Stochastic Processes-1
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Lecture - 01
Introduction and Motivation for Studying Stochastic Processes

Our Lecture is Stochastic processes.

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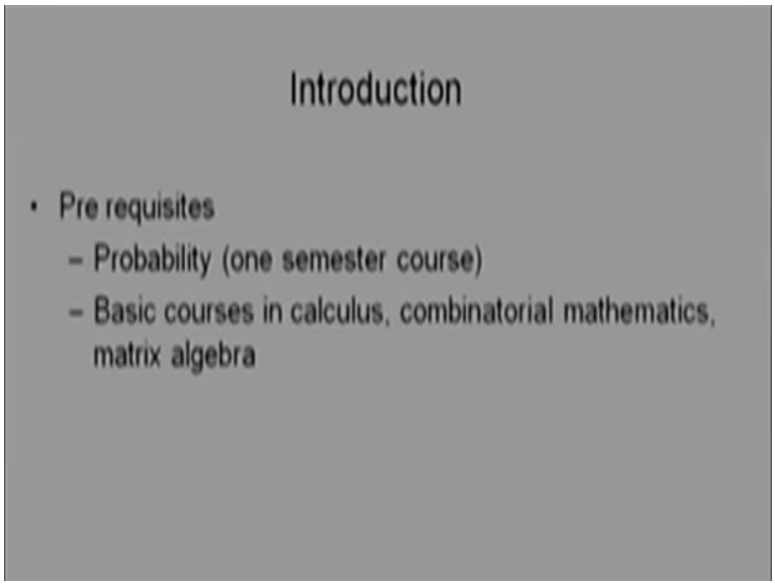

This course needs prerequisite of a probability as a full one semester course. So most of the universities they have a course, probability theory along with the stochastic processes or random process or probability and statistics. So whatever the courses we have, at least some 30 lectures of probability theory is needed for this stochastic process course as a prerequisite.

Other than probability course we need basic course in calculus and some mathematical background over the combinatorial problems and also the matrix algebra. So these courses would have been covered in the maths one or mathematics two courses. So that is enough to understand the stochastic process course.

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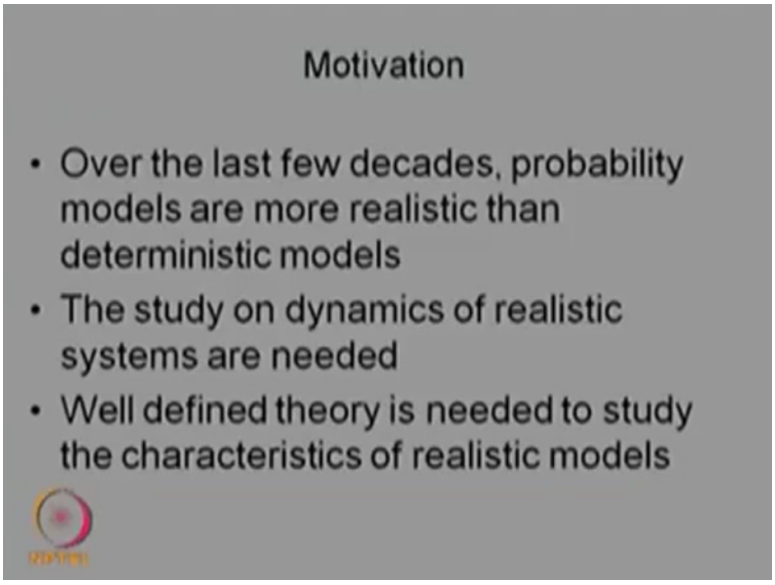

So before we move into the stochastic process, I am going to give what is the motivation behind stochastic process. When we see the last few decades problems, more of the probability models are not deterministic that means you need more probability theory to understand the system then only you can study the dynamics of the model. If you see the, if you want to study the dynamics of the system, then you need more probably theory.

