Stochastic Processes-1
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Lecture – 31
Examples

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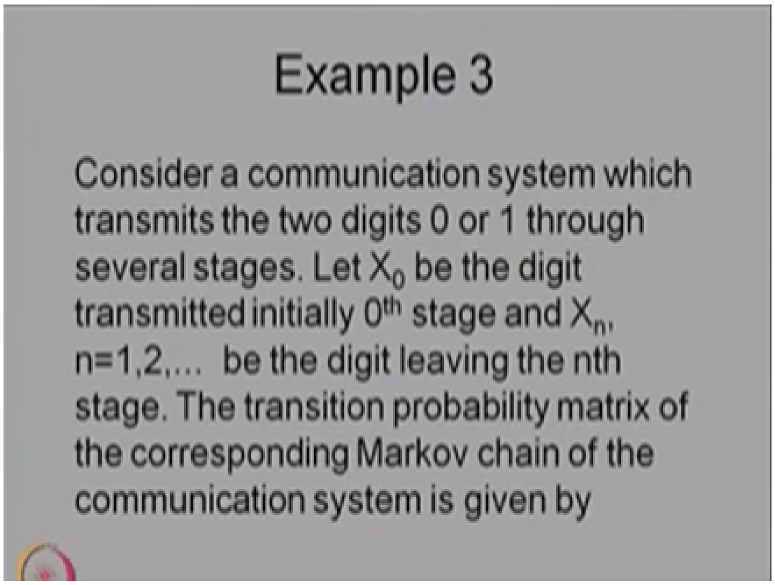

This example talks about the communication system in which whenever the transmission takes place with the digits 0 and 1 in the several stages. Now we are going to define the random variable x not be the digit transmitted initially that is 0th step. Either the transmission digital be be 0 or 1 therefore only two possibilities can be takes place at any nth step transmission either 0 or 1, like that we are making the transmission over the different stages.

Therefore, this xn over the n will form a stochastic process because we never know which digit is transmitted in the nth stage. So nth stage is going to be one random variable and you have a collection of random variable over the stages therefore it is a sequence of random variables so this going to form a is Stochastic process. And this stochastic process is nothing but a discrete time discrete stage stochastic process.

Because the possible values of xn is going to be 0 or 1 therefore the state space is 0 or 1 and it is a discrete state stochastic process. The way the subsequent transmission takes place depends only on the last transmission not the previous stages therefore you can assume that this follows Markov properly. Therefore, this stochastic process is going to be call it as a Discrete-time Markov Chain.

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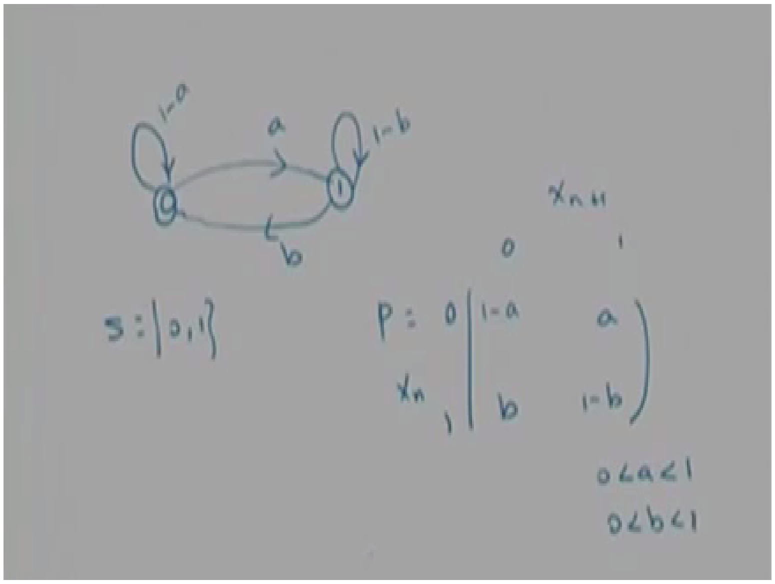

Now our interest is to find out-- so now I will provide what is the one step transition probability for the Markov Chain or let me give the transition diagram for that. So state transition diagram the possible states are 0 or 1 because the state space is 0 and 1. And the probability that in the next step also the transmission is 0 with the probability 1-a. This is a conditional probability; this is the conditional probability of the nth stage the transmission was 0 the n+1th stage is also the transmission 0 with the probability 1-a.

