Stochastic Processes-1
Dr. S. Dharmaraja
Department of Mathematics
Indian Institute of Technology – Delhi

Lecture - 02
Probability Space and Conditional Probability

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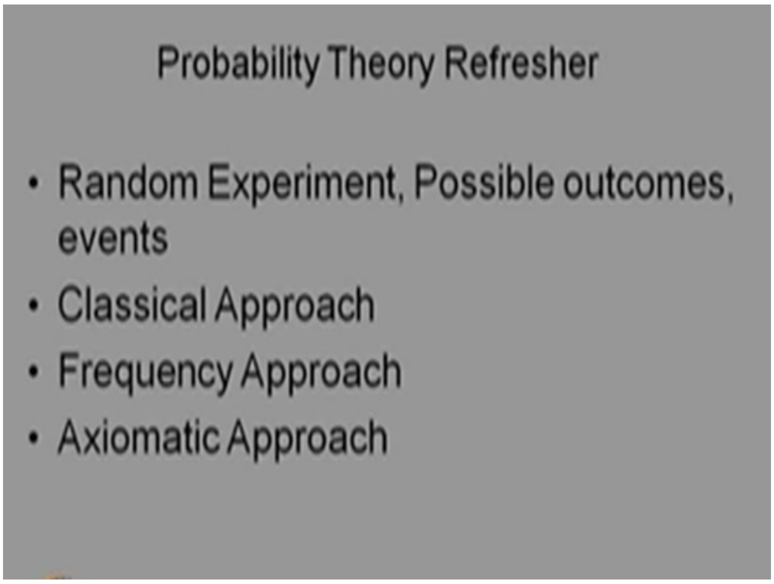

So for that we need the probability theory in detail. So even though we cannot explain the whole probably theory in complete, I am just going to give, I am just making a refresher type of defining what is the probability and what is the random variable and so on and I will cover up whatever the probability theory knowledge is needed for the stochastic process, that I will explain in this lecture as well as the next lecture.

And some of the indetailed probability concepts which will be used later that I am going to explain whenever the problem comes into the picture. So for that first we need what is random experiment? Random experiment is an experiment in which you can able to list out what are all the possible outcomes going to come if that experiment is going to actually takes place. So that means before the experiment takes place you can always able to list out the possible outcomes.

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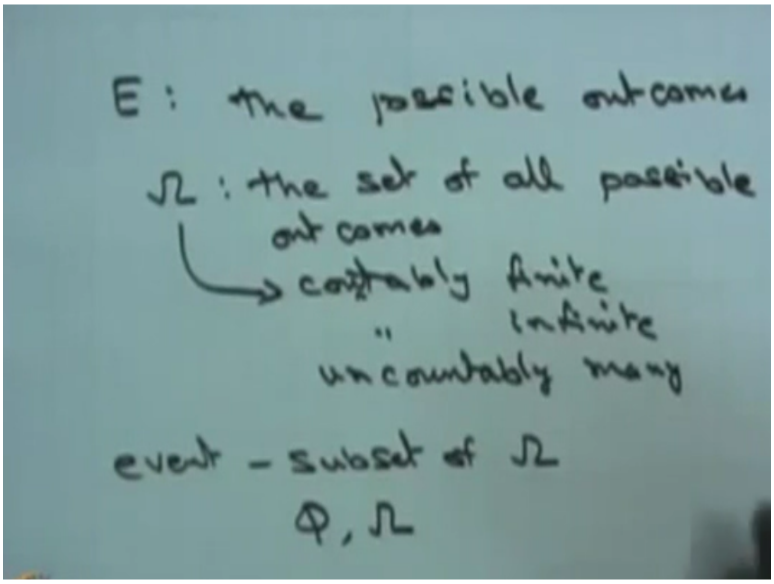

So the possible outcomes that I am going to make it as the collection with the word called omega. So the omega is a set of all possible outcomes. The outcomes could be a numerals or non numerals as well as the outcomes, the omega could be countably finite. It could be a countably finite or it could be countably infinite or it could be uncountably many also. So the way you have chosen the random experiment when you start collecting the possible outcomes that collection I am going to use, I am going to put it in the collection called omega.

Once you have the omega then you can go for creating the event. The event is nothing but the subsets of omega. So the possible events are starting from the empty set as well as these are all the just you can get it like that, it is empty set as well as omega and you can create all the possible subsets of omega that is also going to form events.

