Stochastic Processes - 1
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Lecture – 11
Problems in Random Variables and Distribution (Contd..)

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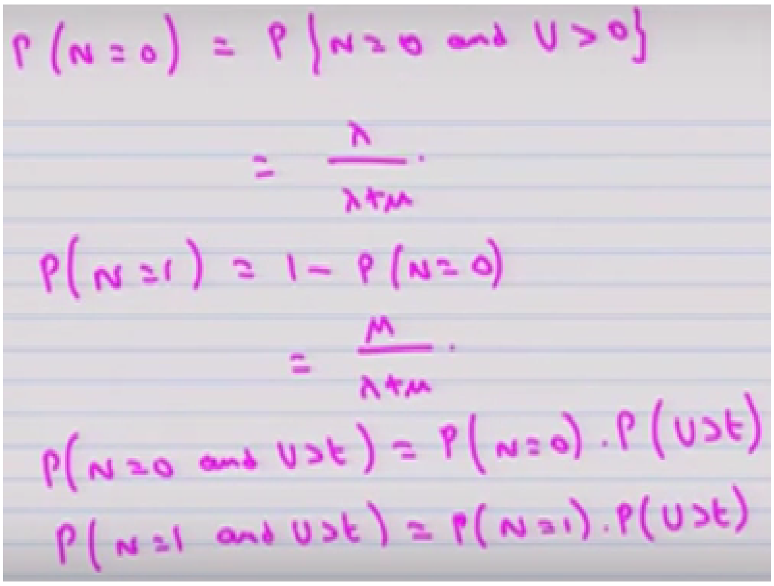

So in this example, we observed that N and U are independent random variables.

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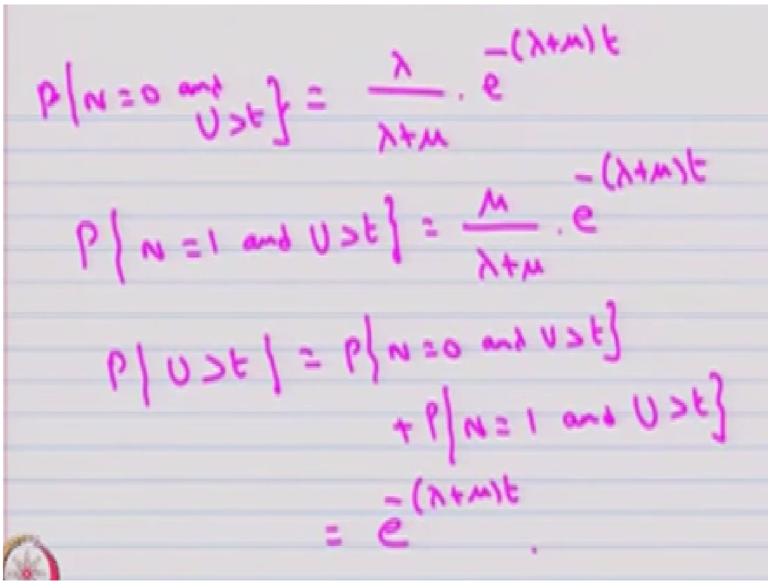

And also by seeing probability of U greater than t that is e power –lambda +Mu t, you can conclude U is the exponential distribution with the parameter lambda + Mu. Now we move into the next example.

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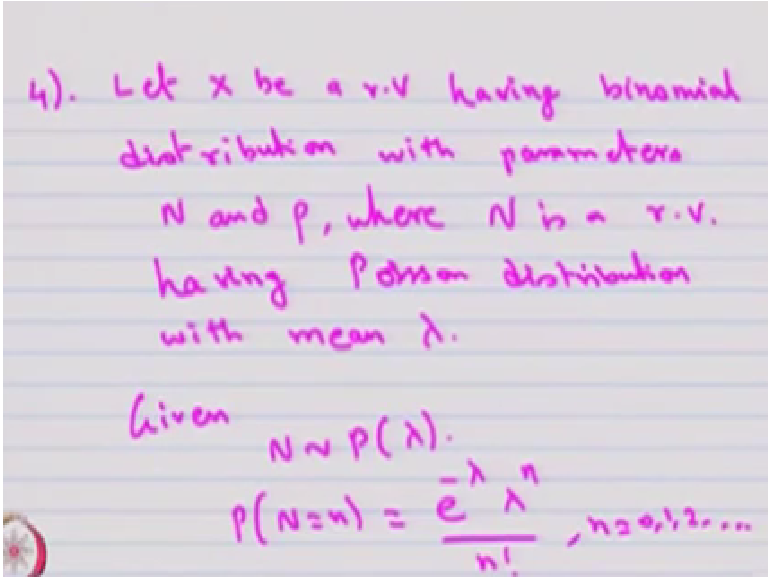

Let x be a random variable having binomial distribution with parameters capital N and P, where capital N is a random variable having Poisson distribution, with mean lambda.

