Stochastic Processes

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Module - 4
Discrete-time Markov Chain
Lecture - 1
Introduction, Definition andTransition Probability Matrix

This stochastic process, in this we are going to discuss the module 4 and Discrete- time Markov Chain. And this is a lecture 1; in this lecture I am going to discuss the introduction about the Discrete-time Markov Chain, then followed by the definition and the important one concept called the one step transition probability matrix.

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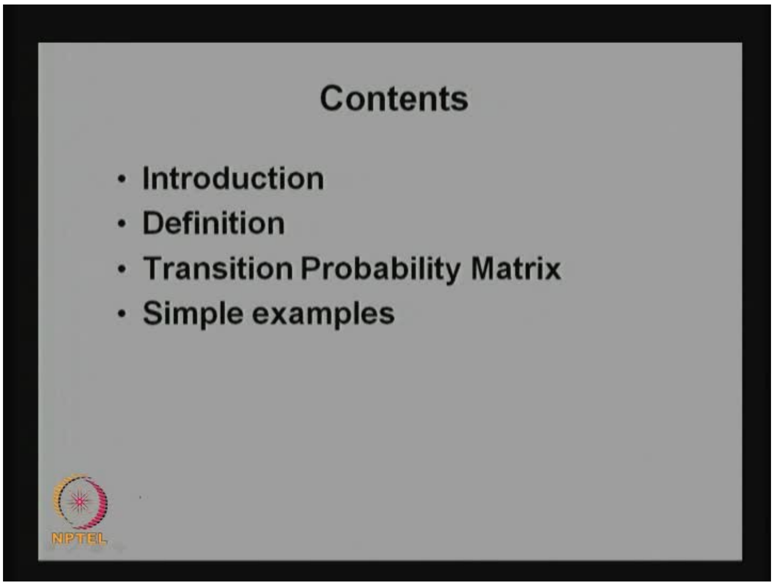

So, this lecture I am going to cover the introduction, definition, transition probability matrix, and few simple examples also.

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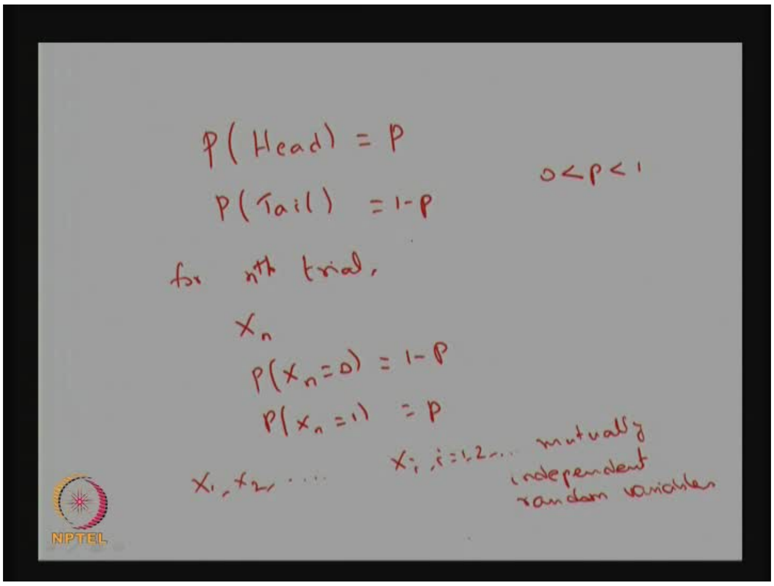

Consider a random experiment of tossing a coin infinitely many times; each trial there are two possible outcomes namely head or tail, assume that the probability of head that probability you assume that that is p, and the probability of tail occurring in each trail that you assume it as 1 minus p. You assume that the p is lies between 0 to 1. Denote for the nth trail, because you are tossing a coin infinitely many times; for the nth trail you denote the random variable X n is the random variable whose values are 0 or 1 with the probability, the probability of X n takes the value 0 that is same as in the nth trail you are getting the tail, that probability is 1 minus p.

And the probability x n takes the value 1 that probabilities make it as p for the heads of s, and already you seen that the probability lies between 0 to 1; thus you have a sequence of random variable x 1, x 2, and so on, and this will form a stochastic process. And assume that all the x i's are, all the x i's are mutually independent random variable independent random variables.

So, this is a random experiment in which a tossing a coin infinitely many times, and for any nth trail you define a random variable x n with the probability it takes the value 0 with the probability 1 minus p, and it takes the value 1 with the probability p. And that is equivalent of appearing head with the probability p, and occurring the tail with the probability 1 minus p.

