Stochastic Processes-1
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Lecture – 15
Problems in Sequence of Random Variables (continued)

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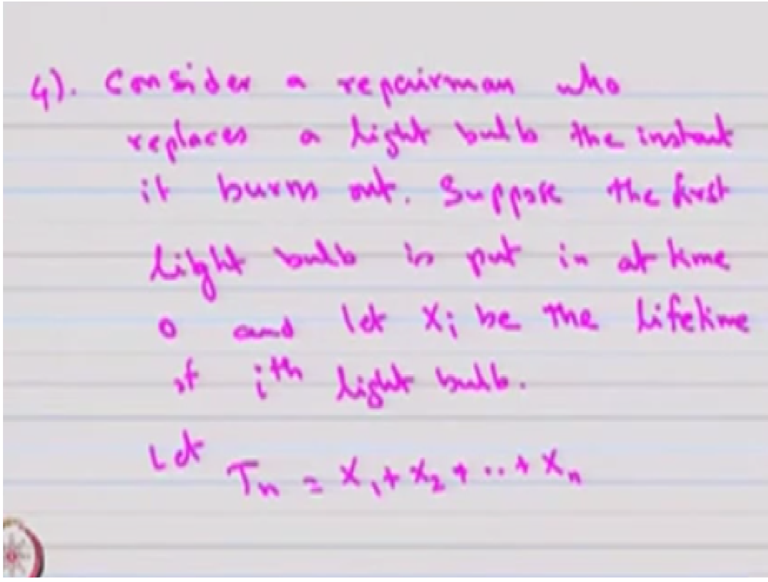

Now I move into the fourth example, consider a repairman who replaces a light bulb the instant it burns out. Suppose, the first light bulb is put in at time zero and let x suffix i either be the lifetime of i-th light bulb okay.

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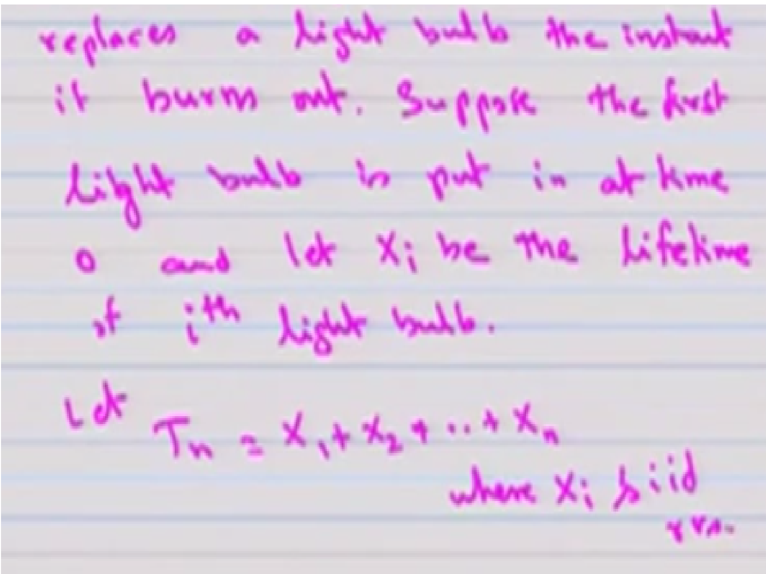

You define the random variable Tn is a sum of n xi where Xi are iid random variables. Xi with a lifetime of the ith light bulb and when Xi are iid random variable you are defining Tn is a x1 plus x2 plus xn and so.

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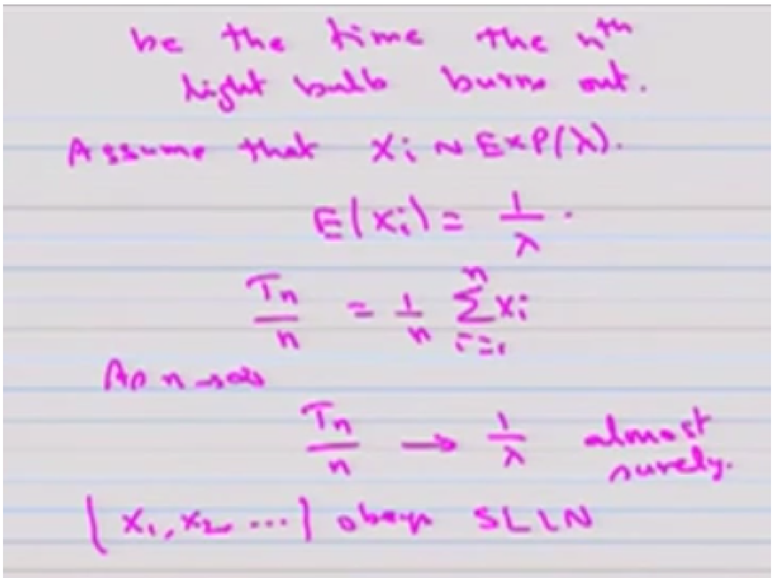

So the Tn be the time of time the nth light bulb burns out because the Tn is a x1 plus x2 and so on till x1 therefore Tn be the time the nth light bulb burns out. Assume that Xi is exponential distribution with a parameter lambda. We know that already Xi are iid random variable, now I am making the further assumption, Xi follows exponential distribution with a parameter lambda. That means, you know what is the mean of this random variable.

Since it is exponential distribution with the parameter lambda, this becomes 1 divided by lambda. Also one can use the result, Tn by n that is nothing but 1 divided by n summation of Xi where i is running from 1 to n as n tends to infinity one can prove Tn by n tends to 1 divided by lambda that is a mean of the random variable Xi almost surely.