So the simple probability theory may not be enough to study the more study on the realistic system. The way this realistic system behaves in a very dynamical way, it is not easy to capture everything through the probabilistic or usual probability models. That means you need more than the probability theory to understand the system or to study the system in a well behaved way. For that one of the important thing is stochastic process.

It deals about the collection of a random variable, so that you can study the dynamics of the system in a better way. Even though I am giving very light way of saying the collection of random variable first we should know how the random variable can be defined, so that you can study the collection of random variable in a better way. So for that we are going to spend few examples through that how the more realistic models need more probability theory other than the usual probability theory.

So that the stochastic process definition and those things I am going to cover at a later part.

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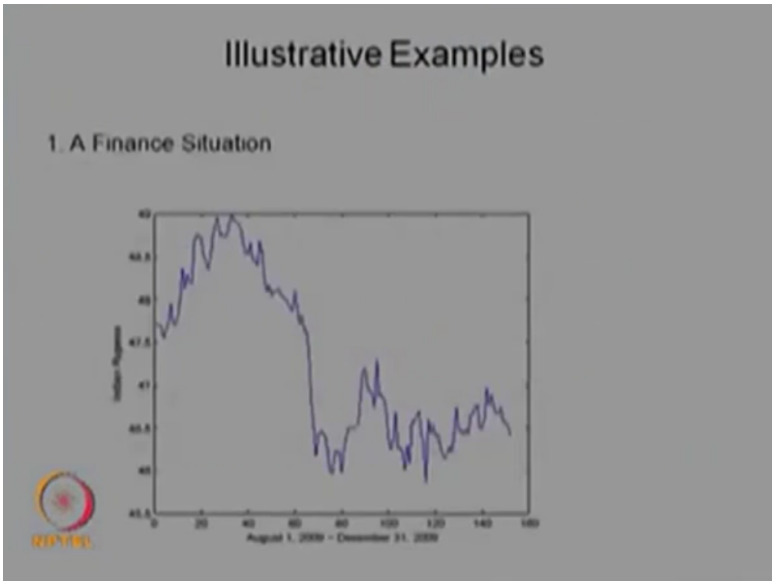

First let us see the first example, that comes in the finance situation. This is the actual data which was captured over the period of time from August 01, 2009 to December 31, 2009 of what is the current price of one US dollar in Indian rupees. So if you see the graph you can make out August 1st, 2009, the price of one US dollar was Rs. 47.57 or Rs. 47.58.

And if you see the dynamics over the years, over the days from August 01st, 2009 to till December 31st, 2009 it keeps on changing and it takes some values higher and after that it goes down and it fluctuates and so on. So this is the actual data which we have captured. And from that our interest will be what could be the US dollar price after sometime. If I know till today what is the price, my interest would be what would be the price after 1 or 2 days or after one month or after 6 months.

That means, I should know how the dynamics keep moving over the days and what is the hidden probabilistic distribution is capturing over the time, so that I can identify what is the distribution behind that. Therefore, I can study the future prediction; I can study the dynamics of this particular model in a much better way. That means I need what is the background or what is the hidden distribution playing or hidden distribution which causes the dynamics of the system.

After identifying what is the distribution, my interest could be, what could be the some other moment over the time. That means what would be the average value or what could be the second order moment if it exists and so on, that can be obtained if I know the actual distribution in the underlying model.