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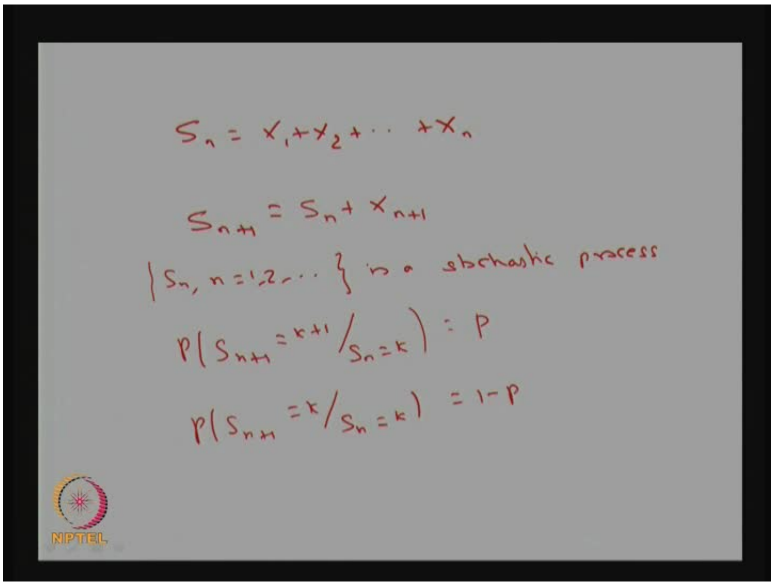

Now, I am going to define another random variable that is partial sum of first n random variables n n x i's, so the S n will be some of the first n random variables, therefore the sum S n gives the number of heads appear in the first n trails. It can be absorbed that S n plus 1 is same as S n plus x n plus 1; since S n is the partial sum of the first n trails outcome so, S n plus one is nothing but S n plus x n plus 1. You can also observe that since S n is the sum of first n random variables, and S n plus one is S n plus x n plus 1, and also all the x i's are mutually independent random variables.

S n is a independent with the x n plus 1, that means here the S n plus 1th random variable is the combination of two independent random variables whereas, the S n is the till nth trail how many heads here appear plus whether, it is head or tail accordingly this value is going to be 0 or 1. Therefore, if you see the sample path of the S n plus 1 it will be increment by 1, if x n plus 1 takes the value 1 or it would have been the same value. Earlier, if this x n plus 1 takes the value 0, and also you can observe that the S n plus 1 is a depends on S n and only on it; it is not dependents on S n minus 1 or S n minus 2 or so on… Because it is accumulated the number of trails values over the n therefore, S n plus 1 is depends on S n and only on it; the S n for different values of n this will form a stochastic process, this will form a stochastic process and now we can come to the conclusion.

The probability of this is a stochastic process, the probability of S n plus 1 suppose this value is k plus 1 given that S n was k, that means the S n plus 1 valued been 1. Therefore, the appearance of the head appearance of n plus 1th trail and that probability is going to p. Similarly, you can make out suppose S n plus 1 value will be k such that S n is also k then, that is possible with the n plus 1 th trail you got the tail therefore, that probability is 1 minus p, this is satisfied for all n.

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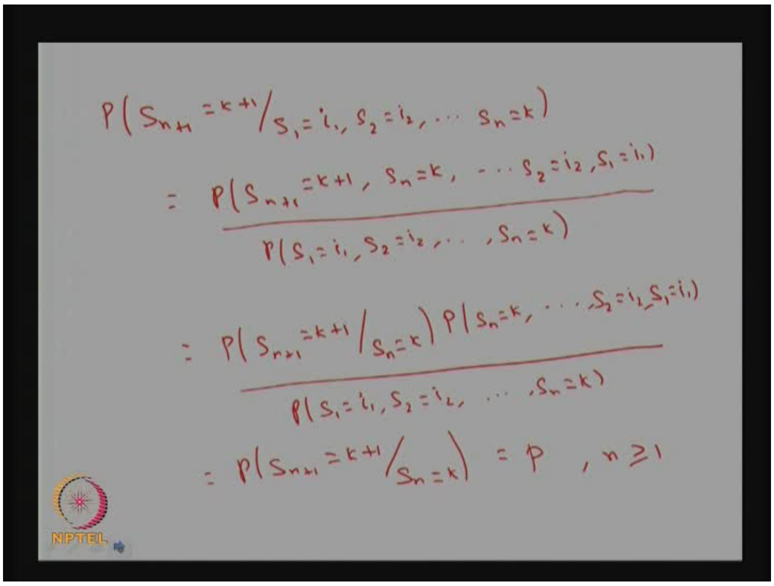

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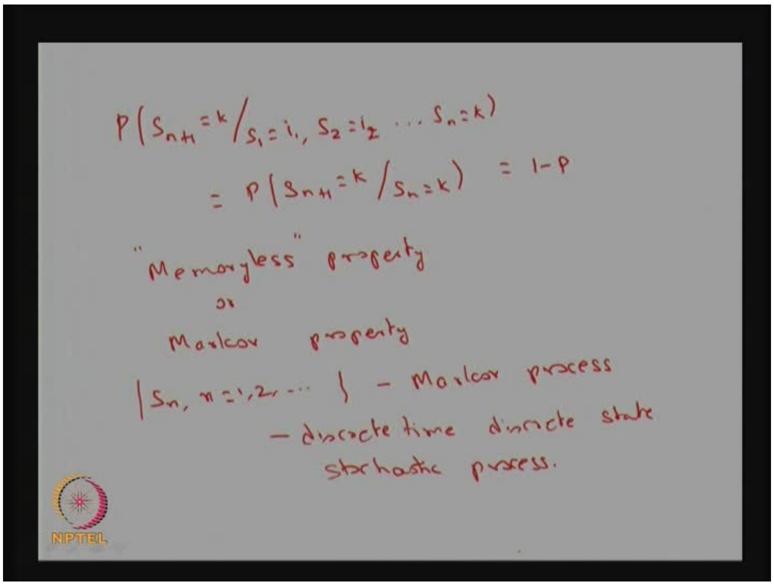

So, you can make out this is satisfied by greater than or equal to even, I can go for n is greater than or equal to 1 not only this. Similarly, I can come to the conclusion the probability of S n plus 1 is equal to k given that s 1 was i 1, s 2 was i 2, and so on… S n was k that is also can be proved, that the probability S n plus 1 is equal to k given that S n is equal to k, that is same as what is the probability that a value was same k in subsequent trails, that is possible of appearing tail in the n plus 1th trail therefore, the appearance of the tail in the n plus 1th trail the probability is 1 minus p, or I can use the notation q.