The one step transition probability of system is moving from 0 to 1 that probabilities is a. That means the nth stage the transmission was the digit 0 the n+1th stage also the-- the n+1th stage the transmission will be digit 1 with the probability a. Similarly, I am going to apply the one step transition probability of 1 to 1 that is 1-b and this is a b that means this 1 to 0 that probability is a 1 to 0 is a b and 1 to 1 is 1-b.

Obviously, this a is lies between 0 to 1 and b is also lies between 1.

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$$P^{(n)} = \begin{matrix} & 0 & 1 \\ 0 & \frac{b+a(1-a-b)^n}{a+b} & \frac{a-a(1-a-b)^n}{a+b} \\ 1 & \frac{b-b(1-a-b)^n}{a+b} & \frac{a+b(1-a-b)^n}{a+b} \end{matrix}$$

$$for \ |1-a-b| < 1$$

So this is the a is the probability that the system is transmitting from the 0th step – sorry nth stage with the digit 0 and the n+1th stage with the digit 1 that probability is a therefore the negation is 1-a because there is the system can transmit either 0 or 1. So once you say that one step transition probability of 0 to 1 is a then 0 to 0 will be 1-a. Similarly, 1 to 0 is given as a probability b and the other digit transmission will be one therefore it is going to be 1 to1 will be 1-b. So this is the state transition diagram.

And this is the one step transition probability for a given time homogenous discrete-time Markov Chain. Our interest is to find out what is the distribution of xn for n. For that you need what is the N-Step Transition Probability matrix. Since the one step transition probability matrix is given you can find out P square P power 3 and so on. By induction method you can find out the P power m but using the P power m you can find out the Pm+n.

Therefore, you can come to the conclusion what is the N-Step Transition Probability of system is moving from 0 to 1 and 0 to 0 and so on. So this is nothing but I am just giving the only the result b + a times 1-a- b power n divided by a + b and this is nothing but a-a times 1-a- b power n divided by a + b. Similarly, if you find out the N-Step transition probability of system moving from 1 to 0 that is b-b times 1-a-b power n divided by a + b.

This is nothing but a + b times 1-a-b power n divided by a + b. So here I am just giving the N-Step Transition Probability in matrix form. By given P you should find out P square P power q by induction you can find out the p power n. And this is valid provided 1-a-b which is less than 1. Because we are finding the P power n matrix, so here it needs some determent also unless otherwise the absolute of 1-a-b which is less than 1, this result is not valid.

So provided this condition, the P of n that is the matrix. So that is same as P power n also. P of n is same as P power n.

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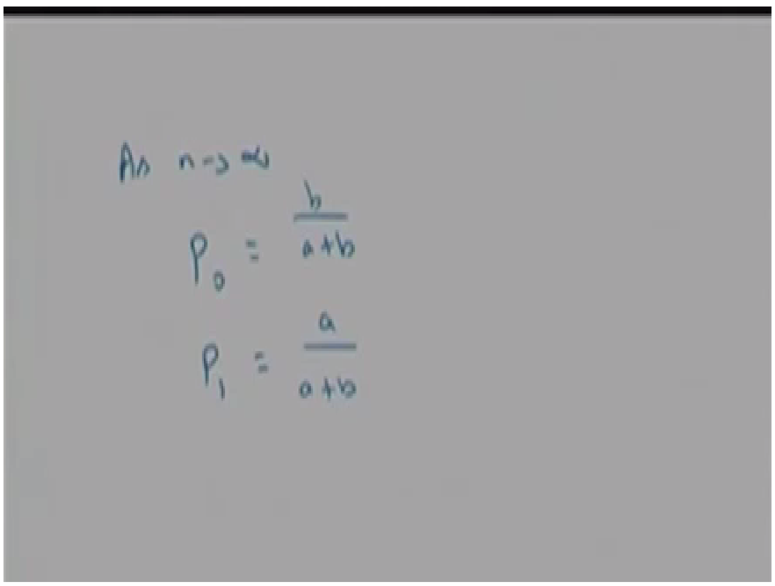

So as an entrance to to infinite you can come to the conclusion what is the probability that the system will be in the state 0 that is same as b divided by a + b. And similarly, what is the probability that the system will be in the state 1 as entrance to infinite that will be a divided by a+ b. This can be visualized from the state transition diagram easily. Whenever the system is keep moving into the state 0 or 1 with the probability a, b and with the self-loop on 1-a and 1-b the subsequent stages the system will be any one of these two states.

