Stochastic Processes-1
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Lecture – 14
Problems in Sequence of Random Variables (Contd…)

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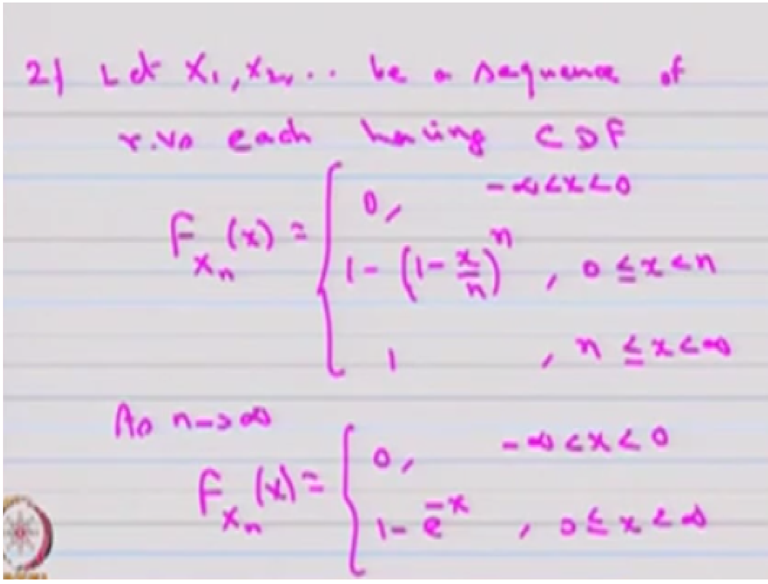

Next example, let x1, x2 and so on be a sequence of random variables, each having CDF cumulative distribution function. F suffix xn of x, it is zero from minus infinity to zero and it takes a value 1 minus 1 minus x by n power n for x is lies between zero to n. From n onwards till infinity the value is 1. So this is the cumulative distribution function for the random variables exercise.

It is a function of n, therefore I have made it F suffix, x suffix n, that means, this is a CDF further random variable n. For every n you have this form. As n tends to infinity, we get F suffix xn of x, that becomes zero from minus infinity to zero and it takes a value 1 minus e power minus x from zero to infinity. As n tends to infinity the CDF of the random variables xn becomes zero between the interval minus infinity to zero.

And the value becomes 1 minus e power minus lambda x where x is lies between zero to infinity.

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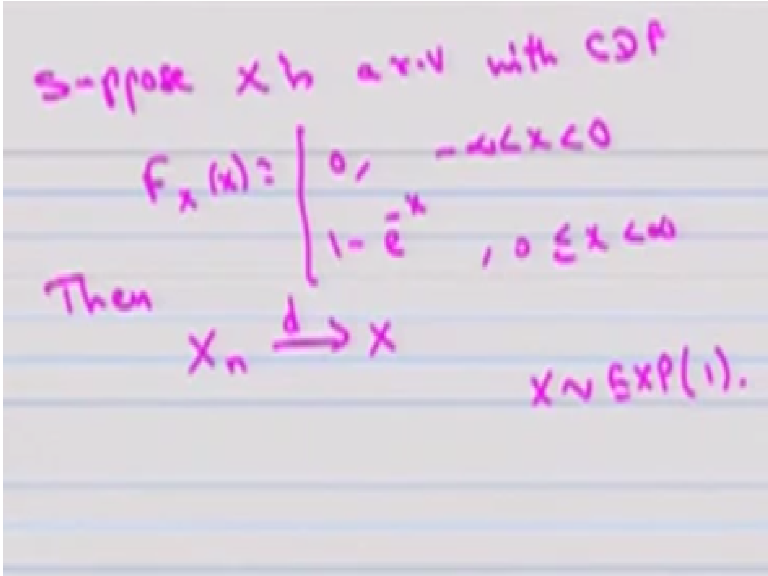

Suppose, x is a random variable with the CDF that is fx of x, that is zero between the interval minus infinity to zero and 1 minus e power minus x where x is lies between zero to infinity then one can conclude xn converges to x in distribution since the sequence of x suffix n of x tends to F of x for x is greater than or equal to zero and the value is 1 minus e power minus x.

Hence, one can conclude the sequence of random variable xn converges to the random variable x in distribution. Here the x is exponential distribution with a parameter 1. So this is the one example of a how the sequence of random variable converges to a random variable in distribution.

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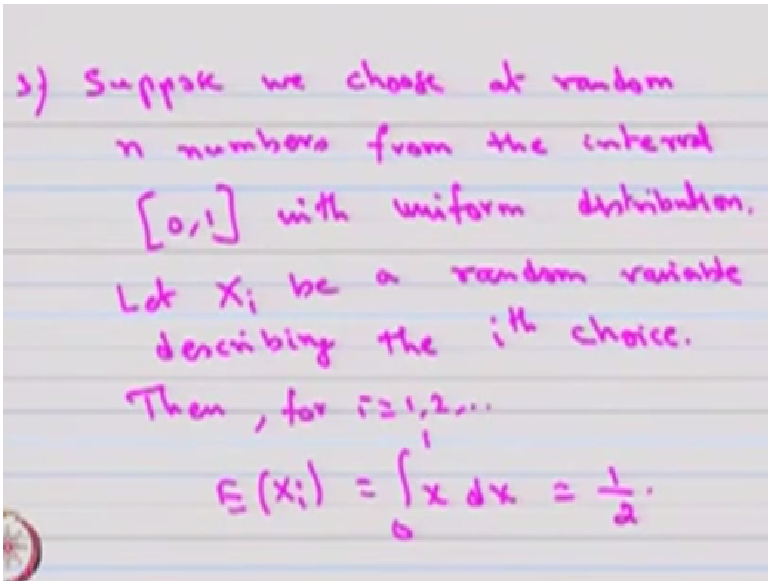

Next I move into the third example, suppose we choose at random n numbers from the interval zero to 1 with a uniform distribution, let Xi be a random variable describing the ith choice. Then for i is equal to 1, 2, and so on, you can find out what is the expectation of Xi is that is nothing but the integration from zero to 1 x times the probability density function, the probability density function for uniform distribution with a interval zero to 1, that is 1 therefore, x into dx, if you compute the expectation of Xi is going to be 1 by 2.

