Stochastic Processes - 1
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Lecture – 12
Problems in Random Variables and Distribution (Contd…)

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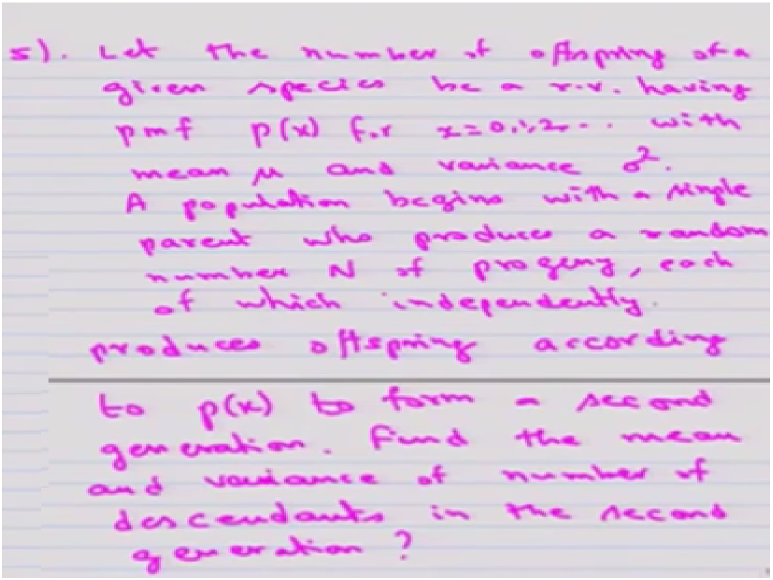

The next example, let the number of offspring of a given species be a random variable having probability mass function p(x) for x =0,1,2, and so on with mean Mu and variants sigma square. A population begins with a single parent who produce; who produces a random number say capital N of progeny, each of which independently produces offspring according to p(x) that is the probability mass function to form a second generation. Find the mean and variants of number of descendants in the second generation?

So let me read out the question again. Let the number of offspring of a given spaces be a random variable having a probability mass function p(x) for x =0,1,2, and so on with the mean Mu and variants sigma square. A population begins with a single parent who produces a random number say capital N of progeny.

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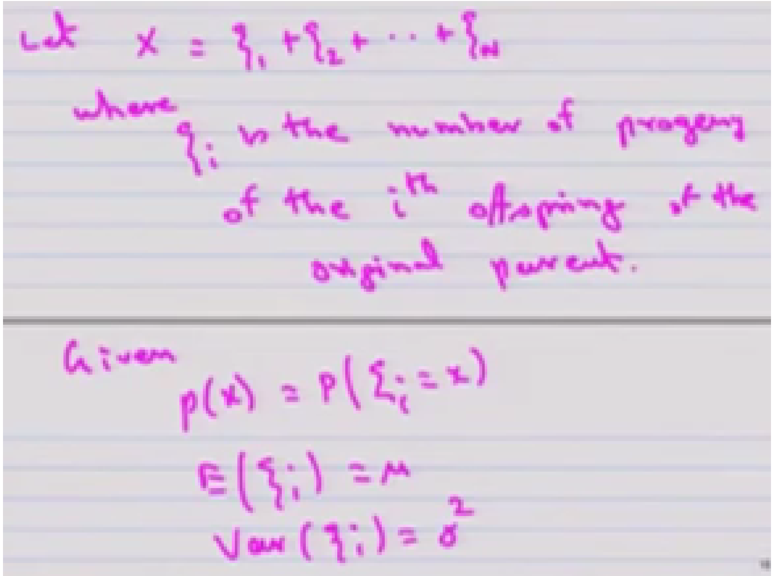

Each of which independently produces offspring according to p(x) to form a second generation. Find the mean and variants of number of descendants in the second generation? Let capital X is psi 1 + psi 2 + psi suffix capital N where, psi i is the number of progeny of the i-th offspring of the original parent. Given the p(x) is nothing but the probability mass function for the random variable psi I.

