Stochastic Processes-1
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Lecture – 30
State Transition Diagram and Examples

Now you are moving to simple examples using the N-Step Transition Probability matrix and the one step transition probability vector how to find the distribution of xn first and simple.

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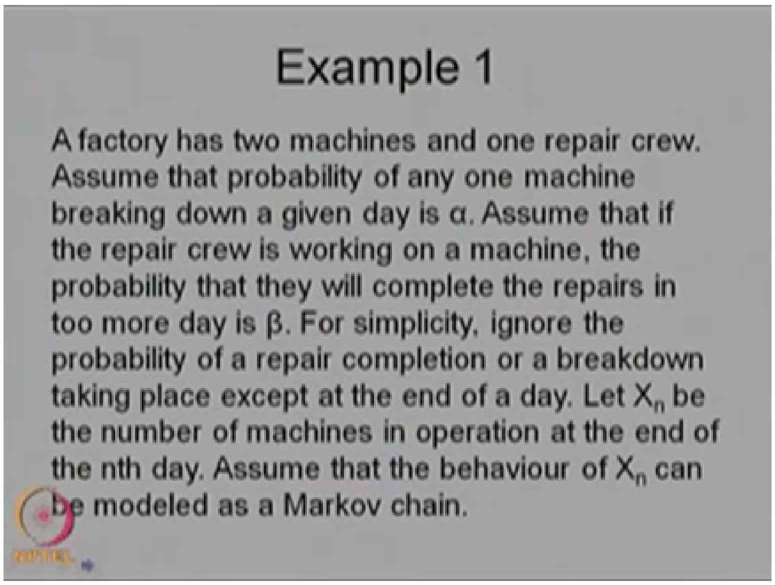

The first example which I have discussed in the lecture one, this is a very simple example in which the underlying stochastic processes is a time homogenous Discrete-time Markov Chain with the state space S is 0, 1 and 2.

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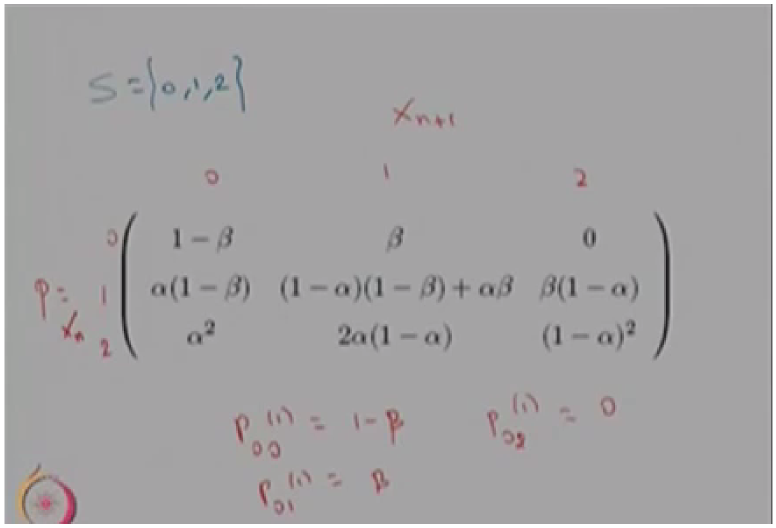

So this is the state space and the information which we have based on that we can make a one step transition probability matrix that is nothing but what is the possible probability in which the system is moving from state i to j in one step that you can fill it up. So this exercise you have done it in the lecture one.

