Stochastic Processes - 1
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Lecture No - 18
Classification of Stochastic Processes

So, based on the values of the way I have explained the random variable or the stochastic processes is going to be x of w, t where w is belonging to omega and t is belonging to T.

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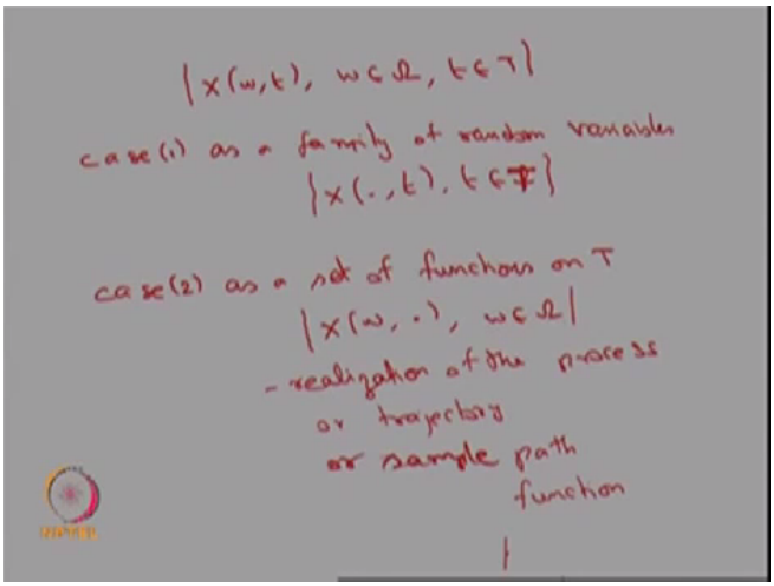

There are two approaches can define the stochastic processes. The first one that is a we name it as a case one. I can say it as the collection of random variable as a family. Family, of random variables as a x dot t where t is belonging to T. So, this is the way I can create random variable and this is the easier approach in the sense once I know the different T or fixed T it is going to be random variable.

And I have collected a family of random variable for different values of t. Therefore, this is the way we can create the stochastic process. This is the easier approach also. The next one that is the case two, that is nothing but as a set of functions on T that is nothing but a collection of x w, for w is belonging to omega that means I have made a function on T and once I fix one w I will have a one function and if I fix another w where w is nothing but a possible outcomes.

Therefore, if I have a different possible outcome that is going to be create a different stochastic process where as therefore I can create a stochastic process of x of w, t either fixing a t or fixing the w accordingly I can have a two different ways of creating a stochastic process. And the case two the way I have made a collection of random variable by fixing the w then I made a function of functions on t.

Therefore, this is going to be the method of realization of the process or it is going to be called it as a trajectory or it can be called it as a sample path or we called it as a sample function also. So, these are all the different ways the case two can be called that means once you know the one possible outcome therefore you are tracing the stochastic processes along the one possible outcomes.

Therefore, that is going to be called it as a realization of the process or the trajectory of the sample path. So, the conclusion is we can always define a stochastic processes as a collection of random variable for different value of t or we can go for a collection of functions on t for a different values of the possible outcomes that is w belonging to omega.

So, these are all the two approaches in creating the stochastic process. Not only we can go for making a one dimensional random variable or one dimensional stochastic process. So, you can create

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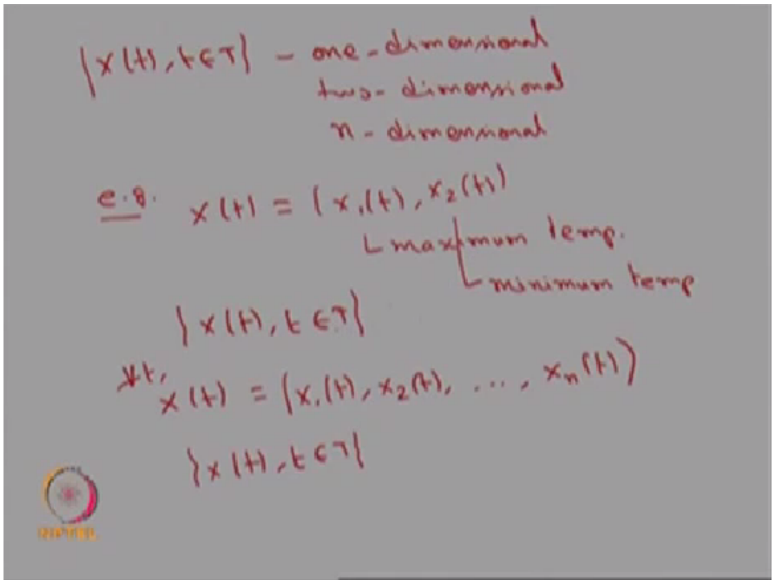

A stochastic process it could be a one dimensional or it could be two dimensional or it could be a n dimensional also. So, first we have discussed what is stochastic process and how to create a stochastic process whether it existed so on then we have given the parameters based on the state space then we have given what are all the ways we can create the two different approaches you can create the stochastic process.

