Stochastic Processes -1

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Lecture - 24
Simple Random Walk and Population

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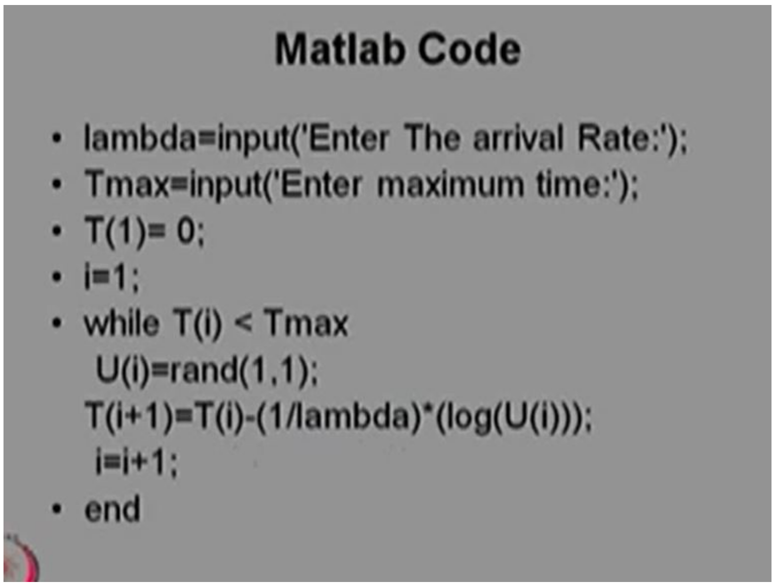

Now I am going to explain how we can create the sample path of the Poisson process using the Matlab code. So since I said the Poisson process is related with the inter arrival times or exponential distribution, so I can start with the time 0 there is no customer in the system and I can go for what is a maximum time I need the sample path then I can keep on create the random variables.

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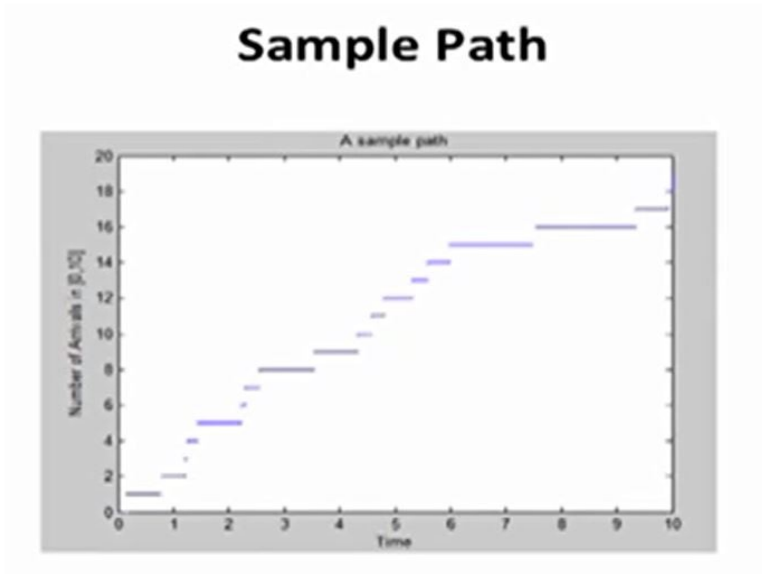

From the random variable I can generate the exponentially distributed the time event then I can shift the time event bye T (i + 1) by adding the next exponentially distributed time event, then I can go for plotting the sample path. So this is the one sample path in which over the time from 0 to 10, the number of arrivals occurs in the interval 0 to 10 in the form of that means there is one arrival occurs at this time, therefore the N (t) value is incremented by 1 and it is taking the same value and when at the second arrival occurs, then the increment is taken by 2 and so on.

So and if you see carefully the sample path you can find out the increment is always by 1 over the time and there is no 2 arrival or more than 1 arrival in a very small interval of time and you can able to see the inter arrival time that is going to be a exponentially distributed with a parameter Lambda whatever the Lambda f chosen in this sample path. So this is the way the sample path of the Poisson process looks like.

