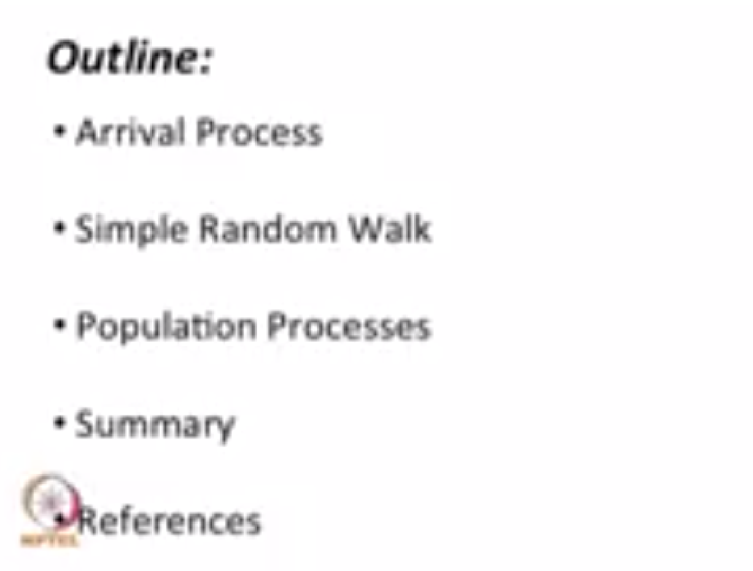
Stochastic Processes - 1
Dr. S. Dharmaraja
Department of Mathematics
Indian Institute of Technology – Delhi

Lecture - 21
Bernoulli Process

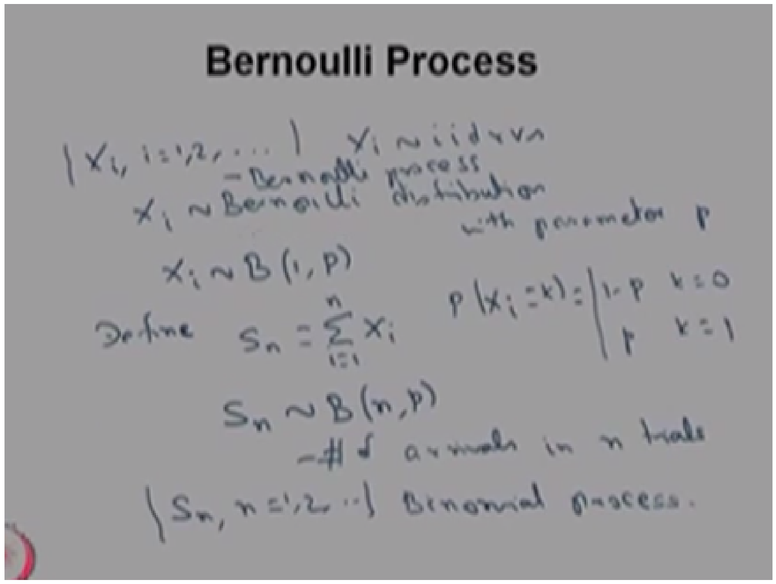
This is module 2, definition and simple stochastic process and today is lecture 2 simple stochastic process. In the lecture 1, we have seen the definition of a stochastic process and the classification of a stochastic process based on a time space and the parameter space. And we have given a few simple stochastic process via the classification.

(Refer Slide Time: 00:50)



In this lecture, we are going to discuss some simple stochastic process starting with the discrete time arrival process that is the Bernoulli process and continuous time arrival process that is a Poisson process. Followed by that we are going to discuss the simple random walk, then we are going to discuss a one simple population process, which arises in the branching process, then we are going to discuss the Gauzier process. So, with that the lecture 2 will be over.

(Refer Slide Time: 01:25)



What is Bernoulli Process? Bernoulli process can be created by the sequence of a random variable. Suppose you think of a random variable Xi, where 'i' is belong, 'i' takes the value one, two and so on, therefore this is going to be a collection of random variable and each random variable are X size, and you can think of X size are going to be an iid random variables. And each is coming from the Bernoulli trials.

That means each random variable is a Bernoulli distributed, each random variable is a Bernoulli distribution and with a parameter p. So the same thing can be written in the notation form Xi takes the, Xi's are in the notation, it is the capital B (1, p). That means it is a binomial distribution with the parameters 1 and p that is same as each Xi or Bernoulli distributed with the parameter 1 and p.