So with the proportion of a b divided by a+b the system will be in the state 1. Similarly, with the proportion a divided by-- sorry a divided by a+b the system will be in the state 1 with the proportion b divided by a+b the system will be in the state 0 in a long run. The interpretation of as an entrance to infinite this probability is nothing but in a long run with this proportion the system will be in the state 0 or 1.

So this state transition diagram will be useful to study the long run distribution or where the system will be as (()) (08:29) to infinity to study those things the state transition diagram will be useful.

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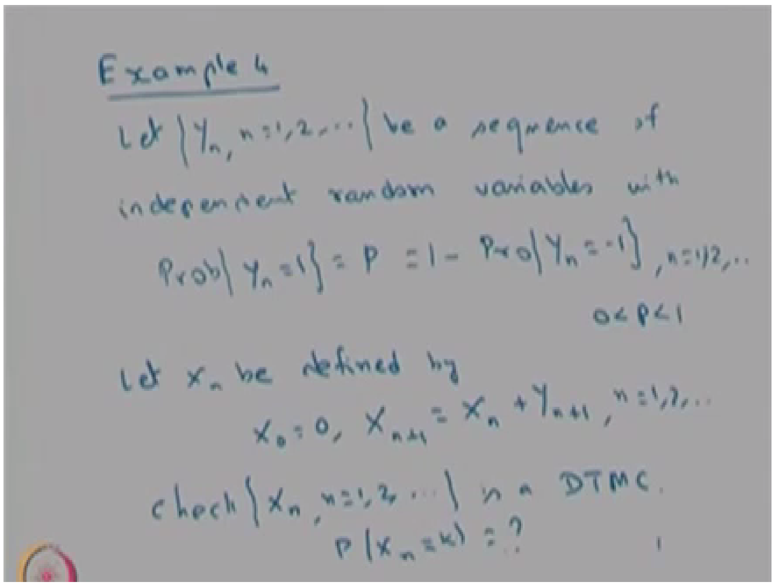

Now we will moving to the next problem that is Example 4. Let it is a sequence of random variable be a sequence of independent random variables with condition the probability of yn takes the value 1 that probability is P that is same as 1- the probability of yn takes the value -1.

We have a stochastic process and each random variable is a independent random variable and the probability mass function is provided with this situation the probability of yn takes the value one is P.

You can assume that P takes the value 0 to 1.That is same as 1- of probability of yn takes the value -1 for all m. Now I am going to define another random variable. Let xn be defined by x not is equal to 0 whereas a xn+1 onwards that is going to be xn + yn+1 for n is equal to 1, 2 and so on. So we are defining another random variable xn with the x not is equal to 0 and xn+ 1 is equal to xn+ yn+1. Now the question is check xn that stochastic process is the DTMC.

If it is a DTMC also find out what is the probability of xn takes the value k. We started with one stochastic process and we defined another stochastic process with the earlier stochastic process and check whether the given the new stochastic process is a Discrete-time Markov Chain that is the default one that is time homogenous Discrete-time Markov Chain. If so then what is the probability of xn takes the value k that is nothing but find out the distribution of xn.

So how to find out this the given or the xn is going to be the DTMC.