I am not proving here the way you do the sequence of random variable converges to another random variable converges takes place in probability or in distribution or in (()) (04:43) or almost surely one can prove this the Tn by n converges to 1 by lambda almost surely. That means we can conclude the random variable x1, x2 and so on obeys strong law of large numbers.

Because the Tn by n that is nothing but 1 by n of summation of Xi that converges to the value 1 by lambda almost surely we can conclude the sequence of random variable Xi obeys a strong law of large number. Even though in this problem, I made the assumption Xi follows the exponential distribution with the parameter lambda, in general the lifetime can be any distribution. So this problem will be discussed in detail in renewal processes.

So as such here we are making the assumption of distribution of Xi is exponential distribution therefore I made it converges takes place almost surely to the value 1 by lambda, this can be generalised. There are many more problems of the similar kind but we are discussing only few problems, therefore we can use the similar logic of finding the moment generating function, then concluding the distribution and finding the limiting distribution.

Or you verify whether the sequence of random variables converges takes place in mean, converges takes place in probability or converges takes place in distribution, or converges in the (()) (06:39) or converges almost surely. This can be used in any problem of at the same way what I have done it here. And I have not discussed any problem in the central limit theorem but that will be used many times therefore I have not given any problems for the central limit theorem.

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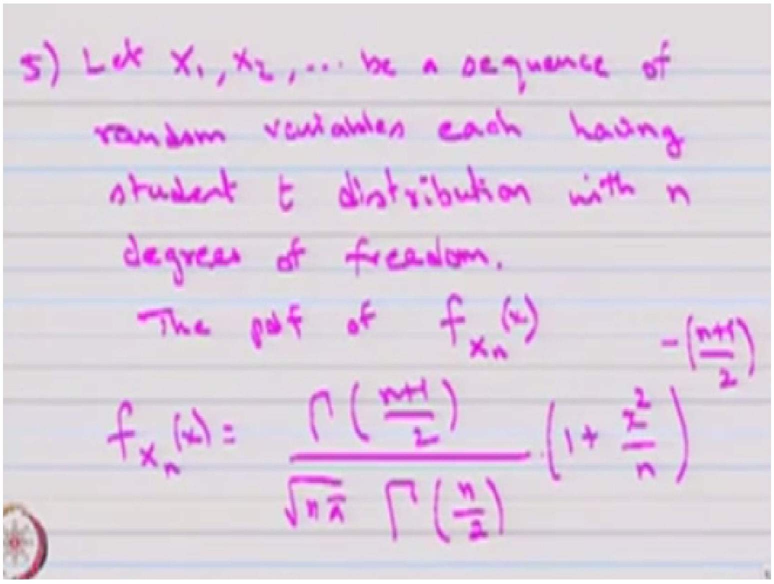

Let x1, x2, so on be a sequence of random variables, each having student T distribution with n degrees of freedom. Our interest is to find out the limiting distribution of a student T distribution. We know that the probability density function of F of x for the random variable xn is given by gamma of a n plus 1 by 2 divided by square root of n times pi multiplied by gamma of n by 2 multiplied by 1 plus x square by n power minus n plus 1 by 2.

So this is the probability density function of a random variable xn.

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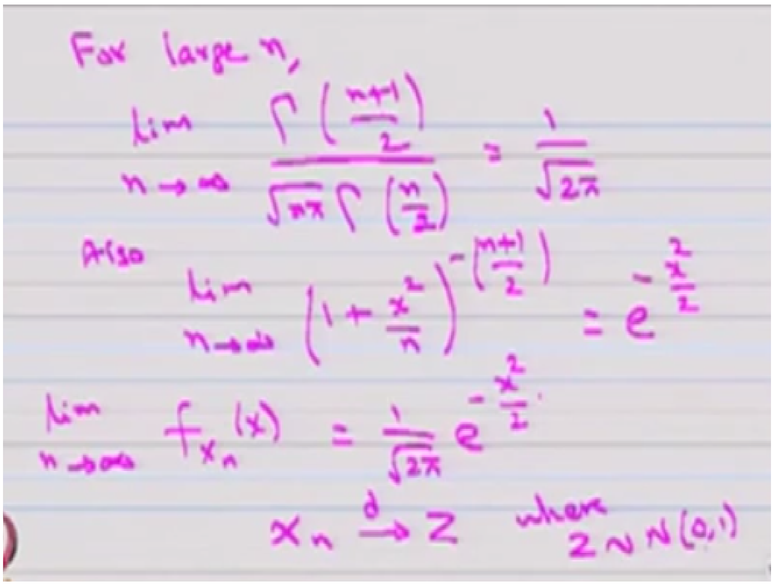

Our interest is to find out the limiting distribution of the random variable xn. For larger, for large n, we have the results limit as n tends to infinity of gamma of n plus 1 by 2 divided by square root of n pi of gamma of n by 2 is 1 divided by square root of 2 pi using Stirling's approximation. And also, limit n tends to infinity of 1 plus x square by 2 the whole power minus n plus 1 by 2 that we know, that is e power minus x square by 2.

Hence, the limit n tends to infinity of the probability density function of the random variable xn becomes 1 divided by square root of 2 pi e power minus x square by 2. Since, right hand side is the probability density function of a standard normal distribution, we conclude for a larger n the sequence of random variables x1, x2, xn and so on that tends to the random variable z and this converges takes place in distribution where z is standard normal distribution.

So this is a simple example of the sequence of random variables each having a student T distribution, the limiting distribution converges to standard normal and that converges in distribution.