Stochastic Processes-1
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Lecture – 16
Problems in Sequence of Random Variables (Contd…)

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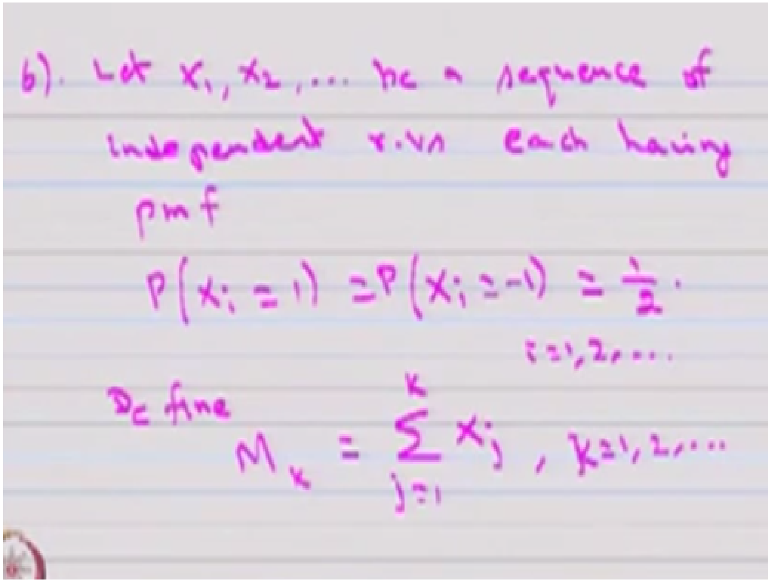

Now we move into next example, example 6, Let x1, x2, so on be a sequence of independent random variables each having probability mass function, probability of xi is equal to 1 that is same as probability of xi takes the value minus 1, the probability is 1 by 2. This is valid for that means it is a sequence of iid random variables and they are discrete type. Define M suffix k as the sum of first k xi random variables.

So this running index is k is equal to 1, 2 and so on. So we are defining a sequence of a random variable Mk by summing first k xi random variables.

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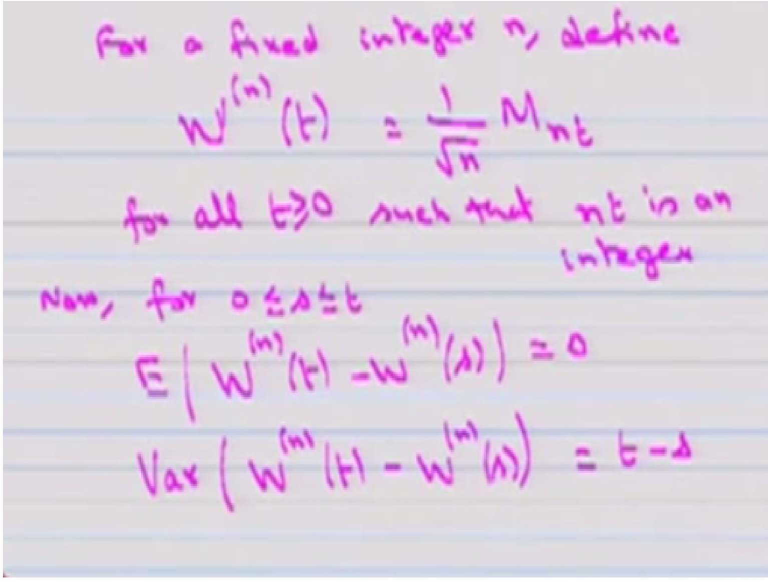

For a fixed integer n, we define another sequence of random variable that is denoted by W superscript n of t that is nothing but 1 divided by square root of n M suffix n times t. This is for all t greater than or equal to zero such that n times t is an integer. So we are defining another sequence of a random variables W superscript n of t that is 1 divided by square root of n times mn of t where n of t is an integer.

So this is valid for all t greater than or equal to zero. If you find out the mean and variants for the difference of the random variable of a n of t minus W n of s for zero less than or equal to s or less than or equal to t, this quantity will be zero. That means W n of t is a 1 divided by square root of n mn of t and the way we define the mn of t that is the summation of xi and the probability of xi is equal to 1 and the probability of x is equal to minus 1, minus 1 is 1 by 2.

Therefore, the mean of xi are going to be zero because of that the expectation of or mean of W n of t minus W n of s that is equal to zero. Also if you evaluate the variants of W n of t minus W n of s by finding first variants of excise using that you find out the variants of mn of t then find out the variants of W n of t minus W n of s that is going to be t minus s.

It need a calculation of expectation of a xi square then using expectation of a xi square and the expectation of xi, you can find out the variants of xi using variants of xi, you can find out the variants of W n of t, then you find out the variants of W n of t minus W n of s.

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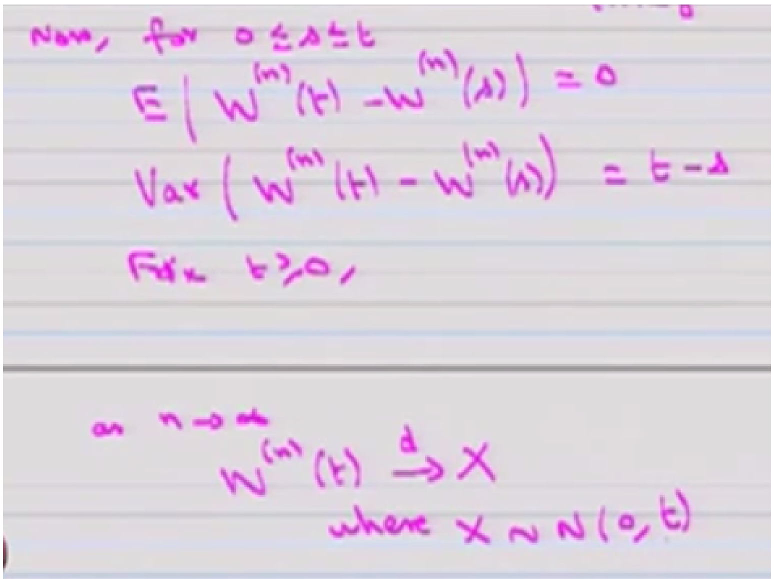

By using mean and variants, for fixing t greater than or equal to zero as n tends to infinity we can conclude W n of t tends to a random variable x and this converges takes place in distribution using CLT, one can conclude W n of t converges to the random variable x and the converges in distribution where x is normal distribution with the mean zero and the variants t.

Using a central limit theorem, one can prove, W n of t converges to x in distribution where x is normal distribution with a mean zero and the variants t. This result is very useful in Brownian motion and this same problem will be discussed in detail when we are discussing the module of a Brownian motion.