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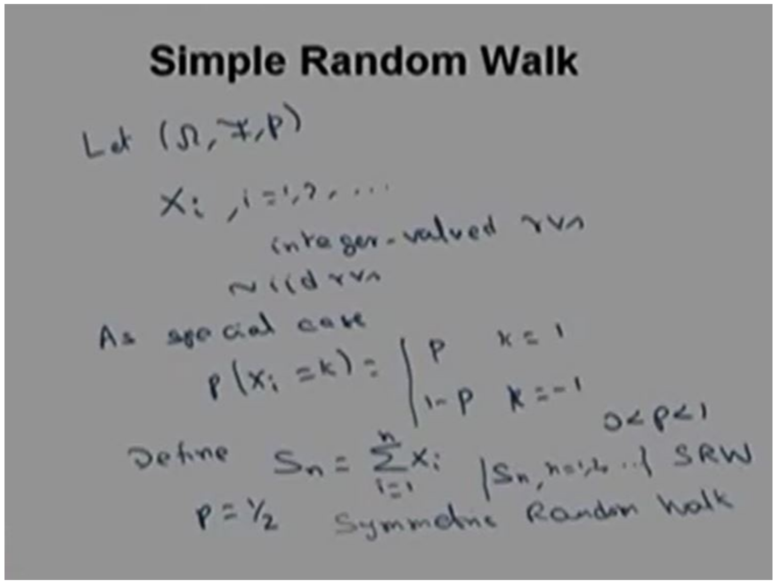

Now we are going to discuss the third type of stochastic process that is a simple random walk. So how we can create the simple random walk let me explain. You have a probability space, from the given probability space, you define a sequence of random variable Xi’s and those random variables are integer valued random variables. Each Xi’s are integer valued random variable not only that all the Xi’s are iid random variables also.

All the Xi’s are iid random variables and each one is integer valued discrete type random variable. As a special case, I can go for the random variable Xi takes a value 1 or -1 with the probability p and 1 - p. This is a special type of random walk in general I am going to define the, in general random walk also as a special case, I will go for the random variable Xi takes a value 1 with a probability p.

And Xi takes a value -1 with the probability 1 – p, where the p can take the value 0 to 1. Now I am going to define the random variable Sn that is nothing but Sum of Xi’s, sum of first n Xi's that is going to form the random variable Sn and the Stochastic process Sn or the Stochastic sequence Sn for different values of n this will form a simple random walk. The Sn is going to form a simple random walk, why it is simple?

Because it is going to take an integer valued random variable and each values are going to take, each random variable is going to take the value 1 or -1, therefore this is going to be called as a simple random walk. In general, the k can take the any integers accordingly you will land up having a Sn's are going to be a random walk.

And I am going to give the another special case, when p is equal to 1/2 that means each Xi random variable takes a value 1 with a probability 1/2 or -1 with a probability 1/2, then that random walk is going to be call it as a Symmetric random walk. Why it is symmetric? Because with the probability 1/2, it takes a forward one step or with the probability 1/2 it takes a backward one step.

Therefore, that type of a random walk is called symmetric random walk. In general, if we take a value 1 or -1 then it is called a simple random walk. If k can take any integers, then it is going to be call it as a generalized random walk.

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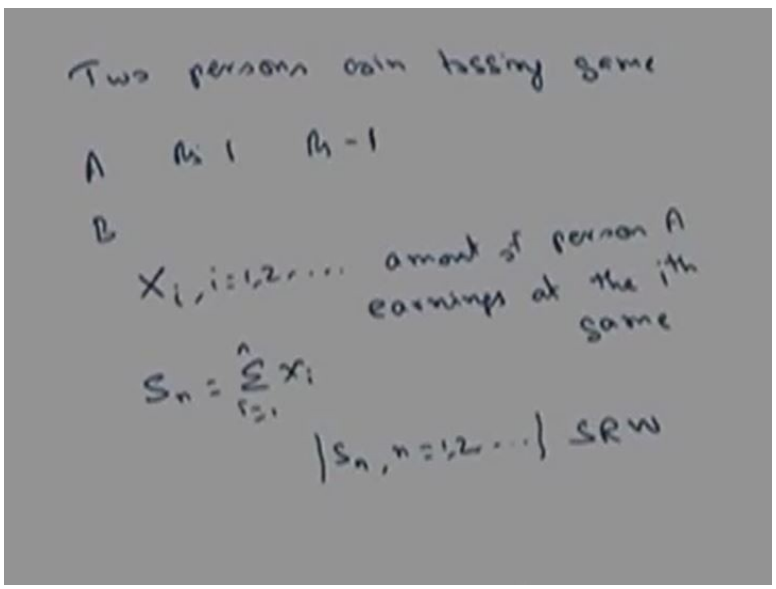