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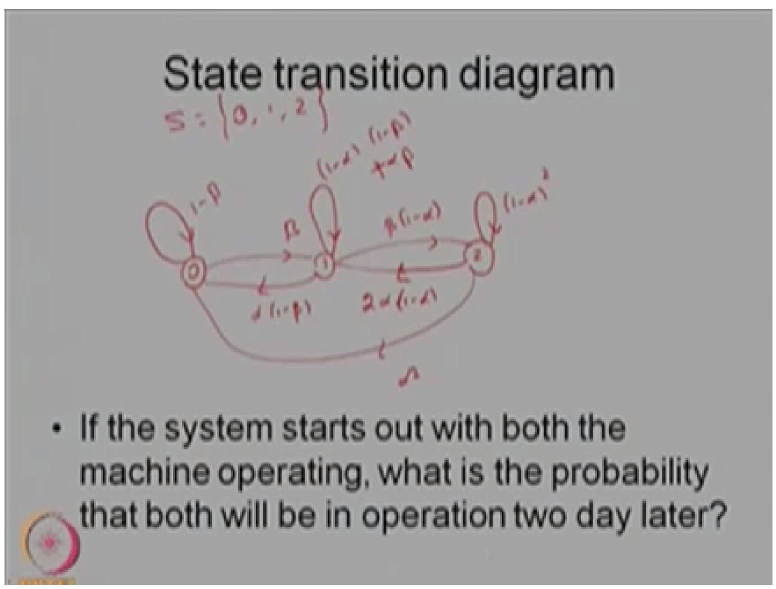

Now our interest is to find out and also we have made the State transition diagram which is equivalent to the one step transition probability matrix and we have got this state transition diagram. Now the question is if the system starts out with both the machines operating, what is the probability that both will be operation two days later?

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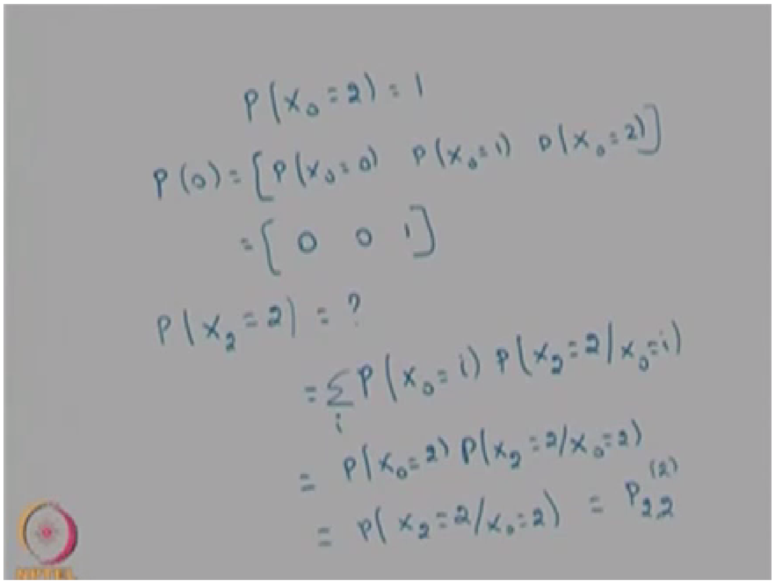

So if you recall what is the random variable xn be the number of machines in operation at the end of the nth day. So the random variable is how many machines are in the operation at the end of nth day. So here the clue is at the time 0 or the 0th step both the machines are operating therefore x0 is equal to 2 with the probability 1. So the given information with the probability 1 both the machines are working at 0th step.

So this can be converted into the P x0 takes the value 2 that probability is 1. Or you can make it in the initial probability distribution or initial probability vector as a that is the probability that at x0 the system was in the state 0 at the 0th step the system was in the state one so this is the initial probability vector. At a time 0 the system was in this state two therefore that probability is one and all other probabilities are 0. So this is the given information about the initial probability vector.

Now the question is what is the probability that both will be operation two days later? That means, what is the probability that we can convert this into what is the probability that x2 in the second step the system will be in state two given that the system was in the state two at the 0th step. So this is what you have to find out that is the conditional probability system, if the system starts with the both machines what is the probability that both will be operation two days later.

So not even this a conditional probability the question is what is the probability that; what is the probability that the system will be in the state. So to find this you can make what is the probability that-- the given information is there x2 is equal to 2 given that x not is equal to i for all possible values of i.

