Stochastic Processes - 1
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Lecture – 10
Problems in Random Variables and Distribution (contd.)

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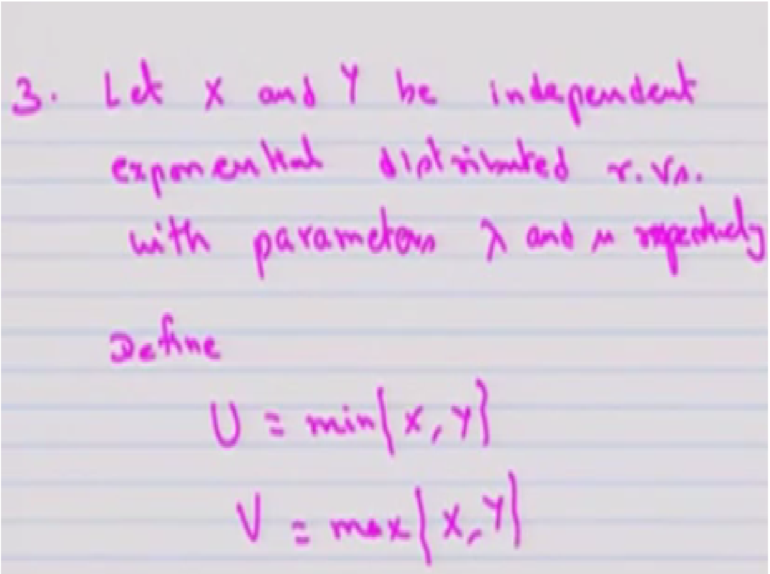

As a third example, we will discuss this one, let x and y be independent exponential distributed random variables with parameters lambda and Mu respectively. Define capital U? that is nothing but minimum of x, y and v is nothing but the maximum of the random variables x, y. The third random variable capital N that is defined as you take the value 0, if x is less than or equal to y, you take the value 1, if x is greater than y.

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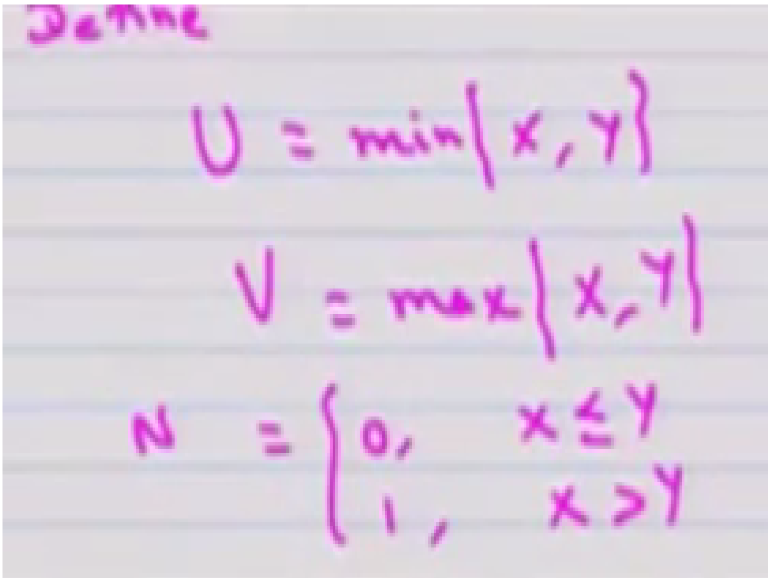

So using the 2 random variables x and y, you have defined a 3 random variables, U, V and N.

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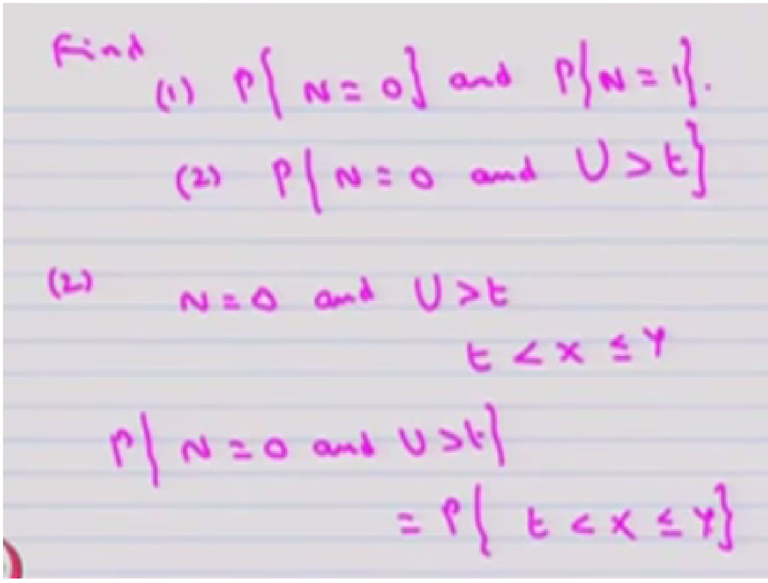

Our interest is to find the probability of capital N takes the value 0 and what is the probability of capital N takes the value 1? The second we are interested to find out the probability of N takes the value 0 and capital U takes the value greater than some t, the t is greater than 0. So let us go for finding the second one first, then we will find out the probability of N=0 and probability of N=1.

So let us start with the 2 first. The event N=0, and capital U takes the value greater then t, that is exactly the event of a t less than capital X, less than or equal to capital Y. the event N=0 and capital U greater than t that is same as t less than X, less than or equal to Y. Therefore, the probability of finding N takes the values 0 and U takes the value greater than t that is same as probability of X takes the value t, t less than X, less than or equal to Y.