So this random walk can be created in a simple example of two persons coin tossing game also, this simple random walk can be explained by the example two persons coin tossing example in which you have a person A and B, if at the end of the coin tossing, if he is going to head then he is going to win Rs.1 or if at the end of the nth coin tossing, if it is going to get the tail then he is going to be lose in this game. If A wins, then B gives Rs.1 to A and if A loses, then A gives Rs.1 to B.

So accordingly, I can go for creating a random variable Xn or Xi for i is equal to 1, 2 and so on. Therefore, Xi denotes what is the amount of the person A earning at the ith game. Similarly, we can construct a stochastic process for player B and calculate the measures of interest. I can go for creating a random variable Sn is nothing but summation of Xi's, where i is equal to 1 to n. Therefore, the Sn denotes what is the amount earned by the person A at the end of nth game, that's what a total amount.

So the Xi denotes how much he is going to earn at the end of each game, whereas the Sn is going to be the total amount earned by the person A at the end of a first n games. Therefore, this Sn is going to form a simple random walk, where Xi's are going to take a integer valued with a value 1 and -1 with the probability p it is going to take the value 1 or it is going to take the value -1 with the probability 1 - p.

So, I am just relating the simple random walk with the simple scenario of a two person's coin tossing game.

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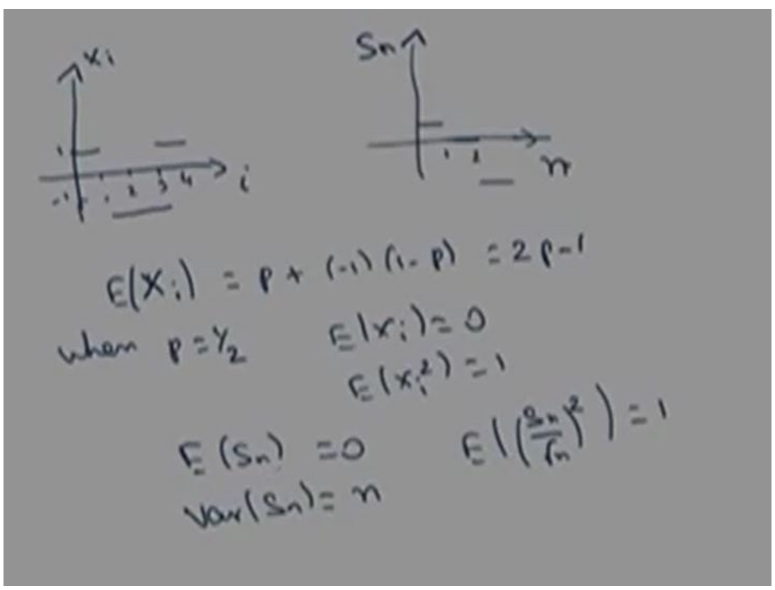

If you see the sample Path of the Sn, first I can go for what is a sample path of each Xi's, each Xi's can take the value 1 or -1, therefore it is going to take the value 1 or -1. Therefore, if X1 takes a value 1, it is 1, if X2 takes a value -1 it is like this if X3 takes the value -1 then it is here. If X4 takes the value 1 then it is like this, so this is a sample path of Xi over the i.

The way I have given the Xi's now I can go for writing what is the possible values of n and what is a possible values of Sn, so since X1 is equal to 1, therefore S1 is going to be 1 and X2 is going to be -1, therefore it takes a value 1+, - 1 therefore it is going to be 0 and X2 is going to be -1, therefore S 3 is going to be -1 and X4 is going to be 1, therefore it is going to be again 0.