This is same as since the initial probability vector is going to be 0, 0 1one so this is land up; what is the probability that the x not is equal to 2 multiplied by what is the probability that x2 is equal to 2 given that x not is equal to 2 and all other probabilities are 0 therefore 0 into anything is going to be 0 therefore it is same as what is the probability that x not is equal to 2 and to the conditional probability.

And x not is equal to 2 is one therefore this is same as what is the probability that x2 is equal to 2 given that x not is equal to 2. So this is a same as what is the probability that 2, 2 in two steps. This is nothing but the system was in the state two at 0^{th} step and the will system being the state two after the two steps. So this is the two step transition probability of system moving from the state 2 to 2.

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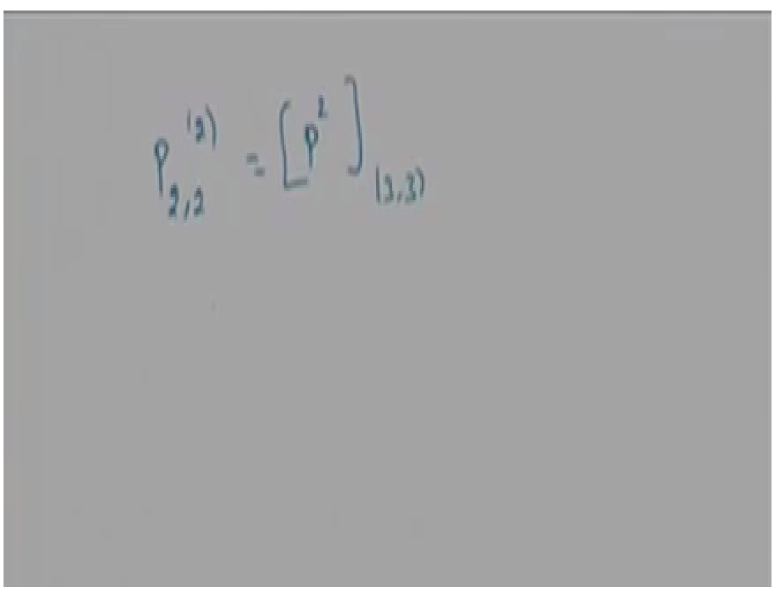

This is same as you find out you find out the P square matrix and from the P square matrix this nothing but the 2, 2 that is going to be the last element, out of that nine elements the third row, third column element that is going to be the element for the-- this probability. So what do you have to find out is that find out the P square; find out the P square so we have provided the P.

So this is the P matrix so from the P matrix you will found out the P square so the P square is also going to be a three class three matrix. So from the P square three square three matrix you take the third row third element third column element and that is going to be the probability for two step transition of system moving from the state 2 to 2 that is going to be the answer for the given question.

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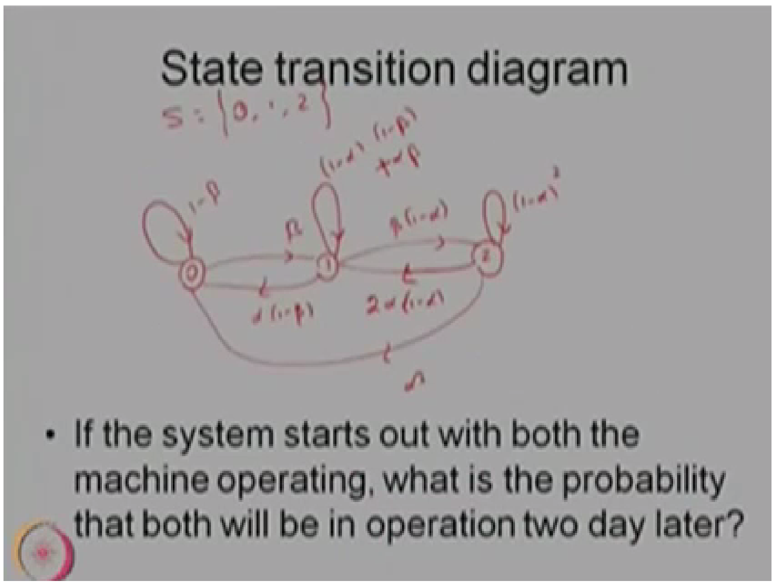

What is the probability that both will be in operation in two days later? Similar to this you can find out the probability for any day or any finite day by finding the P power n matrix then pick the corresponding element and that is going to be the corresponding probability.