That means the probability of n plus 1th trail that distribution given that, I know the value till the nth trail, that is same as the distribution of n plus 1th trail given with the only the n th distribution not the earlier distributions. And this property is called the memory less property. The stochastic process the S n satisfies the Memory less property or the other word called Markov property. The distribution of n plus 1 given that the distribution of 1 first random variable, second random variable, the nth random variable, that is same as conditional distribution of n plus 1th random variable given that with the nth random variable only. And this property is called it as Memory less or Markov property.

This stochastic process this S n satisfying the Markov property or Memory less property is called Markov process; the stochastic process satisfying the Memory less or Markov property is a called Markov process. In this example the stochastic process S n is the discrete-time discrete state stochastic process. Now, I can give based on the state space and the parameter space, I can classify the Markov process, or I can give the name of the Markov process in a easy way based on the state space as well as the parameter space.

So, when the state space S, the S is the state space this is nothing but, the collection of all possible values of the stochastic process, if this of the discrete type that means the collection of elements in state space S is going to be a finite or countably infinite, then we say the state space of the discrete type. So, whenever the stochastic process satisfying the Markov property, then the stochastic process is called as the Markov process, or you can say whenever the state space is the discrete then we can say the corresponding process we can call it as a Markov chain.

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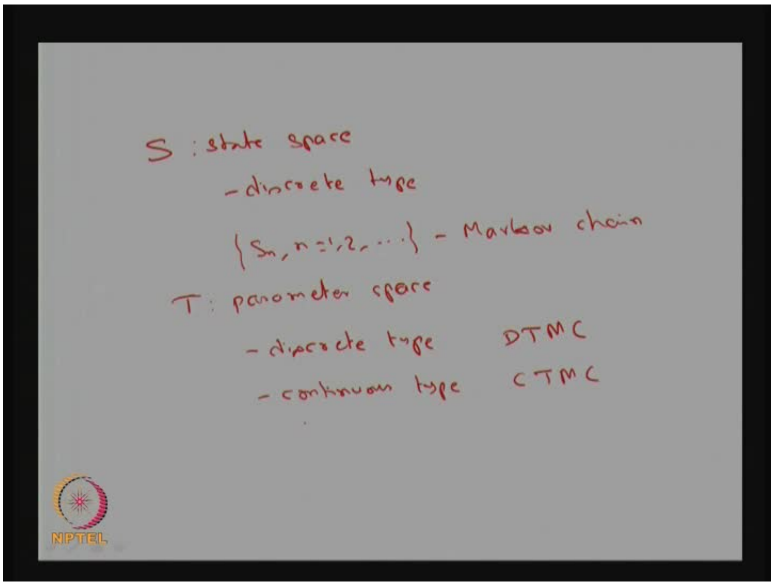

Whenever the state space is said as discrete. Now, based on the parameter space capital T parameter space is nothing but, the possible values of T whether it is going to be finite or countably infinite, then it is going to be discrete space or discrete-time, or it is going to be uncountably many values, then it is going to be call it as a continuous type. So, whenever the T is going to be a discrete type, then the Markov chain is going to be call it as a Discrete Time Markov Chain. Whenever the parameter space is going to be of the continuous type that means, the possible values of capital T is going to be uncountably many, then we say Continuous Time Markov Chain. So, in this example the S n the possible values of S n is also going to be the state space is going to be a discrete type, and the parameter space is also going to be discrete type therefore, the given example the S n is also going to be the Discrete Time Markov Chain.

So, in this module you are going to study the Discrete Time Markov Chain; the next module 4 module 5 we are going to discuss the continuous time Markov chain. So, in general whenever the stochastic process satisfying the Markov property, it will be call it as Markov process. So, based on the state space the Markov process is called as a Markov chain, and based on the parameter space it is call a discrete time Markov chain or continuous time Markov chain accordingly, the discrete type or continuous type. Even though, I have given one simple example and through that I explained the Markov property. Therefore, we land up the discrete time Markov chain, I am going to give the formal definition of discrete time Markov chain.