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$$= \iint_{t<x\le y} \lambda e^{-\lambda x} \cdot \mu e^{-\mu y}\, dy\, dx$$

$$= \int_t^\infty \left(\int_x^\infty \mu e^{-\mu y}\, dy \right) \lambda e^{-\lambda x}\, dx$$

$$= \int_t^\infty e^{-\mu x} \cdot \lambda e^{-\lambda x}\, dx$$

$$= \frac{\lambda}{\lambda+\mu} \cdot \int_t^\infty (\lambda+\mu) e^{-(\lambda+\mu)x}\, dx$$

That is same as the double integration with the t less than x, less than or equal to y of the joint probability density of x and y. Since x and y are independent random variable the joint probability density function is a product of marginal probability, sorry this is the; e power – Mu y, dy dx. So the probability of t less than x, less than or equal to y that is same as the double integration t less than x, less than or equal to y of a integrant is a lambda times e power –lambda x, Mu times e power – Mu y dy dx.

That is same as the inner integral becomes the x to infinity Mu times e power – Mu y dy, then lambda times e power –lambda x integration with respect to x between the limits t to infinity. That is same as, now the inner integration you can integrate and you can substitute the limit x and infinity if you simplify you will get a t to infinity e power – Mu x, then multiplied by lambda times e power – lambda x dx.

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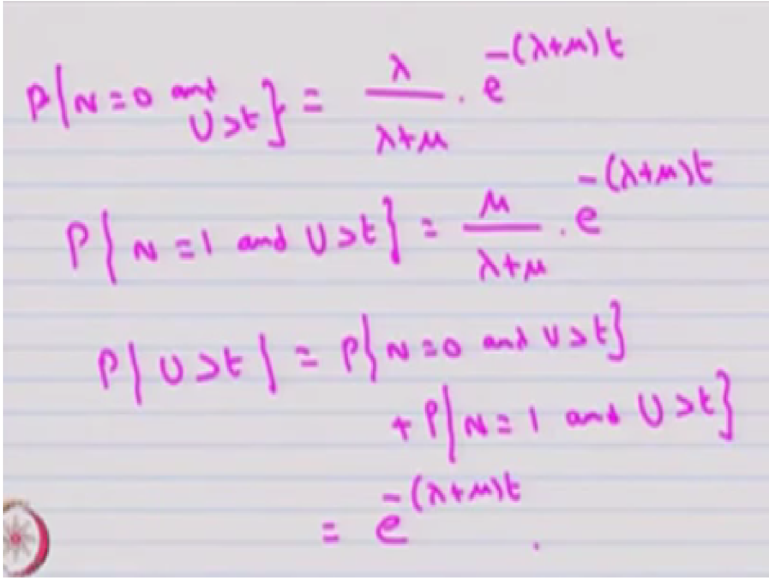

If you do the integration, the interior integration you will get e power – Mu x, then the remaining things are as it is, that is same as; you can keep lambda by lambda + Mu outside, this become the integration from t to infinity of lambda + Mu times e power – lambda + Mu x dx. You know how to do the integration for this, if you simplify you will get the answer that is lambda divided by lambda + Mu times e power – lambda+ Mu times t.

So this is the result for probability of N takes the value 0 and U takes the value greater than, the t is greater than 0. Similarly, you can work out the probability of N takes the value 1 and U takes the value greater than t, that will be Mu divided by lambda + Mu multiplied by e power –lambda + Mu times t. So with this, second part is over. Now we have found the boundary of N= 0 and U greater than t.

Also we will get the probability of N=1 and U greater than t. Now you will go for finding the first result. But before that, you can find what is the probability of U greater than tx? that is nothing but finding out the probability of N=0 and U greater than t + the probability of N takes the value 1 and U greater than t. That is, we can find out the probability of U greater than t.

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$$P(N=0) = P\{N=0 \text{ and } U>0\}$$
$$= \frac{\lambda}{\lambda+\mu}.$$
$$P(N=1) = 1 - P(N=0)$$
$$= \frac{\mu}{\lambda+\mu}.$$
$$P(N=0 \text{ and } U>t) = P(N=0).P(U>t)$$
$$P(N=1 \text{ and } U>t) = P(N=1).P(U>t)$$

You know the result, if you add, you will get the result, that is e power – lambda + Mu times t, that is a meaning of the minimum of 2 random variables takes a value greater than t that is e power – lambda + mu times t. You can find out the probability of a N=0 in using probability of N=0 and U is greater than 0, the t is greater than or =0, therefore probability of N is =0, that you can compute from probability of N=0 and U is greater than 0.

That means you substitute t =0 in the previous result, you will get a probability of N=0, so that is nothing but if you recall probability of N=0 and U greater than t, that is lambda divided by lambda + Mu e power – lambda + Mu t. Here the t can be greater than or = 0, so substitute t =0, that will give probability of N=0. Therefore, this will become lambda divided by lambda + Mu.

Similarly, you can find out a probability of N=1 in probability of N=0 and probability of N=1 and U is greater than 0 or you can find out probability of N=1 by making one of the probability of N= 1- probability of N=0. So 2 ways you can find probability of N=1, so here I am using 1-probability of N=0 that will be Mu divided by lambda + Mu. So in this problem even though we asked only 2 things but here we have got a probability of N=0.

Probability of N= 1 also probability of U is greater than t. If you observe, probability of N=0 and U greater than t, that is same as the probability of N=0 multiplied by probability of U greater than t. Similarly, you will get the observation probability of N=1 and U greater than t, that will give probability of N=1 multiplied by probability of U greater than t. So hence N and U are independent random variables.

So these example is useful in a birth death processes, therefore I am explaining this problem as an illustrative example.