So this is the way the sample path goes over the n. So, this is a one sample path for the possible values of Xi takes a value 1 and -1 accordingly. I have drawn the sample path of Sn over the n. Since Xi's are going to take the value 1 and -1 and with the probability p and with the probability 1 – p, it takes the value -1, I can go for finding out what is the expectation of Xi that is nothing but Xi is equal to p + (-1) times (1 - p), therefore this is nothing but 2p - 1.

So, when I go for discussing the symmetric random walk when the p is equal to 1/2 then the expectation of each Xi is going to be 0 and also I can able to find out what is E of Xi Squares that is going to be 1, not only that when p is equal to ½, I can able to find out what is the expectation of Sn that is going to be 0 and the variance of Sn is going to be n and I can go for writing what is an expectation of Sn / Root n power 2, that is going to be 1.

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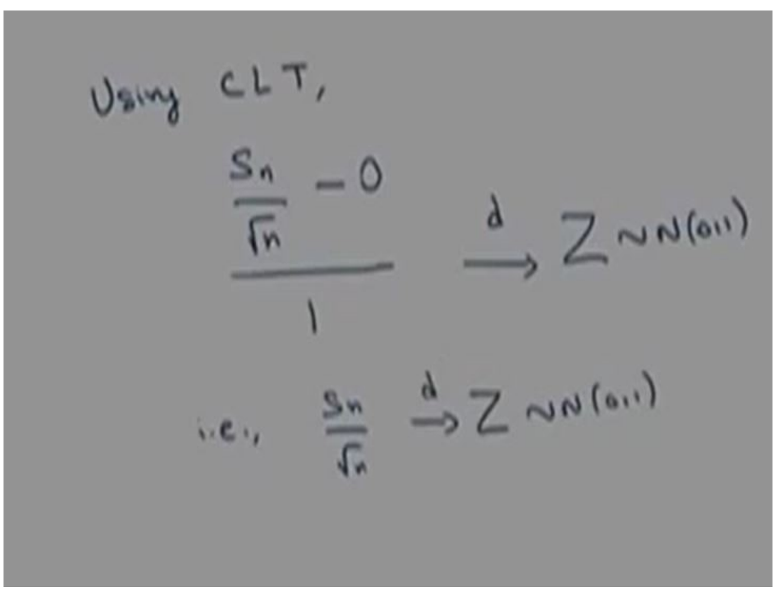

So the way I have got the result for expectation of Xi's and expectation of Sn, I can go for what is the limiting distribution of Sn. So using center limit theorem, I know what it is a mean for each

Sn and I know what is the variance of each Sn also, therefore using a CLT I can able to conclude Sn divided by square root of n - the mean of this random variable is 0 divided by the standard deviation is going to be 1 and this has a n tends to infinity.

This will be a standard normal distribution, where Z is going to be a standard normal distribution has n tends to infinity and this convergence via distribution that means I can able to conclude the distribution of Sn / square root of n has n tends to infinity in distribution this sequence of random variable will converge to the standard normal in distribution.

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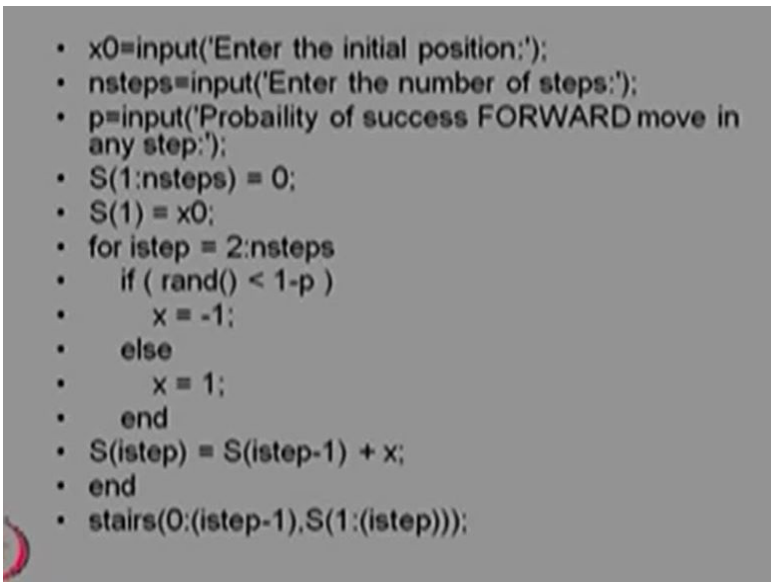