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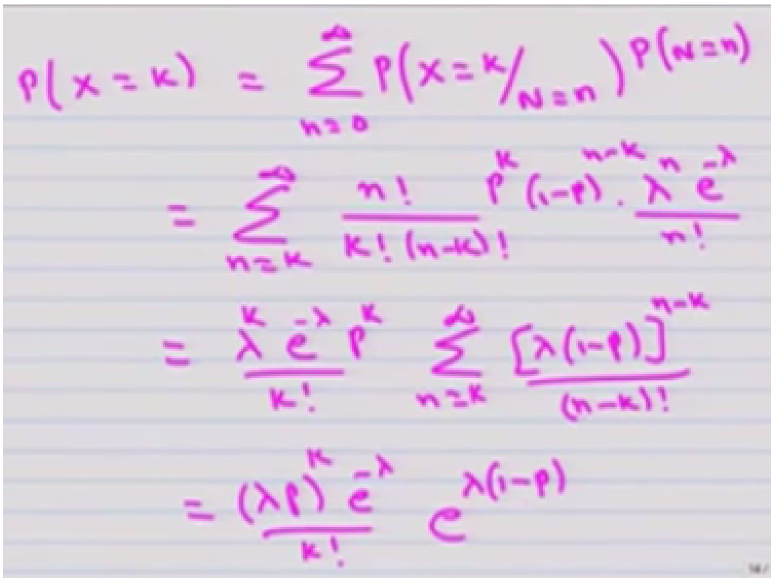

The question is find the marginal distribution of x or find the probability mass function of the random variable x? Given N is Poisson distribution with the parameter lambda that means the probability mass function for the random variable N is; e power – lambda lambda power N divided by n factorial, the possible values of n are 0, 1, 2 and so on. Our interest is to find out what is the probability mass function of the random variable x?

That is same as n=0 to infinity. What is the conditional probability of the random variable x takes the value k given, the other random variable N takes the value small n, multiplied by probability of N takes the value n. That is same as, the n takes the value from k to infinity n factorial divided by k factorial * n- k factorial and p power k 1-p power n-k multiplied by lambda power n e power – lambda divided by n factorial.

So no need of a n=0 to k-1 because the capital N takes the value small n, therefore the running index from k to infinity. That is same as you can take some terms outside that is lambda power k e power – lambda e power k divided by k factorial. The remaining terms that is a n is running from k to infinity. This can be written in the form of lambda times 1-p power n-k divided by n-k factorial.

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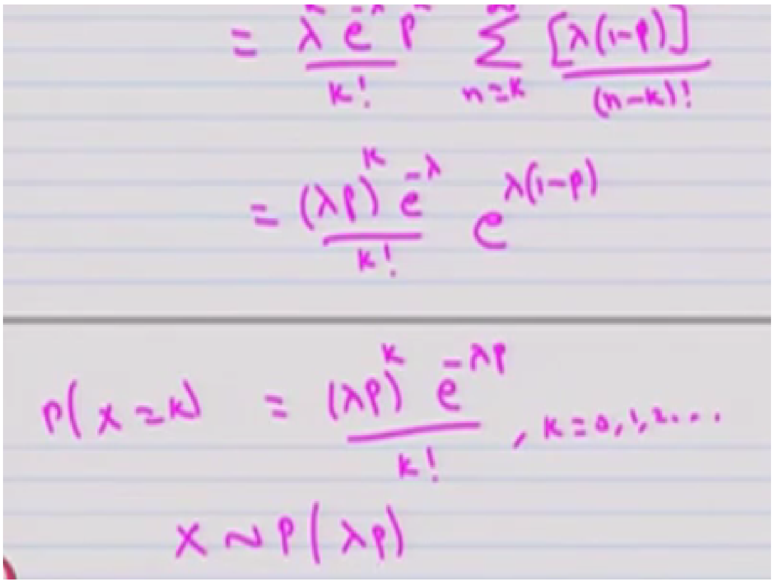

That is same as lambda p power k multiplied by e power – lambda by k factorial, the summation n=k to infinity and so on that becomes e power lambda times 1-p. Therefore, further you can simplify therefore the probability of x takes the value k is same as lambda p power k e power – lambda lambda p divided by k factorial, where k takes the value 0, 1, 2, and so on.

Hence the conclusion is the random variable x which is Poisson distributed with the parameter lambda times p. So this problem occurs in many situations of a stochastic modelling, therefore we have discussed these example in this lecture.