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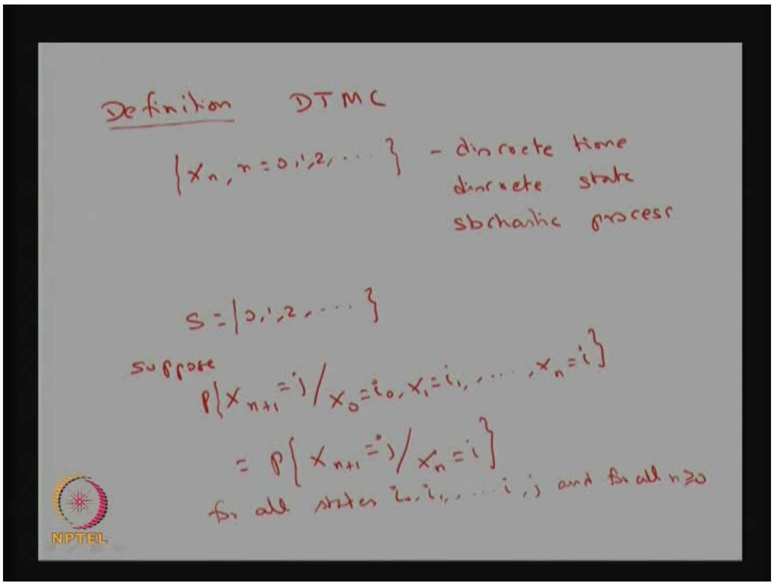

Formal definition of discrete time Markov chain in notation in short we call it as a DTMC. Consider a discrete time a discrete state stochastic process, consider a discrete time this is a discrete time discrete state stochastic process. Assume that the x n takes a finite or countably countable number of possible values, unless otherwise mention this set of possible values will be denoted by the set of non negative integers, s is equal to 0, 1, 2 and so on.. or otherwise measure mention it always assume that the state space s consist of element 0, 1, 2 and so on…

Even, if we takes a other values also you can always make a 1 to 1 corresponding and make the state space is going to be s is equal to 0, 1, 2 and so on... Suppose the probability of x n plus 1 will be taking the value j given that, it was taking the value x naught is equal to i naught x 1 was i 1 and so on.. and x n was I that probability is same as the probability of x n plus 1 will be j given that, x n was i. For all states, for all states i naught whatever value of i naught i 1, i and j and also for all n greater than or equal to 0. If this property satisfy by for all states i naught, i 1 and i comma j as well as for all n greater than or equal to 0, then this stochastic process that is discrete time discrete state stochastic process is going to be known as discrete time Markov chain.

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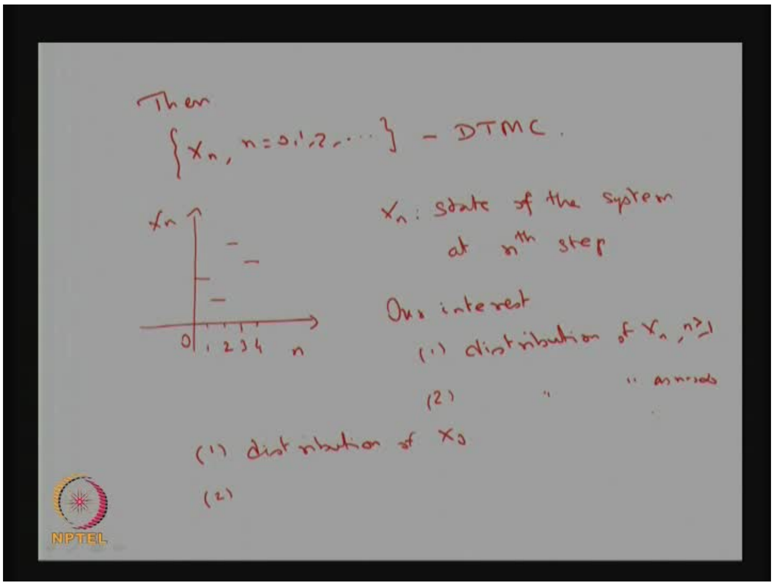

So, basically this is the Markov property. And the Markov property satisfied by all the states, as well as all the random variables so, this Markov property satisfied by any stochastic process then it is called a Markov process. And since, it is a time space discrete and parameter space is discrete therefore, it is called discrete time Markov chain. Then, we say this stochastic process X n takes value n starting with 0, 1, 2 and so on.. this a discrete time Markov chain.

We can just have look of how the sample path look like for the different value of n, and the y axis is x. Suppose at n is equal to 0 it is started with some value at x is equal to n is equal to 1; it would have been different value and n is equal to 2 value, it would have been different value so, this values are the either it could be a finite value or countably infinite number of values therefore, the state space is going to be discrete, and the parameter space is going to be discrete. So, like that it is it taking a different value over the n; so this is going to be this sample path or trace of the stochastic process x n.

Suppose, you assume that X n is the state of the system at nth step or nth time point, and this X n satisfies the devious the Markov property, then the stochastic process is going to be call it as a discrete time Markov chain. And our interest will be supposed to stochastic process satisfies the Markov property, our interest will be to know the two things; one is what is the distribution of X n? for n is greater than or equal to 1, you know where the system starts so, X naught you know your interest will be what could be the distribution of the X n that is nothing but, what is the probability that the X n will be in some state j. And also what could be their distribution of X n as n tends to infinity, as n tends to infinity our interest will be finding out the distribution of the X n.

So, at any finite n as well as n tends to infinity that will be of our interest. To compute these you need two things: one is you need a what is the distribution of X naught? That is a initial distribution vector where the system starts at the 0 th step. What is the distribution of X 0? And also second things of your interest will be, what is the transition distribution? or How the transition takes place? What is the distribution of transition from any n th step to n plus 1th step for all n?

