Stochastic Processes - 1
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Lecture - 28
Examples of Discrete time Markov Chain (Contd.)

Now I am moving into the second example.

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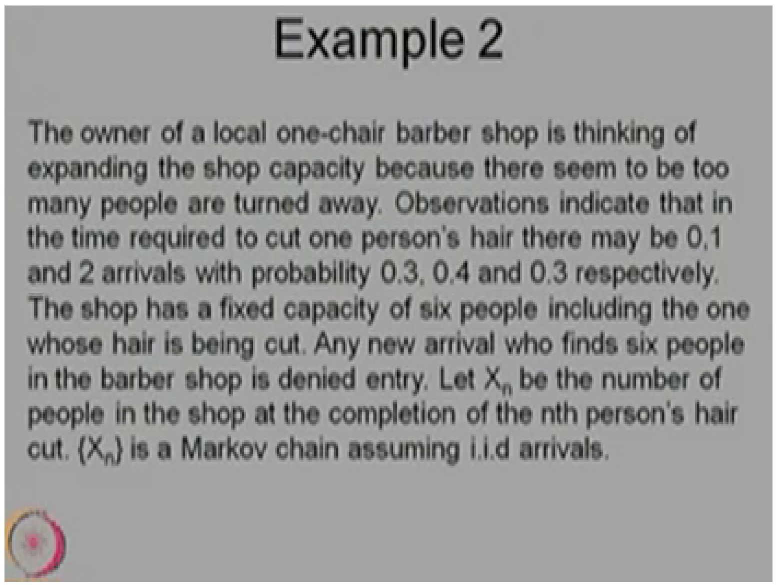

In this example I have taken the barbershop example which I have discussed in the module 1 and also the same example so the owner of a local one share barber shop is thinking of expanding the shop capacity because there seems to be too many people are turned away observation indicate that in the time required to cut one person's hair there may be 0 1 and 2 arrivals with the probability 0.3 0.4 and 0.3 respectively.

So this information is very important that means, during one person's haircut what is the probability that no people turned up with the probability 0.3 and one people may turned up with the probability 0.4 and there is a possibility two arrivals is possible during the one person's haircut with the probability 0.3 therefore the summation of probability is going to be 1.

So during the one person's haircut these are all the only 3 possibilities are possible with the zero arrival or one arrival or two arrival, the shop has a fixed capacity of 6 people including the one whose hair is being cut that means a maximum 6 people can be allowed in the system, so 5 people can wait maximum and 1 people under the service, any new arrival who find 6 people in the barbershop is denied entry that is the meaning of a capacity of the system is finite with the size 6.

Now I am going to define the random variable, let Xn be the number of people in the shop at the completion of the n th person's haircut this is very different random variable or this is very different steps for stochastic process usually the parameter space is a time but here the parameter space is the number of people in the shop the n is the parameter space is the person who leaves after the haircut.

So it's a n th person who leaves the system that becomes the parameter space where as the random variable is how many people in the system when the another person leaves the system that means you should not count that person when you are finding the values of X that means this number is counted at the departure time point so when the n th person leaves how many people in the system.

The system is a maximum 6 people allowed therefore he cannot see more than 5 people in the system when he leaves, so because of this constraint because of during the 1 persons arrival either 0 or 1 or 2 arrivals can takes place and so on based on this information the stochastic process Xn is going to be a discrete time discrete state stochastic process as well as the Markov property satisfied.

That means the probability of Xn + 1 take some value given that all the previous values are known that is same as the conditional probability distribution of Xn +1 take some value given that Xn was some value, so also the future distribution given that present as well as the past information is same as the future distribution given the present, not the whole past information so this Markov properties will be satisfied by this stochastic process.

Therefore, this Xn will form a discrete time Markov chain, obviously it is the time homogeneous discrete time Markov chain also so in this example our interest is to find out what is the one step transition probability matrix.