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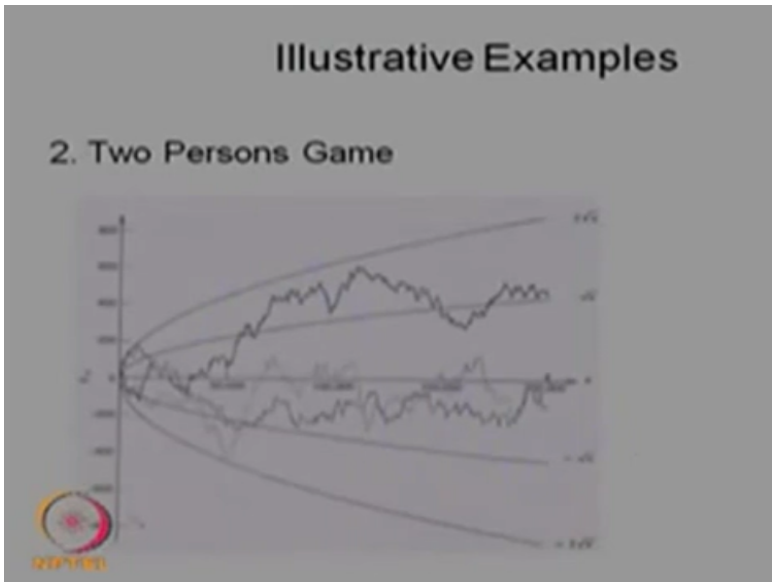

If I see the second example, I am just changing into another model in which there are two people playing a game. The person A and person B.

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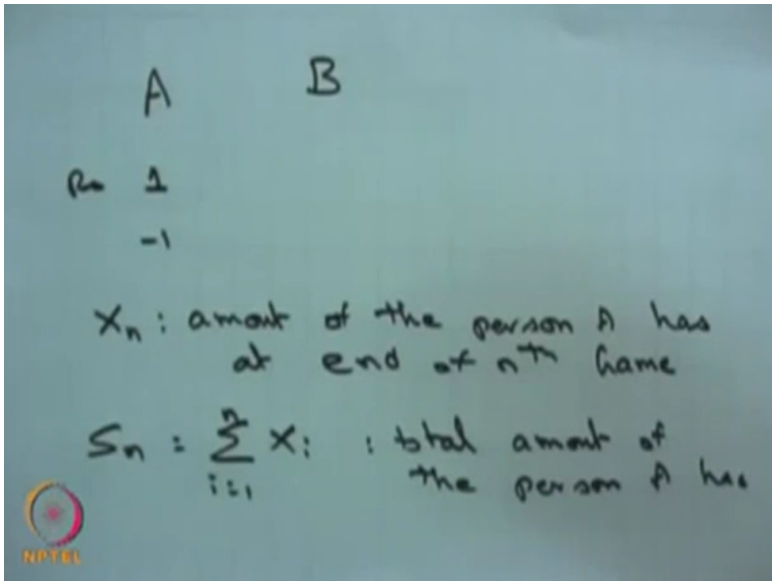

Whenever the person A wins he gets rupees one. suppose the person B wins then he will get the one rupee and at the same time person A uses one rupee on the same day and the play keeps going. Suppose you make the under variable as xn is the amount of the person A has at the end of nth game. If you make out the random variable xn for the person A has the amount at the end of the nth game.

The way the game going on, the value of the xn keep changing and if you make out another random variable, xn is the sum of xi where i is running from one to n. This gives what is the

total amount of the person A has. The diagram in which the xn gives, what is the way the dynamics goes and over the n. And if you see the diagram you can make out the whole dynamics goes.

How the game is going on in the first few games, accordingly it changes the positive side or it goes to the negative side. And if the n goes large then the dynamics of the xn over the n will keep changing over the time and you will get the realization of the xn over the time. And here I have given 3 different realizations and this diagram is taken out from the book by U.N. Bhatt. The title of the book is elements of applied stochastic process.

So this is one of the motivations behind stochastic process. And from these our interest will be after the, what is the distribution of xn at any n and also as n tends to infinity what could be the distribution of x n. That means you need the distribution of the random variable and also you need what could be the distribution as n tends to infinity or the limiting distribution of x n.

If you know the distribution, then you can get all other moments for different n as well as the asymptotic behavior of the random variable x n.

