Stochastic Processes - 1

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Lecture - 17
Definition of Stochastic Processes, Parameter and State Spaces

This is the model two of stochastic processes. In this modal, what we are going to discuss is the affiliation then followed by this stochastic processes. And this modal consists of two lectures. Here, this is the first lecture in which we are going to describe the stochastic process then we are going to discuss the classification of stochastic process followed by a few simple examples which arises in the real world problem. So, the content of this lecture

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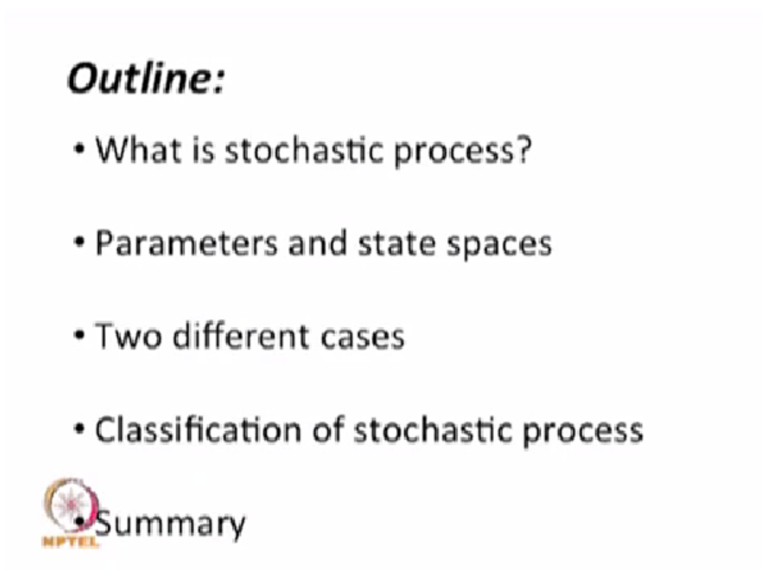

is going to be as I said let me first give that affiliation of stochastic processes then I will explain how to create or how to develop this stochastic process. What is the meaning of a parameter and this takes place then I am going to give what are all the approaches in which stochastic processes can be described and the classification of stochastic processes based on the parameter and the state space. Then, at the end of this lecture we are going to discuss

some of the few simple stochastic processes and the summary of the lecture the lecture one and there are few reference book also listed for this course preparation.

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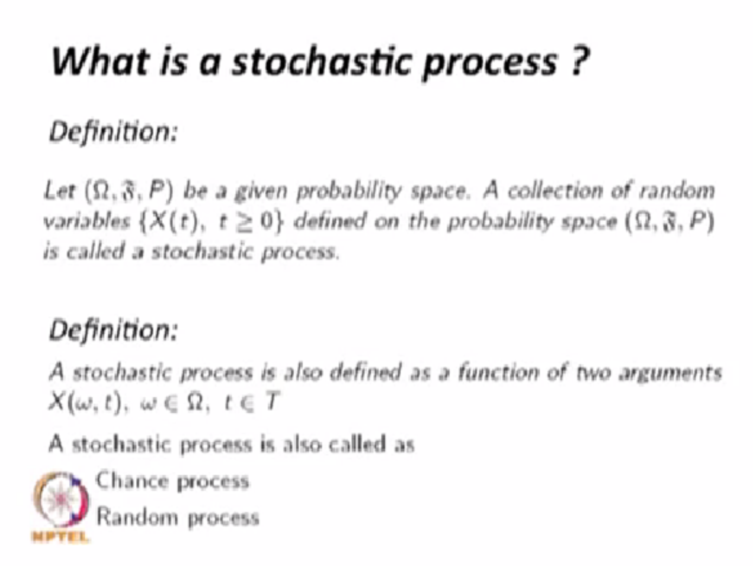

What is stochastic process? Let me give the definition. Let, omega f p be a given probability space. That means you know what is a random experiment? From the random experiment you know what the omega and from the collection of possible outcomes you got that sigma algebra that is capital f and you have probability measure also. Therefore, this triplet is going to be the probability space and you have a given probability space.

From the given probability space, you have the collection of random variables that is x of t where t is belonging to capital T defined on the probability space that is omega e f, P that is called a stochastic processes. That means you have probability space from the probability space you have collected a random variables with the T belonging to capital T and this collection is going to be called it as a stochastic processes.

Now the question whether we can create a only one stochastic processes or how to create a stochastic processes from the sigma algebra that means suppose you have a omega from the omega you can always create a sigma algebra that is a F that is a collection of a subsets of omega satisfying the condition. If you make a union of a few elements, then the union of elements is also belonging to one of the elements and if you take any one of the elements in the F then the compliments is also belonging to F.

So, if these conditions are going to be satisfied then that collection of subsets of omega is going to be called it as sigma algebra. So, from the omega we have created a random variable that is x of T 1 that is nothing but a real valued function which is defined from omega to r such that it satisfies the condition x of T 1 of inverse of minus infinity to the closed interval x that is belonging to f for all x belonging to r.

That means whatever be the x belong to r if the inverse images from minus infinity to some point e x if that is belonging to F then that real valued function is going to be called it as random variable. Like that if you make a different random variable for different T where all the T are belong to so I can go for T I where A is so all the T are belonging to T. So, that means if I have a collection of random variables for the different values of T then that collection is going to called it as a stochastic processes.

Now, the question is whether we can create only one stochastic process from a given probability space or more than one stochastic process can be created from the same probability space. The answer is yes you can always creat more than one random variable from the same probability space that means for a different collection of a T you can have a different stochastic process.