I can go for creating what is a sample path of the simple random walk by using the Matlab code. So for that I have to fix what is the initial position and what is a maximum number of steps I would like to go for finding the sample path and what is a probability of success in each, what is a forward move probability accordingly it is going to take the value 1 with the probability p and it is going to take the value -1 with the probability 1 – p.

So I am giving the value of p only and then I am just going for the possible values of Sn by adding the 1 or -1 accordingly, I am just writing the sample path of Si's.

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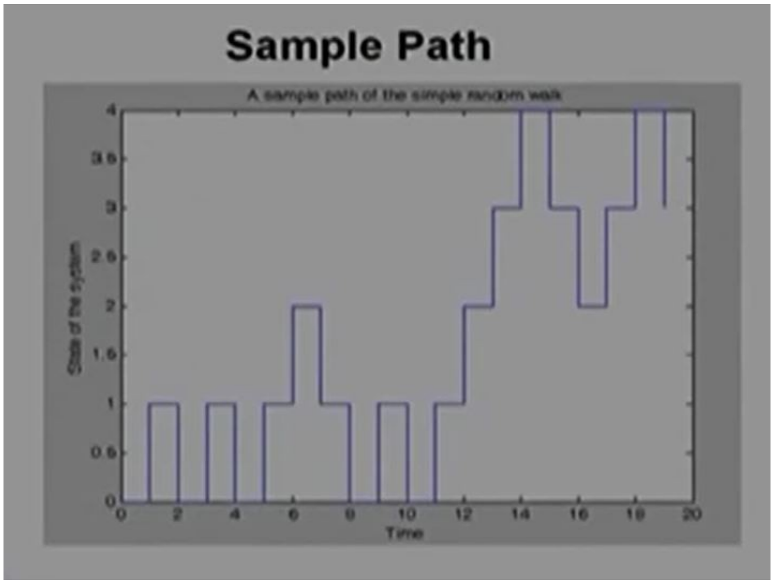

So if you see the sample path over the time 0 to 20 and each Xi’s are going to take the value 1 or -1 accordingly, the Sn is going to take the same value or incremented by 1 or decremented by -1 according to the values of Xi's. Therefore, this is going to be the one sample path, which is depicted using the MatLab code.

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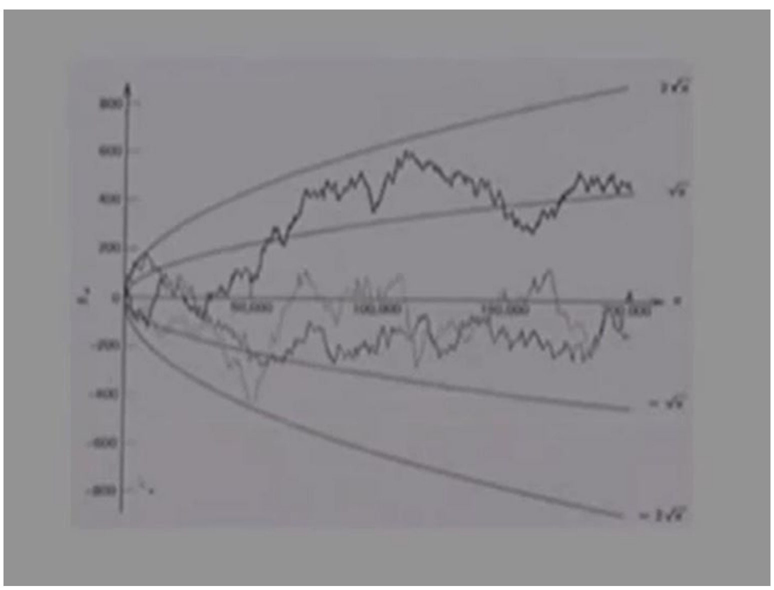

So this is the earlier, I have shown the same graph. This is the Sn as n tends to infinity. Here you can see the different sample path for as n tends to infinity, you can find out what is the distribution of Sn divided by square root of n, as n tends to infinity also and this figures it has a three different sample path and one can observe what is the amount of a person A has n tends to infinity that depends on whether he is going to take the positive value or he is going to have the negative value depends on the 1st few games.