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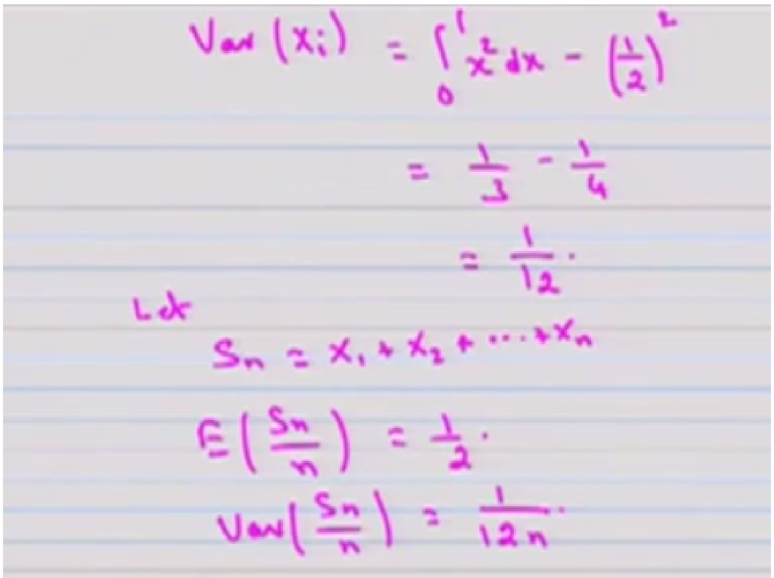

Similarly, one can evaluate the variants of Xi that is nothing but zero to 1, x square dx minus the mean square. Expectation of x square minus expectation of x the whole square, so the expectation of x square is zero to 1, x square dx. So if you evaluate this quantity, that is 1 by 3 minus 1 by 4. So if you simplify you will get 1 by 12.

If you remember the formula of variants of a uniformly distributed random variable between the interval a to b, then the variants of xi, x is nothing but you can get it by substituting the value of a is equal to zero and b is equal to 1, you will get 1 by 2. Let S suffix n be x1 plus x2 and so on till x. One can find mean and variants of S because you know the mean n variants of xi using that you can find out what is the mean of Sn.

But our interest is not the finding the mean of Sn, our interest is to find out the mean of Sn by n. That is basically, suppose Xi are the samples, then Sn divided by n is nothing but the sample mean. So expectation of Sn divided by n that becomes 1 by 2. Similarly, if you calculate variants of Sn by n that becomes 1 divided by 12 times n because the variants of Xi is 1 by 12. So the variants of Sn is a summation of Xi from 1 to n therefore, variants of Sn by n becomes 1 divided by 12 times n.

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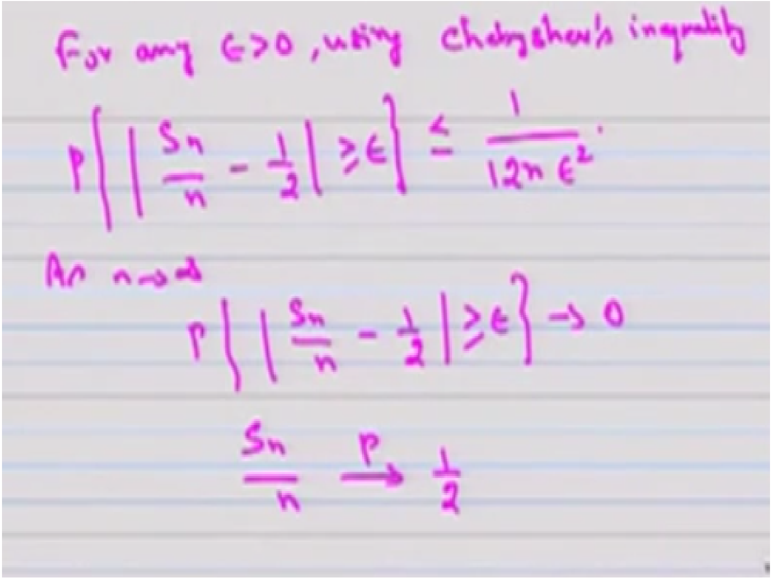

For any epsilon greater than zero, using Chebyshev's inequality, one can conclude the probability of absolute of Sn by n minus 1 by 2 greater than or equal to epsilon that is less than or equal to 1 divided by 12 times n epsilon square. I am using the Chebyshev's inequality by knowing mean of a Sn by n is 1 by 2 and variants of Sn by n is 1 divided by 12n, I get this inequality.

Now as n tends to infinity the probability of absolute of Sn by n minus 1 by 2 which is greater than or equal to epsilon will tends to zero because epsilon is in the, n is in the denominator because n is the denominator as n tends to infinity, this probability tends to zero. That is nothing but Sn by n tends to the value 1 by 2 and this converges takes place in probability.

The sequence of random variable Sn by n converges to 1 by 2 in probability. Therefore, we say the sequence of random variable Xn for n is equal to 1, 2 and so on obeys the weak law of the large numbers because the Sn by n converges to 1 by 2 in probability therefore we say the sequence of random variables Xn’s obeys the weak law of large numbers. So that is the intension of a giving this example.