

Stochastic Processes - 1
Dr. S. Dharmaraja
Department of Mathematics
Indian Institute of Technology – Delhi

Lecture – 11
Problems in Random Variables and Distribution (Contd..)

(Refer Slide Time: 00:00)

$$\begin{aligned}
 P(N=0) &= P\{N \geq 0 \text{ and } U > 0\} \\
 &= \frac{\lambda}{\lambda + \mu} \\
 P(N=1) &= 1 - P(N=0) \\
 &= \frac{\mu}{\lambda + \mu} \\
 P(N \geq 0 \text{ and } U > t) &= P(N=0) \cdot P(U > t) \\
 P(N \geq 1 \text{ and } U > t) &= P(N=1) \cdot P(U > t)
 \end{aligned}$$

So in this example, we observed that N and U are independent random variables.

(Refer Slide Time: 00:08)

$$\begin{aligned}
 P\{N=0 \text{ and } U > t\} &= \frac{\lambda}{\lambda + \mu} \cdot e^{-(\lambda + \mu)t} \\
 P\{N=1 \text{ and } U > t\} &= \frac{\mu}{\lambda + \mu} \cdot e^{-(\lambda + \mu)t} \\
 P\{U > t\} &= P\{N=0 \text{ and } U > t\} \\
 &\quad + P\{N=1 \text{ and } U > t\} \\
 &= e^{-(\lambda + \mu)t}
 \end{aligned}$$

And also by seeing probability of U greater than t that is e power $-(\lambda + \mu)t$, you can conclude U is the exponential distribution with the parameter $\lambda + \mu$. Now we move into the next example.

(Refer Slide Time: 00:26)

4). Let x be a r.v having binomial distribution with parameters N and p , where N is a r.v. having Poisson distribution with mean λ .

Given $N \sim P(\lambda)$.

$$P(N=n) = \frac{e^{-\lambda} \lambda^n}{n!}, n=0,1,2,\dots$$

Let x be a random variable having binomial distribution with parameters capital N and P , where capital N is a random variable having Poisson distribution, with mean λ .

(Refer Slide Time: 02:21)

$$\begin{aligned}
 P(X=k) &= \sum_{n=0}^{\infty} P(X=k/N=n) P(N=n) \\
 &= \sum_{n=k}^{\infty} \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} \frac{\lambda^n e^{-\lambda}}{n!} \\
 &= \frac{\lambda^k e^{-\lambda} p^k}{k!} \sum_{n=k}^{\infty} \frac{[\lambda(1-p)]^{n-k}}{(n-k)!} \\
 &= \frac{(\lambda p)^k e^{-\lambda}}{k!} e^{\lambda(1-p)}
 \end{aligned}$$

The question is find the marginal distribution of x or find the probability mass function of the random variable x ? Given N is Poisson distribution with the parameter λ that means the probability mass function for the random variable N is; $e^{-\lambda} \lambda^n / n!$, the possible values of n are 0, 1, 2 and so on. Our interest is to find out what is the probability mass function of the random variable x ?

That is same as $n=0$ to infinity. What is the conditional probability of the random variable x takes the value k given, the other random variable N takes the value small n , multiplied by probability of N takes the value n . That is same as, the n takes the value from k to infinity n

factorial divided by k factorial * n- k factorial and p power k 1-p power n-k multiplied by lambda power n e power – lambda divided by n factorial.

So no need of a n=0 to k-1 because the capital N takes the value small n, therefore the running index from k to infinity. That is same as you can take some terms outside that is lambda power k e power – lambda e power k divided by k factorial. The remaining terms that is a n is running from k to infinity. This can be written in the form of lambda times 1-p power n-k divided by n-k factorial.

(Refer Slide Time: 05:06)

The image shows handwritten mathematical work on lined paper. The top part shows the simplification of a binomial distribution's probability mass function. It starts with the expression $\frac{\lambda^k e^{-\lambda} p^k}{k!} \sum_{n=k}^{\infty} \frac{[\lambda(1-p)]^{n-k}}{(n-k)!}$. This is then simplified to $\frac{(\lambda p)^k e^{-\lambda}}{k!} e^{\lambda(1-p)}$. A horizontal line separates this from the final result, which is $P(X=k) = \frac{(\lambda p)^k e^{-\lambda p}}{k!}, k=0,1,2,\dots$. Below this, it is concluded that $X \sim P(\lambda p)$.

That is same as lambda p power k multiplied by e power – lambda by k factorial, the summation n=k to infinity and so on that becomes e power lambda times 1-p. Therefore, further you can simplify therefore the probability of x takes the value k is same as lambda p power k e power – lambda lambda p divided by k factorial, where k takes the value 0, 1, 2, and so on.

Hence the conclusion is the random variable x which is Poisson distributed with the parameter lambda times p. So this problem occurs in many situations of a stochastic modelling, therefore we have discussed these example in this lecture.