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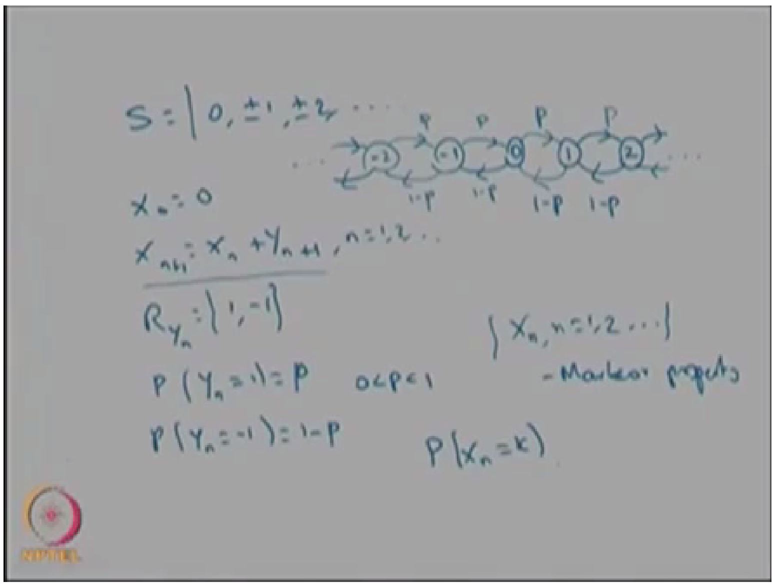

Since yn takes the value 1 with the probability P and yn takes the value -1 with the probability 1-p you can make out the possible values of yn is going to be 0 or plus or minus one; plus or minus 2 and so on. Because the relation is a x not is equal to 0 and the xn is xn+yn+1and the range of yn is 1, -1 therefore the range of xn that inform a state space and x not is equal to 0 therefore xn that relation is a xn+1 -- xn+1 is to xn+ yn+1 so n takes a value 1 and so on.

Therefore, the possible values of xn will be 0 plus or minus 1 or plus or minus 2 and so on therefore that will form a state space. Now the given clue is that probability of yn takes the value 1 is probability P and a probability of yn takes the value -1 that is 1-p and the probability P is lies between 0 to 1. So using this information you can make a state space of the yn that is going to be-- sorry 1, 2 and so on -1, -2 and so on.

Now we can fill up what is the one step transition of system is moving from 0 to 1 that means the x1 – 0 to 1 suppose you substitute 0 here then suppose it takes the value 1 then the system can move from the state 0 to 1 in one step. Suppose you put the value x not xn is equal to 0 suppose you put n is equal to 0 and yn+1 takes the value 1 with the probability P then the xn+1 value will be one with the probability P.

Now you can go for what is the state transition probability of 1to 0. Suppose the xn value was 1; suppose the yn+1 value was -1 then the xn+1 value will be 0. So the one step transition of a system moving from 1 to 0 because of happening probability of yn+1 is equal to -1 that probability is 1-p. So whenever the system is moving from one step forward that probability will be the probability P and one step backward that probability will be 1-p.

So this is the way it goes forward step and this is the way it goes to the backward step so you can fill up all other probabilities forward probability with the probability P and the backward probability with the 1-p. Also, we can come to the conclusion, the way we have written xn+1 is equal to xn+ yn+1 and all the y's are independent random variable the m+1going to take the value depends only on xn not the previous xn-1 or xn-2 and so on.

Therefore, the conditional distribution of xn+1 given that xn, xn-1 till x not that is same as the conditional distribution of xn+1 given xn. That means the xn is going to satisfy the Markov property because of this relation because of xn+1 is equal to xn+ independent random variable. Therefore, the xn = 1, 2, 3 and so on. This Stochastic process is going to satisfy the Markov property therefore this discrete time discrete state stochastic process is going to be the Discrete-time Markov Chain because of the Markov property satisfied.

Once it is Markov property satisfied by using the Chapman-Kolmogorov Equation you can find out what is the distribution of xn takes the value k, that is nothing but where it started a time 0 and what is the conditional distribution of N-Step Transition Probability and the N-Step Transition Probability is nothing but the elemental from the P power n and this—From here you can find out the one step transition probability matrix from the one step transition probability matrix you can find out a P, P square, P cube and so on.

And you can find out the P power n and that element is going to be the N-Step Transition Probability using that you can find out the distribution. And since we do not know the value of where P is lies between 0 to 1 it is-- I am not going to discuss the computational aspect of finding out the distribution.

This is left as an exercise and the final answer is provided. The difference between the earlier example and this example and this example the state space is going to be a countable infinite. Therefore, the P is not going to be a easy matrix it is going to be a matrix with the many elements in it therefore finding out P square and P power n is going to be little complicated than the usual square matrix.