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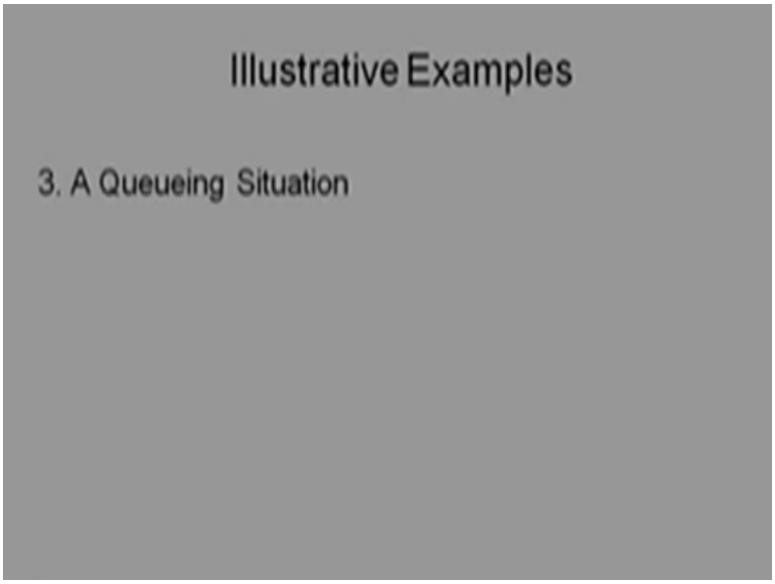

Next I will move into another example in which it is the queuing situation.

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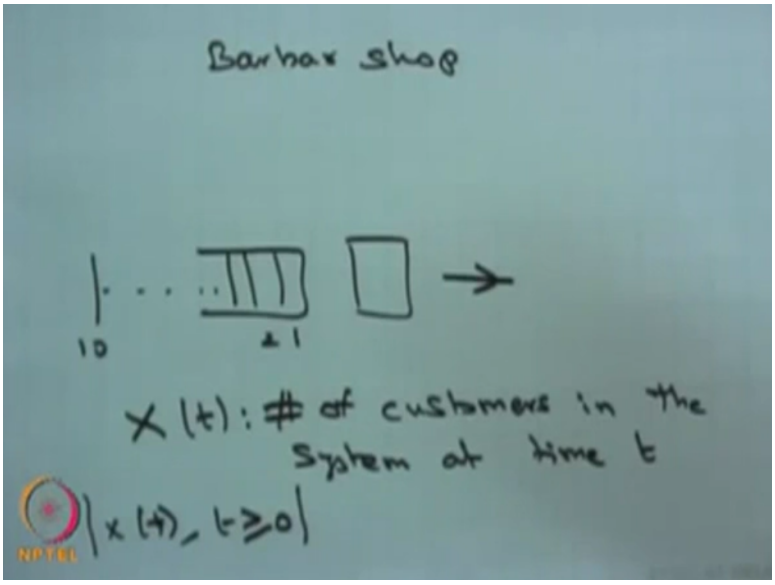

The queuing situation, here I have taken it as, taken a simple example, that is a barber shop example, in which there is only one barber shop person and who does the service for the people whoever entering into the barber shop. And there are only a limiting capacity in which there is a maximum 10 people can stay in their barber shop and one person will be under service.

Once the service is over and the system will be, the customer can leave the system. At any time, maximum 10 people can be in the barber shop and only one person is doing the service for the customer who enter into the barber shop. Suppose you take the random variable as xt is the number of customers in the barber shop or in the system at time t the way the dynamics go; the possible values of xt will be starting from 0 to n.

To study this system, you need what is the way the people or the customers are entering into the system and what is the way the service is going on for the customers and what is the discipline in which the customers are getting served also. Our interest will be, suppose we have the capacity of 10, what could be the waiting time, whenever the customers are entering into the system.

My interest will be one is how to reduce the waiting time on average, this is the customers point of view. As the barber shop point of view how much I can get more revenue, that means how I can increase the capacity of the systems so that I can make more profit over the time. That means if I know the dynamics of xt over t where t is varying from 0 to infinity, I can

understand the system over the time as well as I can whatever the probabilistic measures or whatever the other measures average number of customers or average waiting time and so on.