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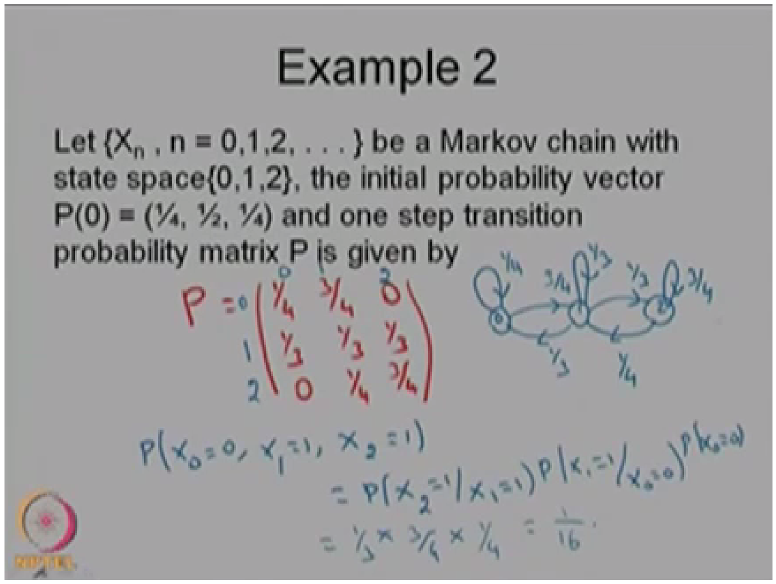

Now, we moving to the next example. This is abstract example in which the xn be the discrete-time Markov Chain the default discrete-time Markov Chain is always it is a time homogenous. So this is the time homogenous discrete-time Markov Chain with the state space 0, 1 and 2. And also it is provided the initial probability vector that is a P0 that is the vector that is one fourth of one fourth. So the submission is going to be one therefore this is the initial probability vector.

That means the system can start from the nth 0th step with the probability 1/4 from the state 0. From, the state one which the probability with the probability one fourth it can start from the state two. And also it is provided the one step transition probability matrix. From the one step transition probability you can draw the state transition diagram also because the state space is 0,1, 2 therefore the nodes are 0, 1 and 2.

And this is the one step transition probability therefore 0 to 0 that probability one step the system is moving from the state 0 to 0 that is 1/4 and the system is moving from the state 0 to 1 in one step that is three fourth and there is no probability from the-- going from the state 0 to 2 therefore you should draw the arch. From one, the one step transition probability of 1 to 1 is one third and this is one third and similarly this is one third.

From the state two, 2 to 0 is 0 and 0 to 1 is one fourth and 2 to 2 is three fourth. This diagram is very important to study the further properties of the states therefore we are drawing the state transition diagram for the Discrete-time Markov Chain. So this is the one step transition probability matrix and this is the state transition diagram. Our interest is to find out the few quantities that is that what is the probability that x not is equal to 0 and x1 is equal to 1 and x2 is equal to 1.

What is the probability that the system was-- it is a join distribution of these three random variable x not is equal to 0 and x1 is equal to 1 and x2 is equal to 1. So this is same as the join distribution the same as you can write in the product of the conditional distribution and a conditional distribution again you can write it using the Markov property the conditional probability of only one step.

Therefore, this is going to be by using the probability theory, you apply the join distribution is same as the product of conditional distribution by using the Markov property you reducing into the another conditional distribution so this is same as what is a probability that x2 is equal to 1 given that x1is equal to 1 multiplied by the x1 is equal to 1 given that x not is equal to 0 and probability of x not is equal to 0.