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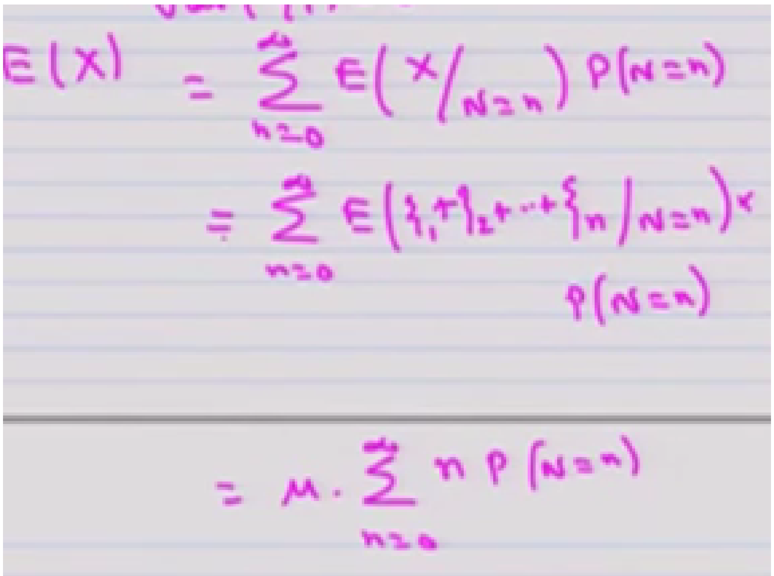

And also the expectation of psi I is that is = Mu and variants of psi I that is equal to sigma square. Psi I are the IID random variables. Our interest is to find out, what is the expectation of capital X, where capital X is a psi 1 + psi 2 + and so on till psi suffix N. Here N is also a random variable that is important. Therefore, this expectation can be computed as n = 0 to infinity.

The conditional expectation of x given capital N takes the value small n multiplied by the probability mass function of capital N, that is same as summation n =0 to infinity that is same as a expectation of psi 1 + psi 2 and so on + psi suffix small n given capital N takes the value small n, multiplied by the probability of N takes the values small n, that is same as; you know that the expectation of psi I are Mu.

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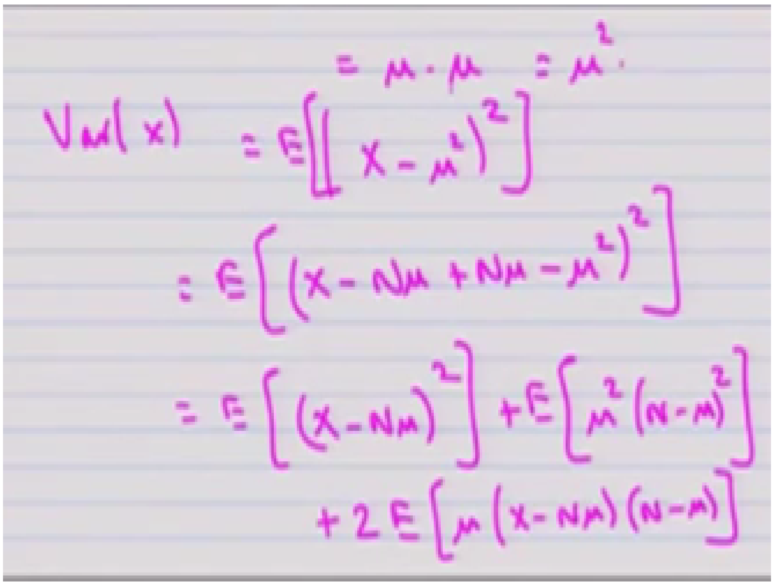

Therefore, this becomes a Mu can be taken out then summation n =0 to infinity, n times the probability of capital N takes the values, that is same as Mu. The expectation of the random variable capital N that is also Mu therefore it is Mu * Mu that is = Mu square. Now we can compute the variants of a x the same way that is the expectation of x- Mu square is the random variable expectation is Mu square the whole square.

Further this can be simplified by expectation of x-N times Mu + N times Nu – Mu square the whole square and you can expand therefore you will get, it is expectation of x- N times Mu the whole square + expectation of Mu square multiplied by n – Mu the whole square and the third term becomes 2 times expectation of Mu multiply by x- N Mu * N- Mu by taking Mu outside.