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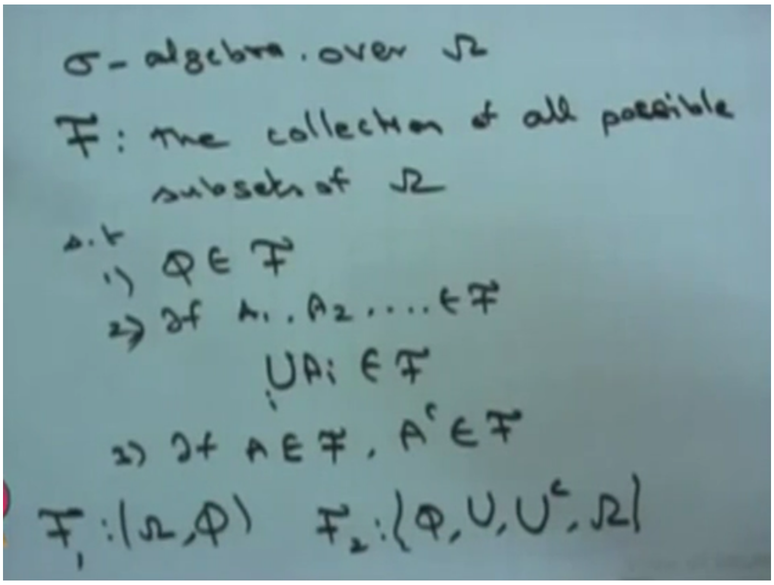

Next we are going to make a probability space, so to define the probability space you need sigma algebra. So what is sigma algebra? What we are going to create sigma algebra over omega. So that I am going to use the word F. F is the sigma algebra over the omega. That is the collection of all possible subsets of omega such that the empty set is belonging to F and if I take then the union of Ai is also belonging to F.

The third condition, if I take one element from F then that complement is also belonging to F. So that means the sigma algebra over the F over the omega that F contains, the collection of all possible subsets of omega such that these three conditions are satisfied. That means you can go for making the trivial F that is going to be contains only in the empty set as well as the whole set. This is also going to be one of the sigma algebras over the omega that is the default one.

Like that I can go for creating many sigma algebra that by making a few elements of possible outcomes that I make it as the set A then I can make it the another sigma algebra that has empty set and I can make a one set called U and U consists of few elements of omega and U complement then I can have omega also. So like that I can keep creating the different sigma algebra over omega and the trivial one is the empty set with the omega set that is going to be the trivial one and now I am going to define the probability space. What is probably space?

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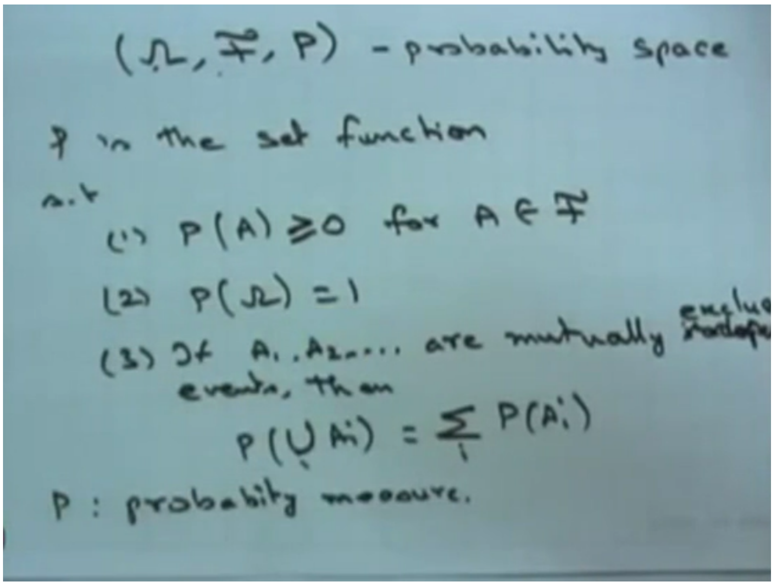

The probability space is a triplet in which the omega is the collection of possible outcomes and F is the sigma algebra over omega and P is the set function. Such that the P of A is always going to be greater than or equal to zero for any A belonging to F. The second condition, the P omega is going t be one always. The third condition if I take few Ai or mutually independent events, then the P of union of Ai is same as summation of P of A i.

Let me just explain the probability space in a better way. This triplet is going to be call it as a probability space as long as you have a collection of possible outcomes and you have a sigma algebra. So this sigma algebra can be anything and you can go for the default one is the largest sigma algebra which you have created and P is the set function such that whatever the element you are going to take it from F, any elements of F is going to be event.

So P of any even that is going to be always greater than or equal to zero and if you take the event is going to be omega, therefore the omega is also one of the element in the F and P of omega is equal to one. And the third condition if you take Ai are mutually exclusive events, then the probability of union is going to be the summation of the probabilities. Summation of P of A.

Then this P is going to be the set function and P is going to be the probability measure. This P is going to be call it as… This P is the probability measure. And this P is the normed measure also because of the condition, P of omega is equal to one. There are many definitions over the probability theory, the classical approach or the frequency approach and what we have given is the Axiomatic approach.