So hence the conclusion is a by knowing the initial probability vector and the one step transition probability matrix or the state transition diagram we can get the distribution of xn for any n. There is a small mistake the running index for xn+1 value is equal to xn+yn+1 that is you starting from 0, 1, 2 and similarly the previous slide xn+1 is equal to xn+yn+1 and the n is running from 0, 1, 2 and so on.

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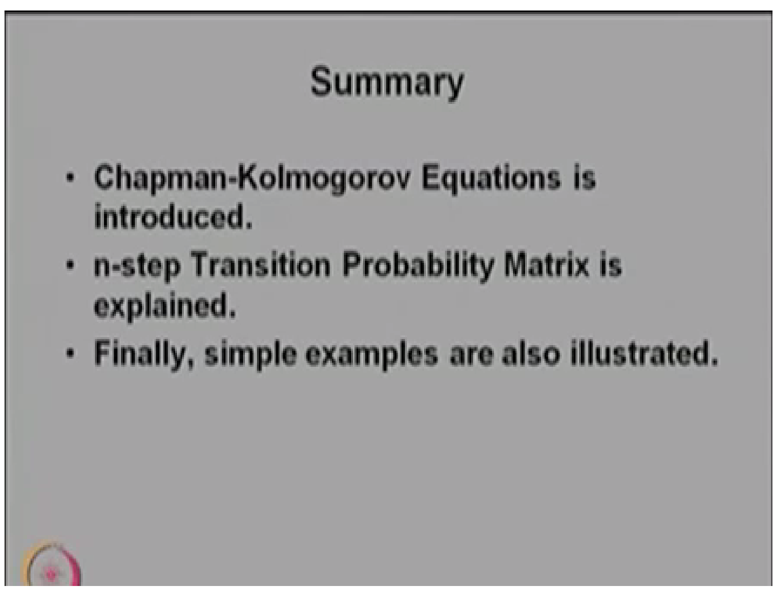

So in this lecture we have discussed Chapman-Kolmogorov Equation and also we have discussed the N-Step Transition Probability Matrix. So the N-Step Transition Probability Matrix can be computed from the one step transition probability matrix with the power of that n. And also we have discussed four simple examples for explaining the Chapman-Kolmogorov Equation and the N-Step Transition Probability Matrix.

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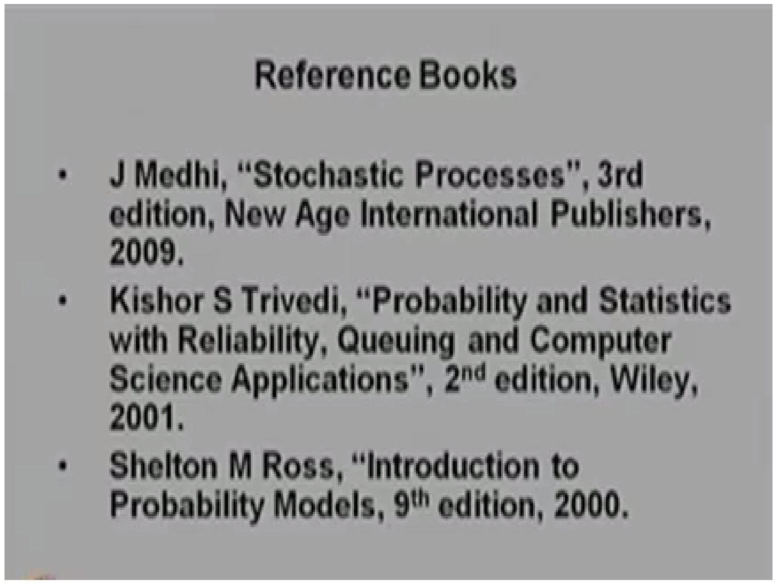

For the lecture 1 and 2, we have used these three books for as a reference the first one is J Medhi, "Stochastic Processes" book. The second one is a Kishor S Trivedi, "Probability and Statistics with the Reliability, Queuing and Computer Science Applications." The third one is a Shelton M Ross, "Introduction to Probability Models." So with this I complete the lecture 2 of Discrete time Makov Chain. Thanks.