So now I can, so this is going to be a stochastic process or we can say it is a stochastic sequence. Now I can define another random variable, for every n, Sn is nothing but sum of first n random variables. Suppose you think Xi is going to be the outcome of the i-th trial, so the Xi can take the value zero or one that means with the probability the X each, Xi can take the value k. If k =0 with the probability one minus p and k can take the value one with the probability p.

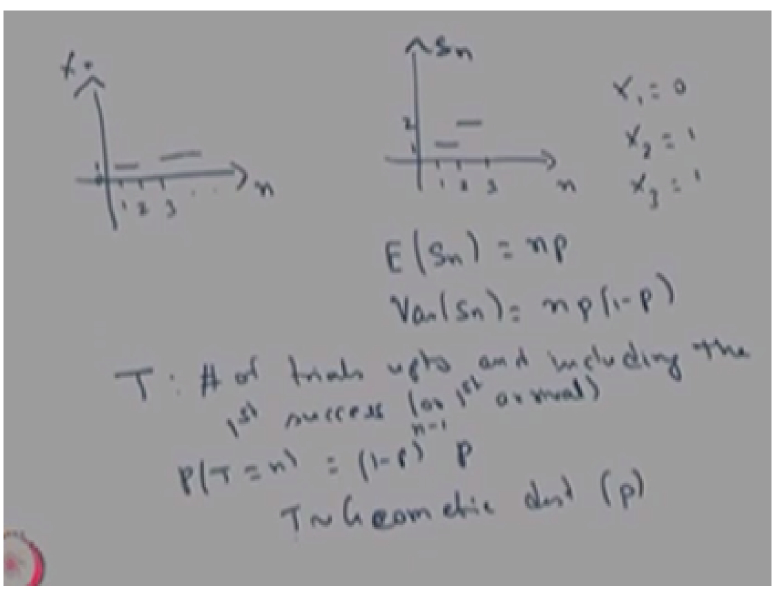
Therefore, since each Xi 's are iid random variable, you can come to the conclusion Sn is nothing but binomial distribution with the parameters (n, p). Suppose you assume that Xi is going to be number of whether the arrival occurs in the i-th trial or not. If Xi takes the value zero, that means no arrival takes place in the i-th trial. If Xi takes the value one that corresponding to the i-th trial, there is at arrival.

So the Sn represents, Sn denotes the number of arrivals in n trials. So now you can create a stochastic process with Sn, where n takes the value one, two and so on, therefore this is going to be a binomial process. So the Xi takes the value zero or one with the probability one minus p and p, each one is going to be Bernoulli distributed, therefore this is going to be a Bernoulli process. This Xi are going to form Bernoulli process.

The way you have created Sn is equal to sum of first n random variable and each Sn is going to be a binomial distribution with the parameters n and p. Therefore, this Sn, that sequence of Sn for n is equal to one, two, three binomial process. Therefore, since you have collected arrivals over the possible values of one, two and so on, therefore this is going to be a one of the discrete time arrival process.

So similarly we are going to explain what is the continuous time arrival process, whereas here binomial process. This is going to be a discrete time arrival process.

(Refer Slide Time: 05:47)



Suppose you like to see the trace of Sn, so before you go to the trace of Sn, we can go for what is a trace or sample path of Xi. For different values of n is equal to one, n is equal to two, n is equal to three and so on, if you see each Xi takes a value zero or one, therefore it can take the value zero or X1 can take the value one or X2 can take the value zero or this can take the value one, again it can take the value and one and zero.

So the possible values of Xi's are going to be zero and one, therefore each Xi's can take the value zero in the horizontal line or it can take the one till you get the next trial. Similarly, if you make the sample path or the trace of Sn, since Sn is going to be sum of first n random variable, therefore based on the Xi takes the value, suppose X1 takes the value zero and suppose X2 takes the value one and suppose X3 takes the value one and so on.

So since X1 = 0, therefore S1 is zero. Then that S2 is same as the X1 plus X2, therefore it takes the value one. And X3 = X1 plus X2 plus X3, therefore that is going to be again, you are adding the values therefore it is going to be a two, therefore this is one and this is two. So based on the X4, it is going to be zero or one, either it can take the value two itself or it can go to the three.