So this is the-- the first term is nothing but the one step transition of system is moving from one to one and this nothing but the system is moving from the state 0 to 1 and this is the initial probability-- you take the probability from the initial probability vector of x not is equal to 0. Now, we are going to label the one step transition probability matrix with the 0, 1, 2 and 0, 1, 2 from this, you can find out this is the one step transition probability of system moving from 1 to 1.

So 1 to 1 is one third into-- this is the system probability of system moving from 0 to 1 so 0 to 1 is three fourth and a system started from the state 0 in the 0th step so that you can take it from the first element that is one fourth. So if you do the simplification we will get 1/16. So this is the join distribution of the system was in state 0 at 0th step; the system was in the state one at the first step and the system was in the state one at the second step their probabilities 1/16.

Similarly, you can find out the other probabilities also that is.

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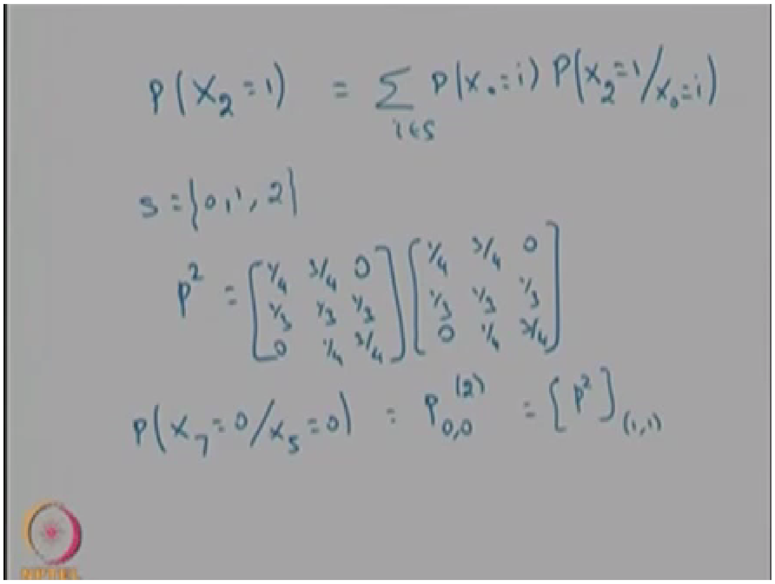

Suppose our interest is what is the probability that at the end of second step the system will be in the state one; that is nothing but what are all the possible states in which the system would have been started from the state i and what is the two step transition of system is moving from the state i to 1. So the i is belonging to S, so here the S is a 0, 1, 2 so already we have given the initial probability vector that is one fourth and one fourth using this.

And you need two step transition probability that means you need to find out what is the P square. So the P square will give the two steps transition probability matrix therefore the P is provided to you so the P is a one fourth, three fourth and 0; one third, one third, one third; 0, one fourth, three fourth, so this is the P. So you multiply the same thing one third, one third, one third; 0, one fourth, three fourth.

You find out the P square so from the P square you pick out the element of x0 is equal to for all possible i then multiply this and this that multiplication will give you probability of x2 is equal to 1. So I am not doing the simplification. So once you know the P square you can find out probability of x2 is equal to 1. Similarly, one can compute the other conditional probabilities also.

Suppose our interest find out what is the probability of x7 is equal to 0 given that x5 was 0. This is same as what is the probability that the system was in the state 0 the fifth step given that what is the probability that this system will be in x0 in this seventh step, that is same as what is the probability of 2 what is the probability of 3. What is the probability of 0, 0 in two steps, that means you find out the P square from the P square the 0, 0 is nothing.

But you take the first row first column element and that is going to be the probability of P of x7 is equal to 0 given that x5 is equal to 0. Similarly, you can find out all other different conditional probability and what do you have to do is always you have to convert because of the given DTMC is a time homogenous, so you convert that into find out the N-Step Transition Probability and the N-Step Transition Probability same as the P power n, so you pick the corresponding element to find out the conditional probability.