That it can be observed from this diagram, the first few results whether he is going to gain by 1 Rupee or he is going to lose by 1 Rupee. Accordingly, the possible values of Sn will go as n tends to infinity. Now we are going to discuss the fourth simple Stochastic process that comes in the population model.

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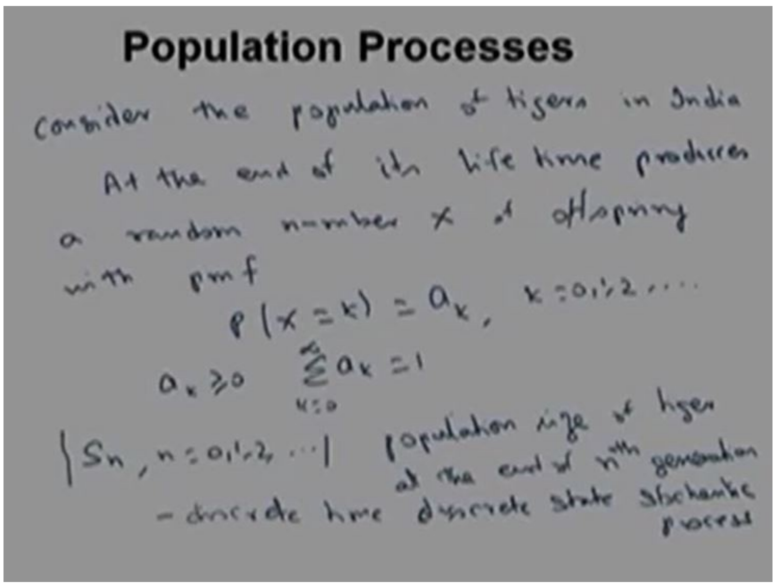

Now we will see the fourth simple stochastic process arises in the population model. Consider a population of Tigers in India. For over the time, this is going to form a stochastic process, so I am going to make the assumption. At the end of its lifetime, it produces a random amount, random number X of off spring with the Probability mass function, that is the probability of X takes the value k that is 'a k', where it satisfies.

a k's are going to greater than or equal to 0 and the summation is going to be 1 and also I am making the assumption, all the offspring's act independently of each other and at the end of their lifetime, individually can have a pregnancy accordance with the probability mass function, the same probability of Xi's takes the value k, with this Sn.

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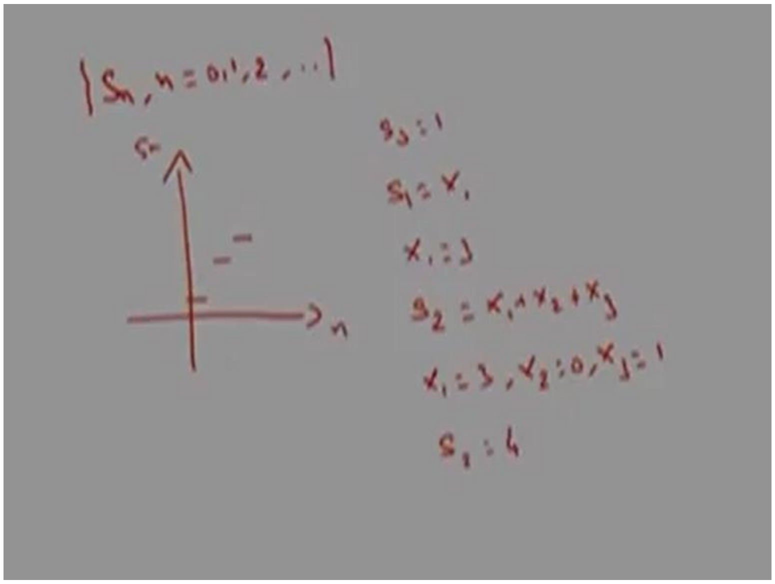

With this, Sn will form a discrete time and discrete state stochastic process, where Sn is the population size of tiger, at the end of the nth generation and if you see the sample path of Sn over the different generation, suppose you make it S0 is equal to 0 and suppose you make it S1 is equal to X1 and suppose X1 takes the value 3 and then the second generation S2 is going to be X1 + X2 + X3 and suppose you make it X1 takes the value 3 and X2 takes the value 0 and X3 takes a value 1, then we have a S2 is going to take the value 4.