Now, we are discussing what is the dimension of the stochastic process? Whether the default it would be a one dimensional or it could be two dimensional or it could be n dimensional? Let me give a one simple example in which it is going to be two dimension that means I have a random variable x of t that is going to be x1 of t, x2 of t in which x1 of t is nothing but the maximum temperature and the x2 of t could be minimum temperature.

The maximum and minimum temperature possible of a place at any time t and this together is going to be a one random variable that means this is a random vector which consist of two random variable x1 of t and x2 of t that means for fixed t you have a one random vector and some t and therefore you have random vector for over the t and this random vector will form a stochastic process therefore, this is going to be a two dimensional stochastic process.

Therefore, in general, you can define a n dimensional stochastic process with the far fixed for every t you have a random vector at some t that is going to be a x1 of t, x2 of t and so on. It is going to be the n th element is the xn of t that is going to be n different in which each one is going to be random variable for fixed t and this is going to be random vector for fixed t and this is going to be n dimensional stochastic process in which each one is going to be one dimensional random variable for fixed t.

So, that means you can go for making a one dimensional random variable then you have a collection of random variable form a one dimensional stochastic process or you can have a two dimensional like that you can have a n dimensional stochastic process. In the course what we are going to discuss always it is a one dimensional stochastic process.

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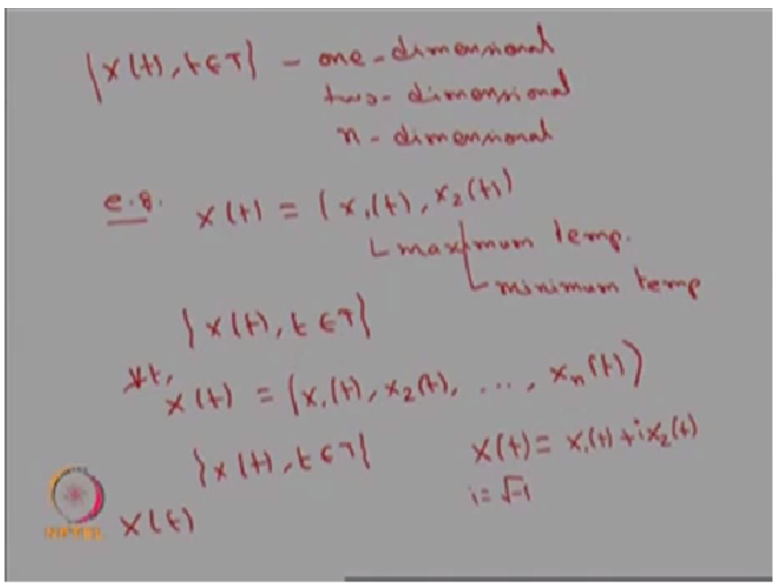

We can always create a complex valued stochastic process also in the form of x of t. If I need it here the x of t is going to be x1 of t plus I times x2 of t where i is nothing but the complex quantities square root of minus one. That mean the x1 of t is a real valued random variable for fixed t and x2 of t is also a real valued the random variable of x t. The way I have meet it the x of t.

This is going to be complex valued random variable for fixed t therefore the x of t over the t that is going to be form a complex valued stochastic process because for fixed t x of t is going to be complex valued random variable. The corresponding stochastic process is called complex valued stochastic process with the one dimensional form like that you can go for the multidimensional complex valued stochastic process and so on.

But in this course here what we are not really discussing is only the real valued one dimensional random variable most of the times. Sometimes we are discussing real valued two dimensional or n dimensional stochastic process that too with the real valued random variable not the complex valued. So, now we are going for classification of stochastic process.

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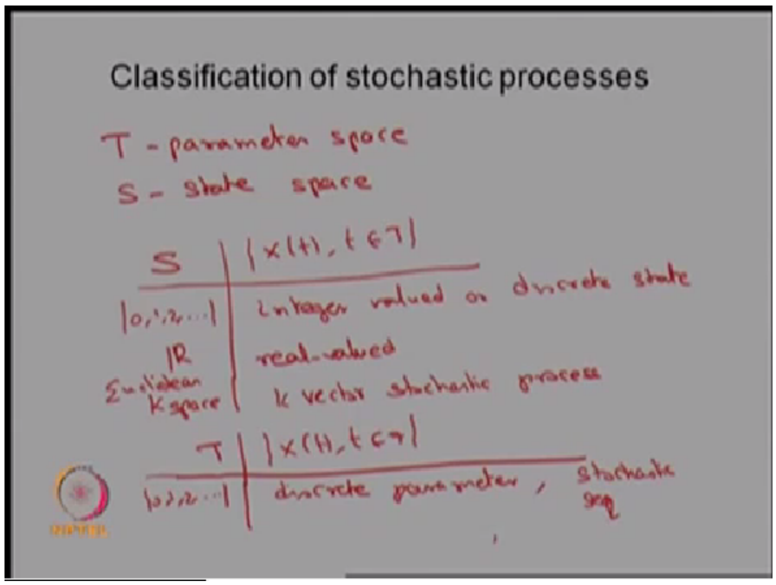

The way, I have explained the parameter space T. The capital T is parameter space and S is going to be the state place that is nothing but the collection of a possible values of x of t and the possible values of a small t belonging to the T that form a parameter space. Some books, they use the notation parameter Z also and capital S is going to be the state space. Now based on these we are going to classify the stochastic process. Suppose let us start with the S.