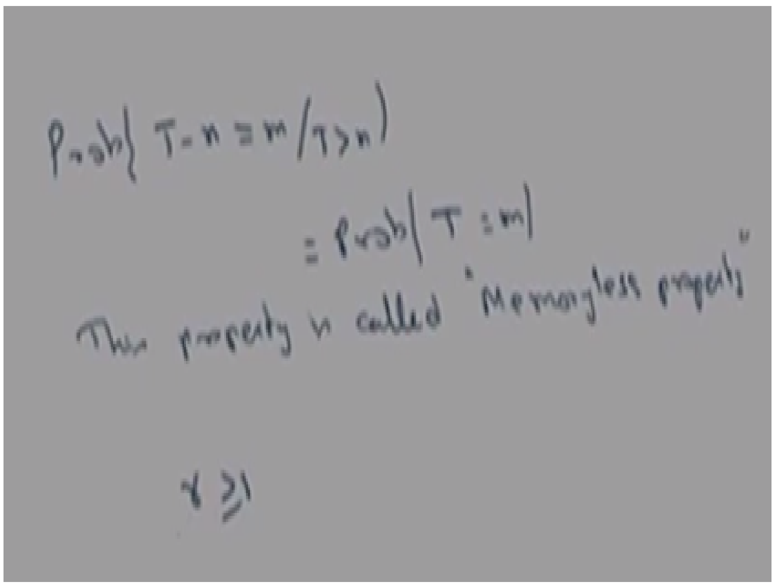
Therefore, if you see the sample path of Sn, it is going to be either incremented by one or it takes the same value till the next n. Therefore, not only you can find out the Sn, you can, not only you can find out the sample path of Sn, you can get the mean and variants because each Sn is going to be a binomial distribution with the parameters n and p, therefore the expectation of Sn is going to be n times p and the variants of Sn is going to be n p (1 – p).

So you can able to see the sample path of Xi's as well as Sn over the different values of n. In discrete time sample paths are sequences. I can also define the new random variable capital T is nothing but number of trials up to and including the first, that means suppose it takes a value n that means for subsequent n minus one trial so I got the failures or no arrival takes place in subsequent n minus one trial and at the n-th trial I get the first arrival.

That means the Ts are under variable to denote how many trials to get the first success or the first arrival. So if it is going to take the first arrival in the n-th trial then the probability of T takes value n that is same as (1- p) (n- 1) p because all the trials are independent and subsequent n- 1 trial gives no arrival and the n-th trial you get the first arrival. Therefore, this is going to follow a geometric distribution with the parameter p.

So since you know the distribution of T, you can find out the mean and variance because the mean of a geometric distribution is going to be 1 / p and the variance of T is going to be (1- p) / p square.

(Refer Slide Time: 10:05)



Similarly, I can go for finding out what is a probability that till n-th trial, I did not get the first or I did not get the first arrival. So if n plus m th trial, if I am getting the first arrival, what is the probability that it is going to take after m trials, it gets the first arrival that probability you can able to get that is same as the probability of the T takes value m. So this property is called memory less property.

Since T is geometrically distributed and the geometric distribution satisfies the memory less property that can be visualized in this example, the probability of T minus n is equal to m given that the T takes the value greater than n that is same as what is the probability that the T takes the value small m. That means the right hand side result is independent of n and it is same as the distribution of that means the residual arrival, number of arrivals that is same as the original arrival distribution.

Therefore, this satisfies the memory less property. So this is the geometric distribution satisfies the memory less property in the discrete time and there is another distribution satisfies the memory less property in the continuous time that is the exponential distribution. So the way I have related the binomial distribution from the Bernoulli process, then I get the binomial process, also I was able to create the geometric distribution.

You can create the or you can develop the Pascal distribution or negative exponential distribution. The way I have defined the capital T is going to be the number of trials to get the first success or first arrival, instead of that if I make another random variable to go for, how many trials are needed to get the r-th success, where r can take the value greater than or equal to one.

If it is the r-th first success is going to happen in the n-th trial, if r is greater than one, then I can go for defining what is the negative binomial distribution for that particular random variable. If r is equal to one, then that is landed to be the same random variable capital T.