More than one stochastic process can be created from one probability space. Now, the next question if I change the sigma algebra what happens? If I change the sigma algebra F then I may land up collecting some other stochastic processes in which those real valued function is going to be random variable for that particular omega and F and P for a given probability space the stochastic processes is going to be changed for a different collection of a T belonging to T.

That means once you know the F then you will have some collection of random variable that will form a stochastic processes. If you change the another F, then you may get the different stochastic processes and also for a given probability space you can have more than one stochastic processes by the way you define a collection of random variable. The way you have a T accordingly you will have a different stochastic processes.

Now the way I have given the collection of random variable I can say it in a different way that is a stochastic processes also defined as a function of two arguments that is x of w, t where w is belonging to omega and T is belonging to T that means the same way I can define the collection of random variable as a collection of w, t where w is belonging to omega and t belonging to T and this is also going to be form it as a stochastic processes.

That means always the W is belonging to omega that means W is belonging to the possible outcomes and the t is belonging to capital T and this is going to set the given probability. This is going to set the stochastic processes. The other names for the stochastic processes are going to be John process. There are some others use the word John's process. There are some others they use the notation that is called a random process.

So either the stochastic processes can be called it as John's process or the random process also. Now, what we are going to see once you have a collection of random variables so based on x of the values of x of t and the values of the different values of T we are going to define what the parameter space and what is state space. What is the meaning of parameter space?

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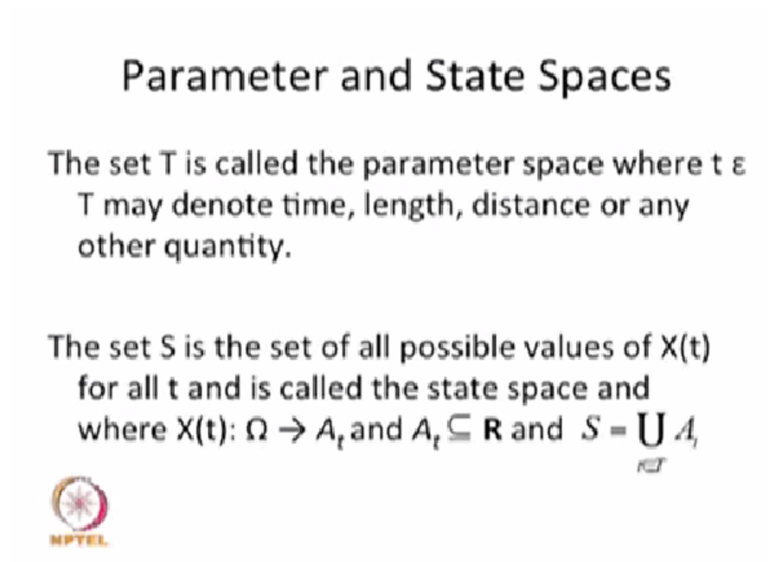

The set we use the notation D that is called the parameter space. The set D is called the parameter space and it is usually represented as the time most of the time or it can be represented as the length or it can be represented as the distance and so on. So, we usually we go for t as a time. So, the set T is called the parameter space. Similarly, I can define the state space as the set

S that is nothing but all possible values of x of t for all t.

So, this set is called the state space. x t is a random variable from omega into A suffix t where A suffix t is a subset of R then the A t are going to be the elements of it is going to be contained in the real line then the S is nothing but union of t belonging to T all the A t that is going to form a state space. That means for fixed T you will have a collection of a possible values that is going to be the A t and for variable T you collect all the union and that possible values of X t is going to form a set and that set is called the state space.

Similarly, the all possible values of a t belonging to T and that set is going to be call it as a parameter space. So, based on the parameter space and the state space we can go for classification. Now, I can explain what are all possible values of S can take.

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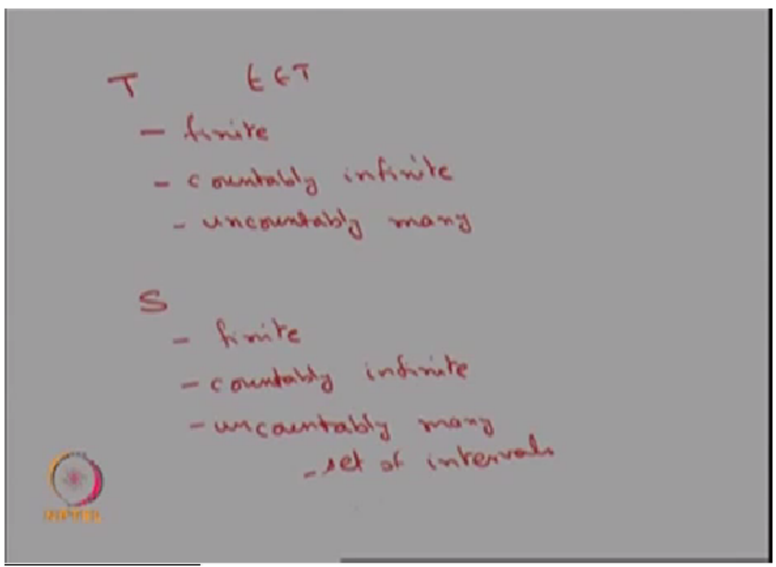

So, this T is going to be the collection of T therefore this can be a finite that means countably finite or it could be countably infinite also or it could be uncountable many elements of t. So, that set can be a finite set or it could be countably infinite or it could be uncountably many elements also. T can also be multidimensional set. Similarly, the state space the S that can be a same way it could be a finite or it could be accountably infinite or it could be uncountably many elements.

So, since the state ways are going to be the collection of all possible values of x t and x t is a real valued function and then it is going to be random variable therefor these elements are going to be always the real numbers. So, either it could be a finite element or it could be accountably infinite elements and it is going to be uncountably many elements that means it could be a set of intervals on a real line or it could be a the whole real line itself.