So, if you see the sample path of Sn over the n it is going to take the value 1, then it is going to take the value 3, then it is going to take the value 4 and so on. And this is the sample path of the population size of a Tiger over the nth generation and this is going to form a discrete time, discrete state stochastic process and there is an another stochastic process, Gaussian process that I will discuss in the later lectures.

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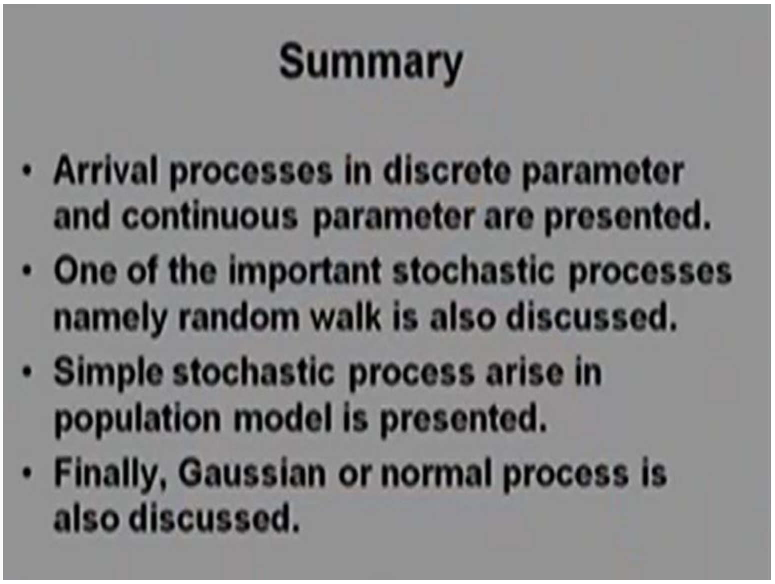

And in this lecture, we have covered the arrival process of the two type 1 is the discrete time and the another is the continuous time arrival process and we have also discussed the random walk and we have discussed a simple stochastic process arises in the population model and the Gaussian process that I will discuss later.

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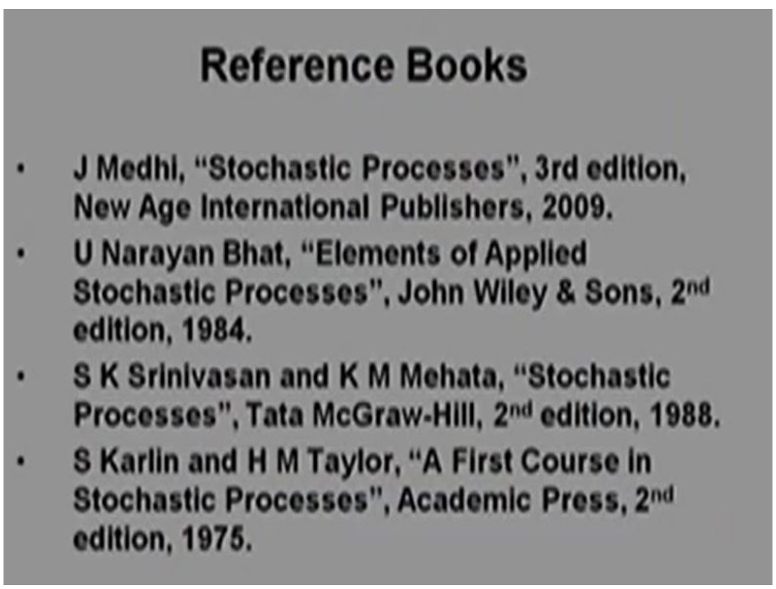

And the references books are which, so with this I complete the model 2 of a definition and the simple stochastic processes. Thank you.