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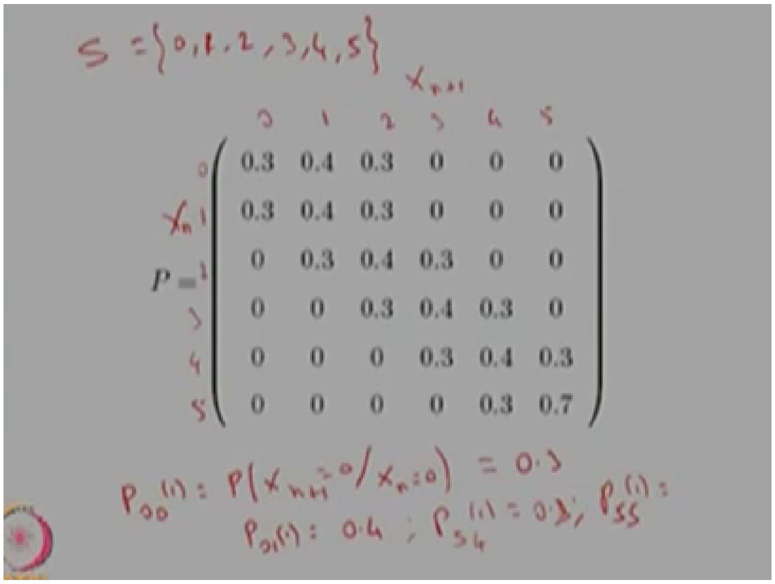

This is going to be the one step transition probability matrix and the possible states S is going to be 0 1 2 3 4 or 5, because the capacity of the system is 6 and whenever he whenever the n th persons leaves the first person second person third person leaves how many people are in the system, therefore the maximum will be 5 and there is a possibility when he leaves and no one in the system also.

And this is the one step process transition probability matrix and this is also going to be a square matrix because it is going to be a countably finite number of elements and this is 0 1 2 3 4 5, now we can discuss what is the probability that 0, 0 in one step that is nothing but in the nth - when the n th person leaves no one in the system, when the n + 1 th person leaves no one in the system.

What is the probability for that? That is the Xn + 1 is equal to 0, given that Xn was 0, so one step transition probability its independent of n, because it's a time homogeneous, it's a one step transition probability matrix, so this is possible at some person leaves whatever be the n nobody in the system when the next person leaves nobody in the system.

So that is possible by when some person leaves the system was empty for some time you don't know how much time it was empty then the n + 1 th person enter in to the system and during his haircut no one turned up or no arrival takes place during his or n +1 th haircut is going on, therefore when he leaves no one in the system, so we are not bothering when he entered into the system and so on.

Our interest is how many numbers of people in the system when the n + 1 th person leaves and this probability is n + 1 th person leaves is 0 people in the system and given that when the n th person leaves also 0 person in the system, so that that is possible with a explanation I have given no one enter into the system during the n + 1 th person's haircut.

And the information is provided indicate that - that time required to haircut one-person haircut there may be a 0 1 or 2 arrivals with the probability 0.3, so no arrival takes place during the one person’s haircut is a 0.3, therefore this probability is possible with the probability 0.3, whereas P 0 one of one step, the same way you can write probability that Xn + 1 is equal to 1, given that Xn is equal to 0, that is possible then the n th person leaves no one in the system.

When the n + 1 th person leaves one person in the system that means during his haircut one person enter into the system that is possible with the probability 0.4, similarly from 0 to 2 in one step that is going to be 0.3 with the probability two arrival takes place during the n + 1 th persons haircut. Now the second row, second row what is the probability that when the n th person leaves one person in the system when n - n + 1 n th percent leaves 0 person in the system.

That is possible, during the n + 1 th the person leaves - haircut there is no one in the system no - arrival takes place, therefore the probability is 0.3 and from 1 to 1, that is possible with one person arrived during the n + 1 th person haircut, therefore that probability is 0.4, and going from the state 1 to 2 that is possible two persons arrived during the n + 1 th person haircut.

Whereas from 2 to 0 that is not possible because when n th person leaves two person in the system therefore n - n + 1 th person in the leaves definitely he will see one person in the system because - because of no arrival and one arrival two arrival, therefore it will be shifted by one column and it will be keep continuing till the end.

Whereas the last one what is the probability that - what is the probability that the 5 people in the system when the n th person leave and 4 people in the system when the n + 1 th person leave that is same as no arrival takes place during the n + 1 th arrival, n + 1th haircut going on so therefore this is going to be 0.3.