So the way I have given the definition that is the probability space with the omega F and P and this is called the Axiomatic approach. And we are going to use the Axiomatic approach on the frequency approach or the classical approach and you should note that the classical approach is going to be the special case of the Axiomatic approach in which you make the collection of a possible outcomes are going to be equally likely then the classical approach is going to be the special case of Axiomatic approach.

Therefore, throughout our course we are going to use the Axiomatic approach not the classical approach.

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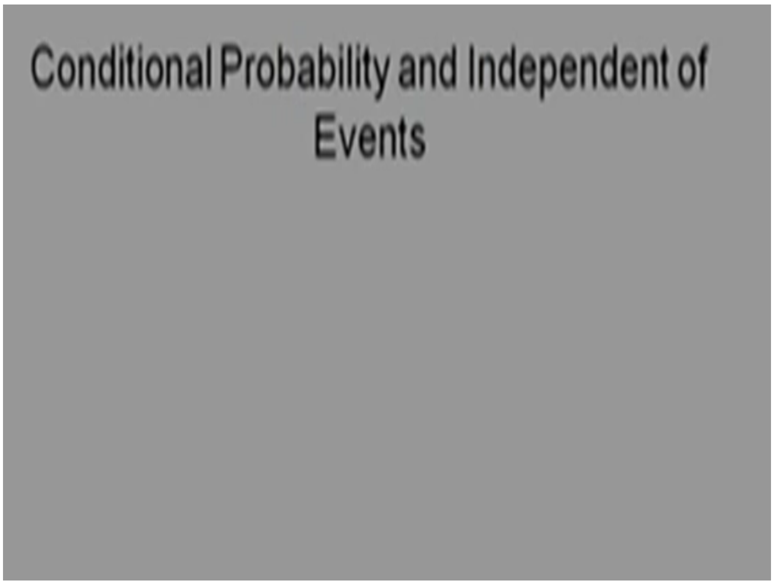

Next I am going into the concept called conditional probability.

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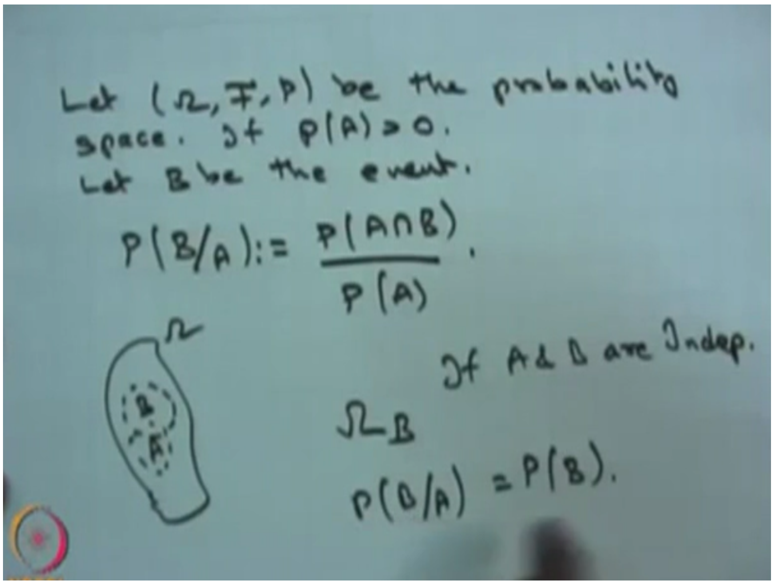

So suppose you have a probability space, if you P of A is greater than or equal to zero, then let B be the event, you can define the probability of B given A is same as the probability of A intersection B divided by probability of A. That means, if already the event A occurs with the positive probability then you can find out what is the probability of the event B given that already the even A occurs that is same as what is the probability that A intersection B divided by probability of A.

So this is by the definition and this can be visualized from the reduced sample space also, that means you have a sample space omega and from the omega you take even A, suppose this is going to be the event A, what you are saying is the event A has already occurred. That means in this given condition and suppose you make another event that is a event B and you are asking what is the proportion in which already the event A occurs.

And you are asking what is the probability of event B, that means you find out what is the reduced sample space omega B and you find out what is the proportion in which or what is the probability of event B occurs in the reduced sample space is same as by using the definition of probability B given A. That means you find out what is the intersection, A intersection B.

That means, you find out what is the event which correspond to A intersection B and what is the ratio in which the probability of A intersection of B with the probability of A that gives the conditional probability. If the even A and B are independent, if A and B are independent event, then there is no way of relating the probability of B given A then the probability of B given A is same as probability of B.

That means, there is no dependency over the event B and A. Therefore, it is not going to cost anything with the even B by occurring the event A. Therefore, the probability of B is same as probably of B intersection A.