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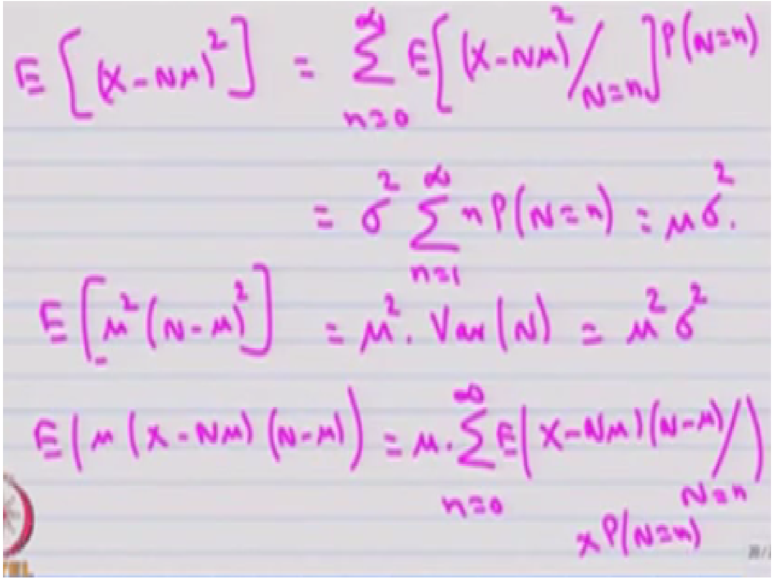

Now you can evaluate the first quantity that is expectation of x – N Mu the whole square suing the conditional expectation by making summation n = 0 to infinity expectation of x- N Mu the whole square given N takes the value small n, multiplied by the probability of N takes the value small n. If you substitute the way you have done the expectation and so on, finally you will get the answer.

That is sigma square summation n =1 to infinity n times the probability of N = n. That is N as n; the summation N times P of N is n is the mean that is Mu, therefore, this becomes Mu times sigma square. Even though I have skid 1 or 2 steps, one can get the sigma square summation N times P(n) that is same as Mu therefore it is Mu sigma square. Similarly, you can work out the second that expectation that is an expectation of Mu square multiplied by N-Mu whole square.

That is same as Mu square is constant the expectation of a N – Mu whole square that is nothing but the variants of the random variable N and a variants of a random variable N is a sigma square. Therefore, it is a Mu square sigma square. Now we have to evaluate the third expectation; that is the expectation of Mu times x – N Mu multiplied by N-Mu. Here also one can use the conditional expectation.

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$$= \mu . \sum_{n=0}^{\infty} (n-\mu) E\left(X - n\mu \,/\, N=n\right) P(N=n)$$

$$= 0 \quad \because E\left(X - n\mu \,/\, N=n\right)$$
$$= E\left(\xi_1 + \xi_2 + \cdots + \xi_n - n\mu\right)$$
$$= 0$$

$$Var(X) = \mu\sigma^2 + \mu^2\sigma^2$$
$$= \sigma^2 \mu (1+\mu).$$

That is Mu times summation and n = 0 to infinity, expectation of x- N Mu multiplied by N-Mu condition N takes the value small n multiplied by probability of N takes the value n. So if you simplify you will get a Mu times summation n= 0 to infinity, n-Mu, expectation of x-n Mu given N takes the value small n multiplied by probability of N takes the value small n that is same as 0.

Because expectation of x- n Mu given N takes the value small n that is nothing but expectation of psi 1, psi 2 and so on + psi n – n Mu that expectation quantity is going to be 0. Because expectation of psi I are going to be Mu and this is the expectation of a psi 1+ psi2 and so on + psi n- n times Mu therefore that becomes 0. Hence the variants of x substitute all these 3 variants results all these expectation results in the above this expression.

Therefore, you will get a variants of x is going to be Mu times sigma square + Mu square sigma square and third term is going to be 0, therefore you will get a sigma square Mu multiplied by 1+ Mu. This problem is useful in branching process therefore I discussed in this lecture. Even though there are many more problems of a similar kind, we have chosen a few problems to further illustrate, illustrative purpose.

And we are going to come across the similar problems in the course also therefore I have chosen some 5 problems to discuss as an illustrative example. Here is the reference for this lecture.