I can find out using this type of random variable. So later we are going to say, this is going to be one of the stochastic process for this example.

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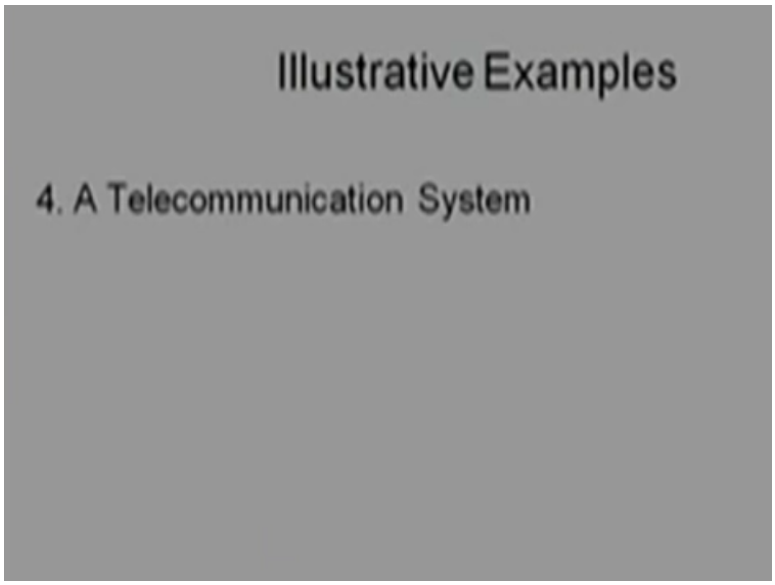

Next I am going to consider the fourth example as the telecommunication system.

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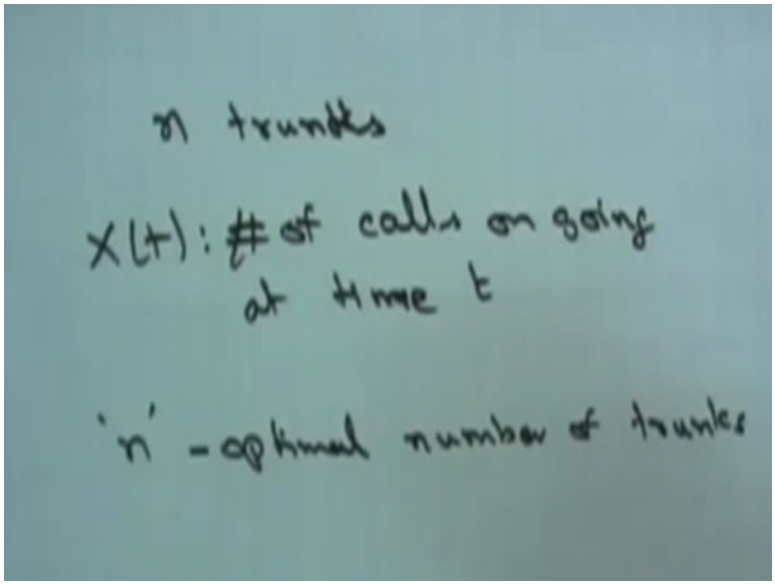

Suppose you think of a system in which you have n trunks are there. Trunks are nothing but it is a maximum number of calls will be allowed at anytime. Whenever a call entering into the system and you have given one trunk to the call and at the end of the call is over the trunk

will be back. So you have a telecommunication system in which n trunks are available. So you have a telecommunication system in which n trunks are available.

Suppose I make a random variable xt as the number of calls on going, at time t, see here also the dynamics of xt is going to be keep changing from 0 to n over the time and my interest will be how I can do the service such a way that more calls will be entertained as well as how I can find out the optimal n such a way that what is the optimal number of trunks such that I can minimize the waiting time or I can maximize the revenue.

So this is also one of the problems which we come across in the daily life and so on. So my interest is to introduce the stochastic process so that I can study this type of system in a better way.