So, if you know the two things the initial distribution vector as well as the distribution of that transition from nth step to n plus 1th step using these two quantity, you can find what is the distribution of X n? for any n as well as find on the distribution of X n as X tends to infinity. For that, I am going to define few condition probability distribution as well as the marginal distribution for the random variable X n through that, we are going to find out the distribution of X n.

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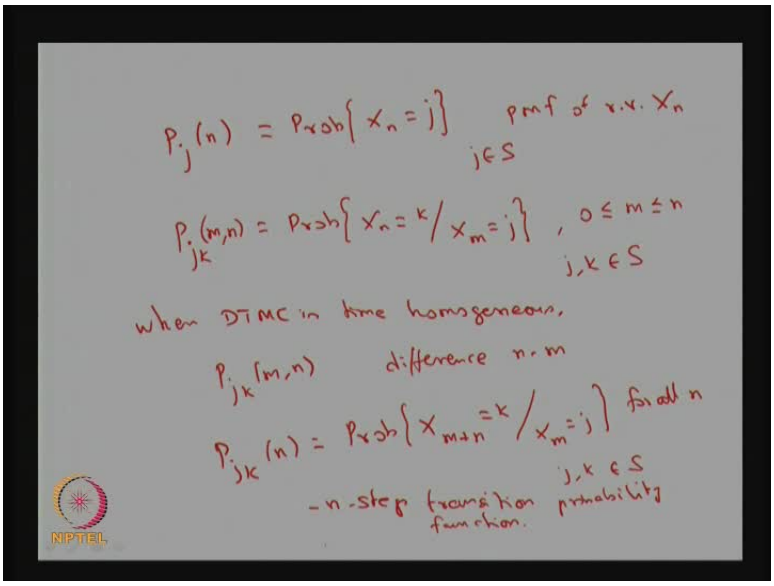

For any n as well as n to infinity. So, the first 1 I am going to define the probability mass function has the p suffix j of n that is nothing but, what is the probability that X n takes a value j, so this is the probability mass function of the random variable X n. What is the probability that the X n takes the value j that I am going to denoted as, that p suffix j of n where j is belonging to the state space capital S; this is a probability mass function of the random variable X n.

Similarly, I am going to define the conditional probability mass function as a p suffix j k of n that is nothing but, what is the probability that X n takes the value k given that X sorry, I need two suffixes two indexes for here with the two variables m comma n. What is a probability that X n will be the state k given that X m was j, obviously the m is lies between 0 to n whatever m and every n, and j comma k is belongings to capital S. So this is a conditional probability distribution of the random variable X n with X m, and at the m th step.

The system was in the state j, and the n th step the system is in the state k. And this is the condition probability with the two arguments m comma n; so, this is the probability that the system makes a transition from the state j at step m to the state k at the step n; This is called transition probability function of the discrete time Markov chain.

When the DTMC is the time homogeneous this is very important, when the DTMC is a time homogeneous that means, it satisfies the time invariant property, that means the p j k of m comma n depends only on the time difference n minus m whenever, the DTMC is a time homogeneous that means the time invariant so, the actual time is not a matter only the time difference is the important. Therefore, this is going to depends only on the time difference n minus m. In this case I do not want the two arguments m comma n, I can go for writing p j k of n that this nothing but, what is the probability that the m plus nth step the system will be in this state k given that, the mth step it is in the state it was in the state j, for all n and here j comma k belonging to s. So, the m does not matter; only the interval are the interval length of step n is matter; So that means the system was in this state j, and it is transient into the state k in n steps, because the DTMC is a time homogeneous, so the X m to X m plus n it is valid for all m.

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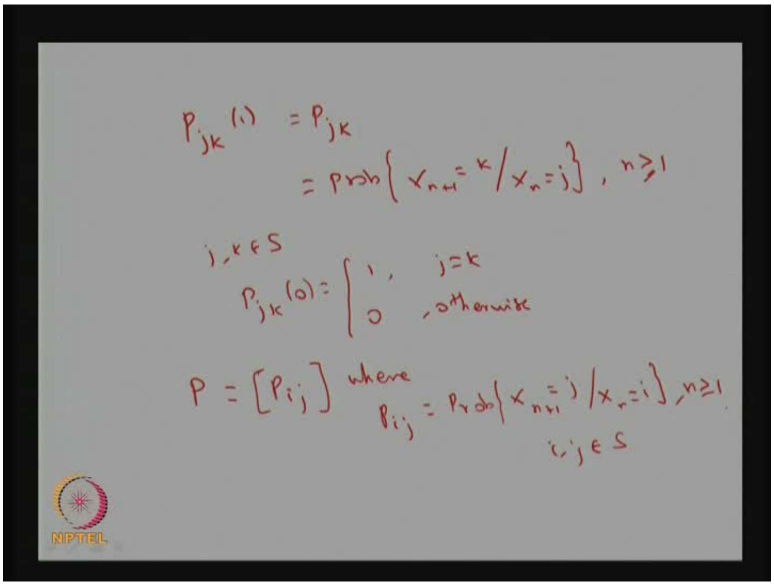

For all n we are finding for the n step transition, this is called the n step, because the DTMC is a time homogeneous and this called a n step transition probability function, this is a n step transition probability function. Using this we can define 1 step transition probability that is denoted by p suffix j k of 1 or you can avoid the bracket 1 also so, you can write it as a p j p j k that is nothing but, what is the probability that the X m plus 1 is equal to k given that X n is equal to j, for all n greater than equal to 1; obviously for j k belonging to S if you find the 0 step transition probability, that values is going to be 1 for j is equal to k otherwise is going to be 0.