Suppose, the possible values of s and what is the name of the stochastic process if s is going to take the only countably infinite or countably finite values? Then it is going to be called as the corresponding stochastic process is going to be called as a integer valued stochastic process or we can call it as a discrete state stochastic process. So, whenever the possible values of S is going to be accountably finite or accountably infinite then we say it is a integer valued stochastic process for a discrete state stochastic process.

Suppose, the possible values of S is going to be the real values then we call it as a real valued stochastic process. Suppose, if it takes euclidean space with the k dimensional euclidean k dimensional space then we call it as a k vector space, k vector stochastic process that means the each random variable going to have a one dimensional random variable and like that you have a k and the variables of our fixed T.

Therefore, you have a k vector stochastic process therefore it is going to be call it as a k vector stochastic process in which each element is going to be one dimensional random variable for fixed T. So, the collection that the k (()) (11:09) value is stochastic process is going to be call it as k vector stochastic process. Similarly, you can go for based on the T what is the name of the stochastic process for different values of T.

That means if it is going to take the value countably finite or countably infinite or it is going to take a only the integer values then we say it is a discrete parameter stochastic process or there is another name it is called the stochastic sequence also whenever the possible values of the T is going to be accountably finite or accountably infinite then we call the corresponding stochastic process as the stochastic sequence of it is a discrete parameter stochastic process.

Otherwise, if it takes uncountable many values in the T then it is going to be call it as a continuous parameter or it going to be called it as a stochastic process itself. Therefore, based on that discretize it uses the world sequence or if it is going to be a uncountable many values of T then it is going to be called it as a stochastic process.

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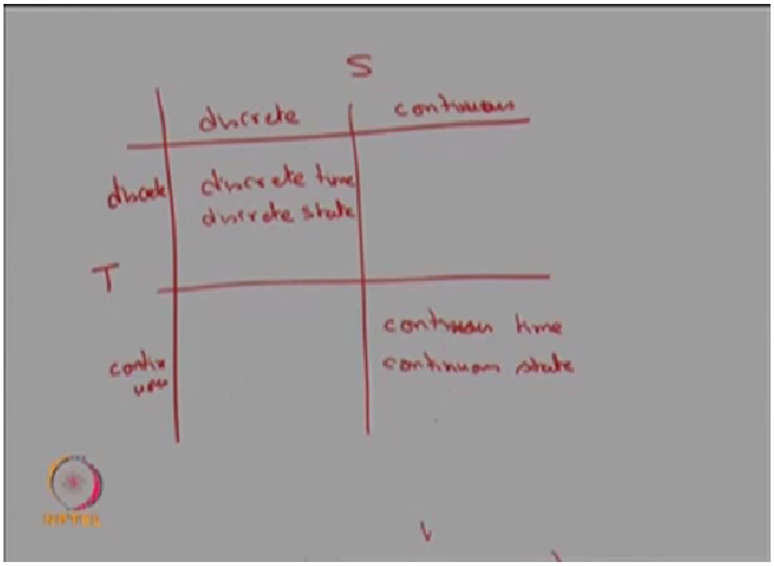

So, based on the classification I can go for making a one table in which the possible values of S will take a column and the possible values of T will make row. So, either it could be a countably finite or countably infinite data uses the word and discrete. If the possible values of T is going to be uncountably many either it is set of all intervals or it would be whole real line itself or it is going to be a union of many intervals in that case it is going to be call it as a continuous parameter.

Similarly, if the possible values of S are going to be countably finite or countably infinite then the state space is going to be call it as a discrete. Similarly, if it is going to be uncountably many values then it is going to be called it as continuous. So, accordingly you can classify the stochastic process into the four type in which if the T is going to be discrete as well as S is going to be a discrete.

Then it is going to be a discrete time or discrete parameter both are one and the same. So, discrete time, discrete state stochastic process. Similarly, if the D is a discrete and the state space is continuous then we can call it as a discrete time continuous state stochastic process. Similarly, this is going to be a continuous time discrete state stochastic process and this is going to be a continuous time, continuous state stochastic process.

That means based on the possible values of a T the possible values of S any stochastic process can be classified into the four types in which it is going to be a discrete, discrete or continuous,

continuous, or discrete continuous, continuous, continuous, based on the time and the state space.