Whereas P 5 to 5 in one step that is possible if the combination of one person arrived the system or two person arrived the system this system size is going to be maximum 6, therefore when n + 1 th person haircut is going on, if one person arrives then he will be entered, if two person arrives then he cannot be accommodated therefore he will he won't join the system therefore the system the number of customers in the system in the n - Xn that is going to take the value 5.

And the combination of a 0.4 as well as 0.5, therefore this probability of system is moving from 5 to 5 is a 0.7, because of 0.4 + 0.3. Now I can give the state transition diagram for this example.

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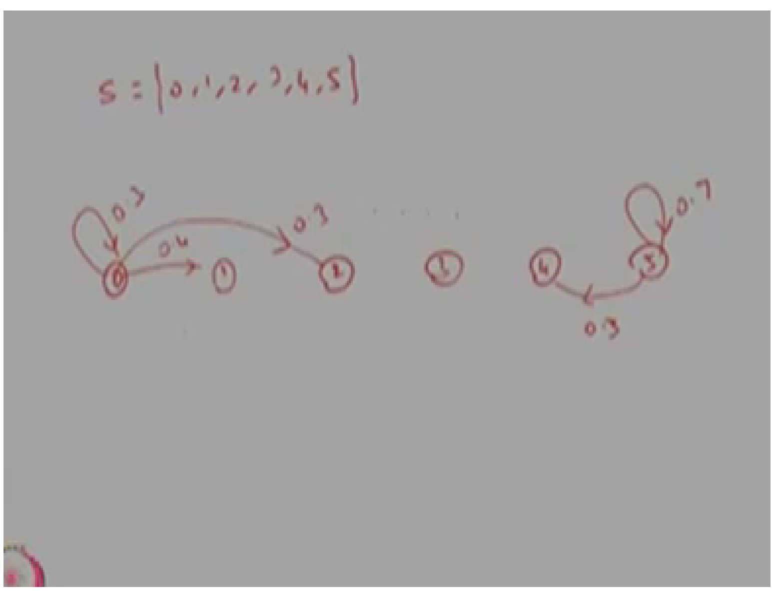

Because S is going to be 0 1 2 3 4 5, therefore the nodes are going to be 0 1 2 3 4 and 5 and possible values from the state - from the one step transition probability matrix I can make out so 0 to 0 that probability is 0.3 and 0 to 1 is 0.4 and 0 to 2 is 0.3, similarly, I can fill up the all other things and 5 to 5 that is very important and 5 to 4 that is possible with the probability 0.3 and 5 to 5 is possible with the probability 0.7.

So this is the state transition diagram I did not complete the state transition diagram you have to fill up all the arcs with the weights going from one arc to other arc with the positive probability wherever there is a probability zero we should not draw the arc for it.

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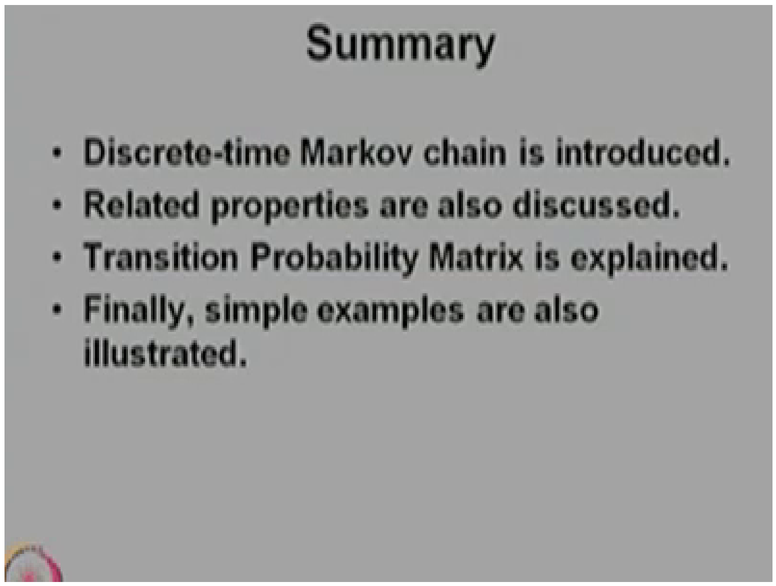

So in this lecture, we have discussed the discrete time Markov chain then we have given the few important properties followed by we have explained the one step transition probability matrix and also we have given two simple examples with this, the lecture one is over for the module 4. Thanks.