This 1 step transition probability I can make it in the matrix form as the capital P is the matrix, and that consist of P i j where the P i j is nothing but, 1 step transition probability mat elements of X n 1 is equal to j given that X n is equal to i, here i comma j belonging to the state space S; you should remember the state space S is consist of finite elements or countably infinite number of elements. Accordingly, this matrix is going to be either when S is going to be a finite elements, then P matrix is going to be square matrix.

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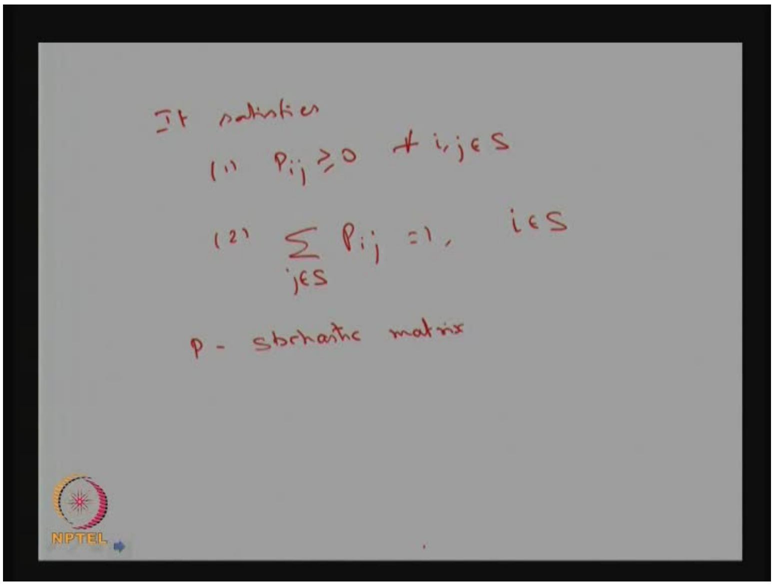

Since, the P i j is the 1 step transition probability of the system moving from the state i to j in 1 step, and since it is a time homogeneous; this is valid for all n, this is valid for all n greater than or equal to 1. And this satisfies the 1 step transition probability matrix satisfies two properties, the each entity will be greater than or equal to 0 for all i comma j belonging to S, because this are all only the conditional probability of the system moving from the state i to j in 1 step therefore, either it will be a 0 or greater than 0 for all possible values of i comma j.

The second condition if you make the summation over j for fixed i then, that is going to be 1 i belonging to be S that means the rho sum is going to be 1, because it is a conditional probability of system moving from 1 state to all other state, if you add a all other possible probabilities then that is going to be 1 since, this 1 step transition probability matrix satisfying this two properties and this matrix P is known as a stochastic matrix. Because of satisfying this two condition the matrix 1 step transition matrix is also called a stochastic matrix; Now, I am going to explain what is the pictorial way of giving the 1 step transition probability matrix or the stochastic matrix, that is provided by State transition diagram, or the other word it is called a directed graph.

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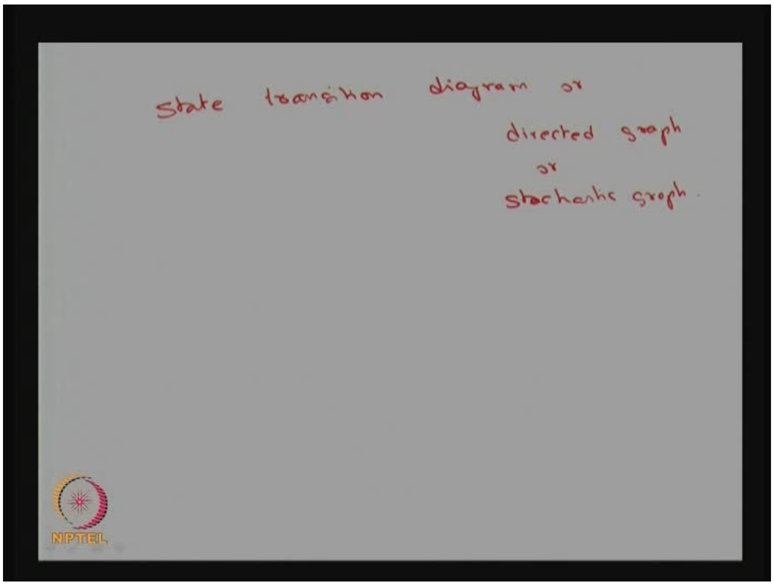

The DTMC can be viewed as a directed graph such that the state space S is a set of vertices or nodes, and the transition probability that is 1 step transition probability, or the weight of the directed arcs between these vertices or nodes. Since, the weights are positive and sum of the arch weights in each node is unity, this directed graph is also called the stochastic graph. It is useful to visualize discrete time the Markov chain and it is also useful to study the properties of the discrete time Markov chain.

