even one step the system moves into the same state I that probability is one. If this probability is one, then we say that state is going to be absorbing state.

In the other way round you can go for defining the absorbing state by a closed communicating class has only one element also. So there are two ways we can say the absorbing state.

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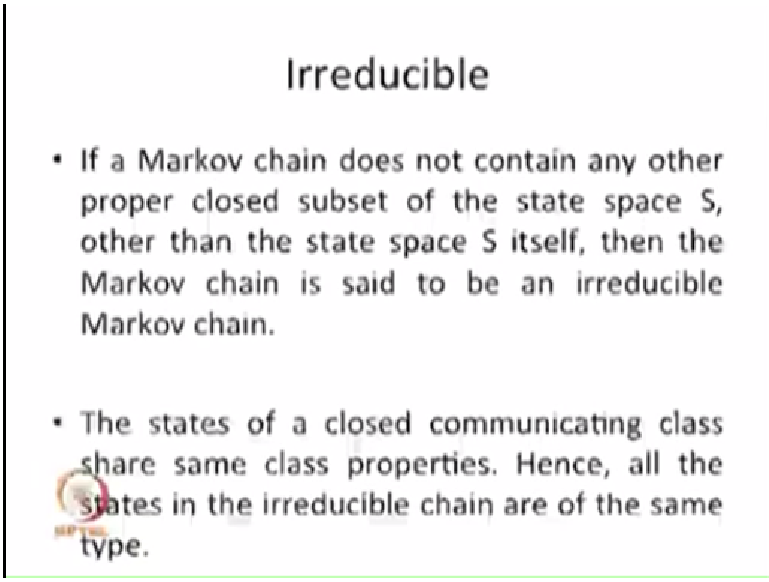

Using this concepts, I am going to develop the next concept called the Irreducible Markov Chain. We are discussing a Time Homogenous Discrete Time Markov Chain whereas this concept called the irreducible that is valid for the Discrete Time Markov Chain as well as the Continuous Time Markov Chain so that we are going to discuss later. Now I am defining the

Irreducibility for a Time Homogeneous Discrete Time Markov Chain.

If the Markov Chain since the irreducible concept comes for the Discrete Time Markov Chain and a Continuous Time Markov Chain, we use the word called the Markov Chain that is valid for both. If the Markov Chain does not contain any other proper closed subset other than the state space S, then the Markov Chain or in short we can use the word MC or Markov Chain then the Markov Chain is called Irreducible Markov Chain.

Whenever the state space cannot be partitioned into more than one closed set the proper set that means you can have only one closed set and that is same as the capital S. All the elements in the state space is going to form a only one closed set in that case that Markov Chain is going to be call it as a Irreducible. Irreducible means you cannot partition the state space.

If more than one closed proper closed subsets are possible from the state space, then that Markov Chain is going to be call it as a reducible Markov Chain. If more than one or we can able to make the partition of the state space into more than one closed set as well as the few transition states and so on that I am going to discuss later. So whenever if you are able to partition the state space then that is going to be a Reducible Markov Chain.

If you are not able to partition the state space and the whole state space is going to be a only one closed proper closed subset then that Markov Chain is going to be call it as a Irreducible Markov Chain. In this case all the states belonging to that class is going to form one class and since it is going to have a only one class all the states is going to have if one state as the period something then all the other states also going to have the same period because you are not able to partition so you have only one class/

Therefore, one state has the period or some number or some integer then that same period will be for the all other states also. So the Markov Chain which are not irreducible or said to be reducible or Non Irreducible Markov Chain.

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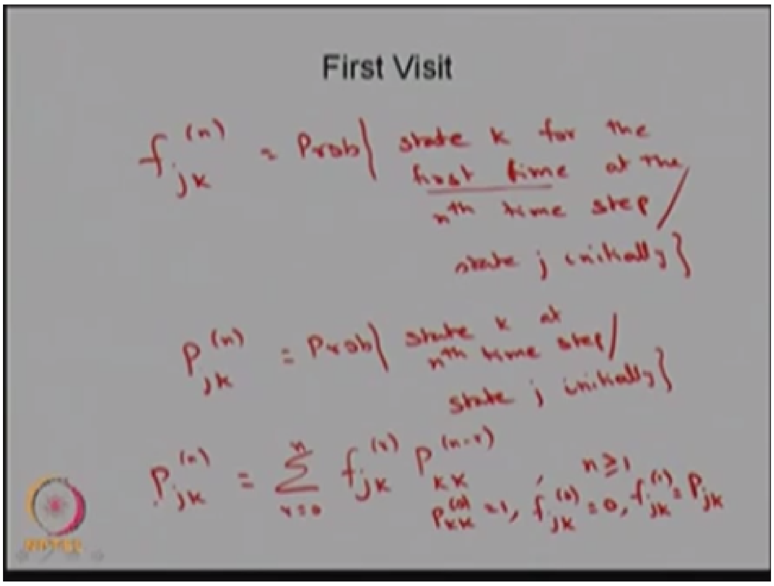

Now I am going to give the next concept called the first visit. We did not come to the classification of a state before that we are developing a few concepts using these concepts we are going to classify the states. The next concept is called a first visit what is the meaning of a first visit? I am going to define the probability mass function as the F suffix JK with superscript N that means that what is the probability that the system reaches the state K for the first time that is important.

For the first time at the Nth time step given that the system start the state J initially. This is a conditional probability mass function of a system moving from the state J to K. And system reaching the state K at Nth time step for the first time that is important. So this is the first time the system reaches the state K at the Nth step exactly at the Nth step and this conditional probability mass function that I am going to write as the F suffix JK of N.

This is different from the PJK of N. This is also conditional probability whereas this probability is defined what is the probability that the system reaches the state K at the Nth time step. Given that it was in the state J initially. This is also conditional probability the only difference is the first time that means that is a possibility the system here the P suffix JK of N means that is a possibility the system would have come to the state K before Nth step also.

So that probability is included whereas as the F suffix J, K at the Nth step means that this is the only the Nth step it reaches the state K. Therefore, the way I have given the first time conditional this probability and this is not necessarily the first time this is also conditional probability I can relate the F suffix J, K with the P suffix J, K both are in the N-step transition probability, but one is for the first time the other one is not necessarily.

I can relate both in the form of P, J, suffix K Superscript N that is the N step that is same as F suffix J, K of R steps and P suffix K, K of N minus R steps and R can be vary from 0 to small N or N is greater than or equal to 1. This means if the system is moving from the state J to K in the N step not necessarily the first time that can be written as the union of mutually exclusive events for different R in which the system moves from the state J to K in R steps for the first time and the remaining N minus R steps there is a possibility the system would have move the state K to K not necessarily the first time.

And the possible R can be 0 to small n and this n can vary from 1 to infinity. Obviously, we can make out I can give the P, K, K of 0 step that is going to be one and similarly you can make out a F suffix J of K that is 0 steps also O and F, J, K of one step that is nothing, but the P, J, K. The first time the system is moving from the state J to K in one step that is same as the one step transition probability/

The first time and one step transition probability is same whereas for N is greater than or equal to 1 then it is going to be combination of the first time with the not necessarily the first time N minus R step transition probability that all possible events that will give the all together final probability. So here we have use the total probability rule as well as the Chapman Kolmogorov equation for the Time Homogeneous Discrete Time Markov Chain to land up giving the relation between the P, J, K with the F, J, K.