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Example 1

A factory has two machines and one repair crew. Assume that probability of anyone machine breaking down on a given day is α. Assume that if the repair crew is working on a machine, the probability that will complete the repairs in a day is β. For simplicity, ignore the probability of a repair completion or a breakdown taking place except at the end of a day. Let X_n be the number of machines in operation at the end of the nth day, Assume the behavior of X_n can be modeled as a Markov chain.

I am going to explain the discrete time Markov chain with three simple examples: the first example is as follows. A factory as a two machines and one repair crew. Assume that probability of any 1 machine breaking down a given is alpha, so alpha is the probability. Assume that if the repair crew is working on a machine, the probability they will complete the repairs in a two more day is beta. For simplicity, ignore the probability of a repair completion or a breakdown taking place except at the end of a day.

That means we observed the system at the end of the day how many working machines in the systems? Let X n be the number of machine in operation at the end of the nth day, Assume that the behavior of X n can be modeled as a Markov chain. So based on the information available here the machine can be breakdown, and we have only 1 repair person, and the probability of he can do repair in a day that probability is beta. And 1 minus beta is a probability that he cannot complete the repair of the machine in a day.

And the random variable X n is a its denotes how many machines in the operation at the end of the day? Therefore, the possible values of X n since, we having a two machines the possible values of X n will be 0, 1 or 2. So, this will form a state space capital S. So, the S consist of the element 0, 1, 2; and the X n over the n it is form the discrete time Markov chain, because this a discrete time discrete state stochastic process, and also based on the clue the number of machines working in any day, depends how many machines working in a previous day and how many things are under repair and so on..

So, dynamics of the number of machines in operation depends only on the number of machines working in the previous day, not all the pervious earlier days. Therefore, the memory less property satisfied by the stochastic process X n therefore, it is called a discrete time Markov chain. Our interest is to find what is the 1 step transition probability matrix with the assumption is that X n is the time homogeneous also since, it is time homogeneous DTMC. Therefore, we are trying to find out what is a 1 step transition probability matrix for the given time homogeneous DTMC.

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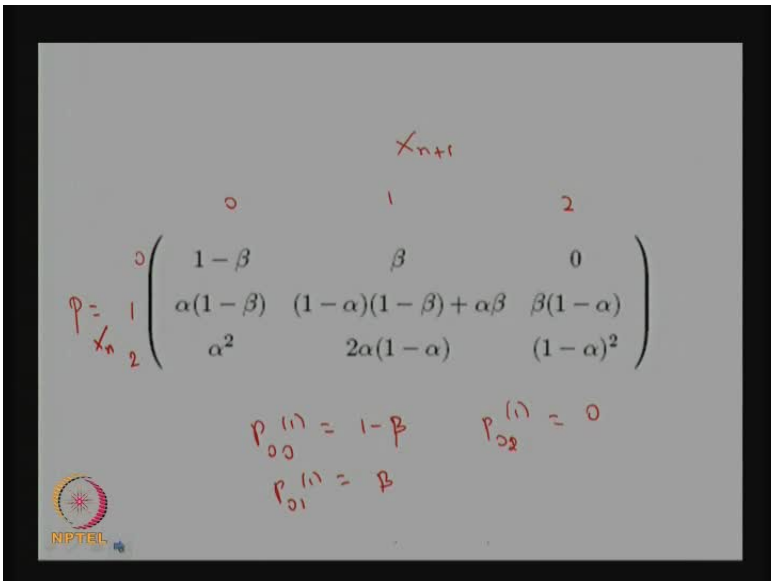

So, this is the 1 step transition probability matrix P n and the possible states are 0, 1 and 2. Suppose the system was in the state 0, 1 or 2 in the nth step where, the system will be n plus 1th step therefore, this is the possible values of X n plus 1 and this the possible values of X n, and this 1 step transition probability matrix will give suppose, the system was in the state the nth step, what is the probability is that it will be in these states in the n plus 1th step, so the first index will give what is the probability 0 comma 0 in 1 step that means, in the nth step number of working machines are 0.

And what is the probability that in n plus 1th step, also 0 machine are in the working condition that means all are under repair, all two machines are under repair. And the probability of the crew is going to be not repair, that going to be 1 minus beta therefore, the probability is 1 minus beta in 1 step, what is the probability that the number of working machines going from 0 to 1; that is because of the crew is finishing the repair in a day and that probabilities is beta.

And since, he can do 1 repair in a day so, the possibility of the repairing more than 1 machine in a day, it is not possible the rare event and the probability is going to be 0 therefore, the P 0 2 is going to be 0. Similarly, now we can visualize what is the probability? that number of working machine is 1 in the nth step, and what is the probability is that in the 0 machines working in the n plus 1th step.

That is possible with the two independent things: the 1 machine can be failed and the other machine cannot be finishing the repair therefore, the crews is not finishing the repair, that probability is 1 minus beta multiplied by 1 machine is going to be repaired. Therefore the total number of machine working will be 0 in the n plus 1th step, that is alpha times 1 minus beta and similarly, you can evaluate the other element also and for example, the system is going from the state 2 to 0.

That is nothing but, at the nth step two machines are working condition, and the n plus 1th step 0 machines are the working condition, that means both the machines are got repair got failed in the same day. Therefore, that probability is alpha times alpha that is the probability both the machines got failed in the same day. Therefore, in the next day the number of working machine is going to be from 2 to 0 like that, we can visualize the other elements also. The same 1 step transition probability matrix can be visualize with the state transition diagram.

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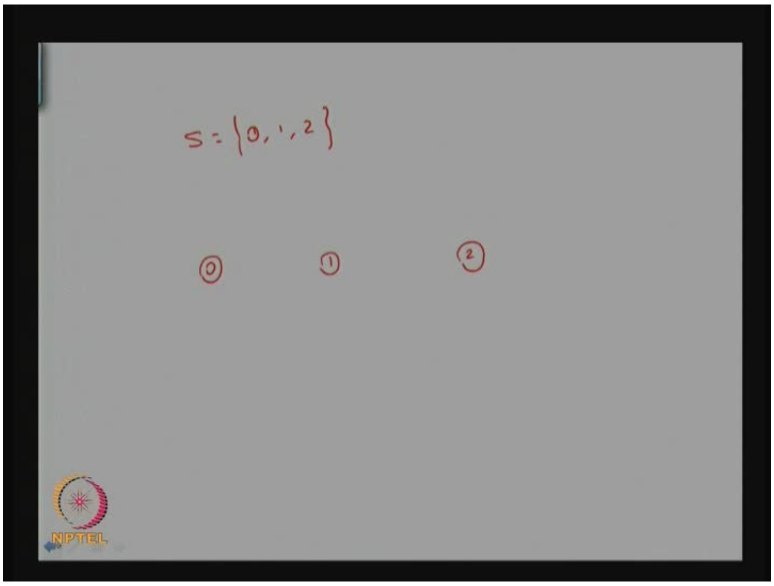

And state transition diagram you have to make state space as a vertices or the nodes, and the weight of the directed arc are nothing but, the 1 step transition probability that is the system is moving from the 1 states to other states, those are going to be the weights.

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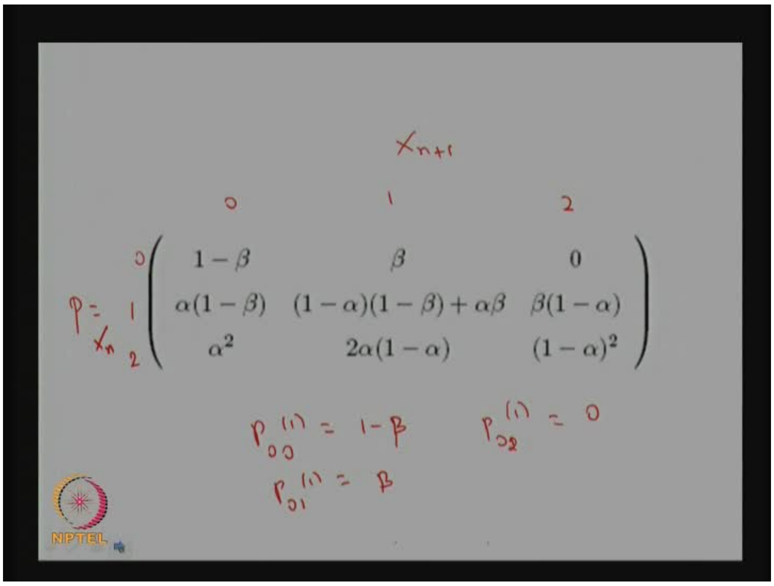

If the probabilities are zeros, then no need to draw the directed arc from that particular node to that destination node. So, first we start with the nodes as the possible values of the states space; so you list out all the states space as a node. Now, by seeing the 1 step transition probability matrix you should make the arc from 0 to 0 self look is allowed; if the probability is going to be greater than zero. So, you should draw the self loop from 0 to 0 as a arc value minus beta, you should draw the arc from 0 to 1 with the arc weight beta, and you should not draw any arc from 0 to 2, because that probability is going to be 0.

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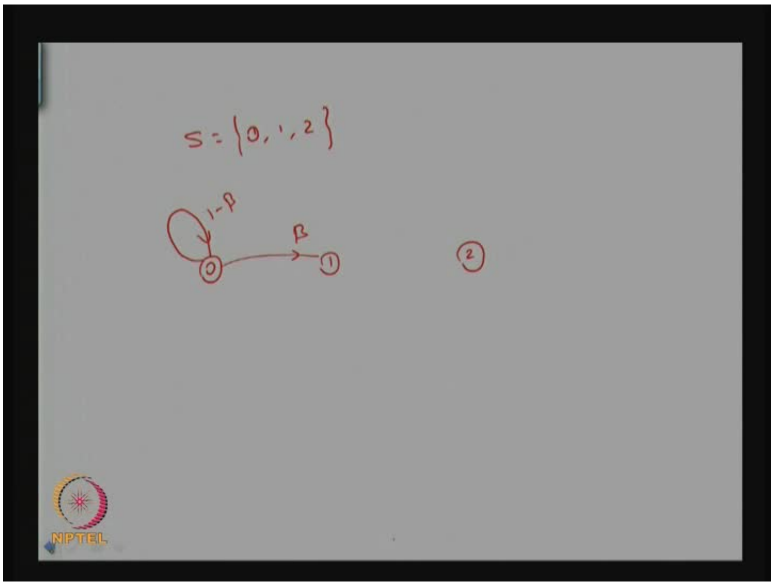

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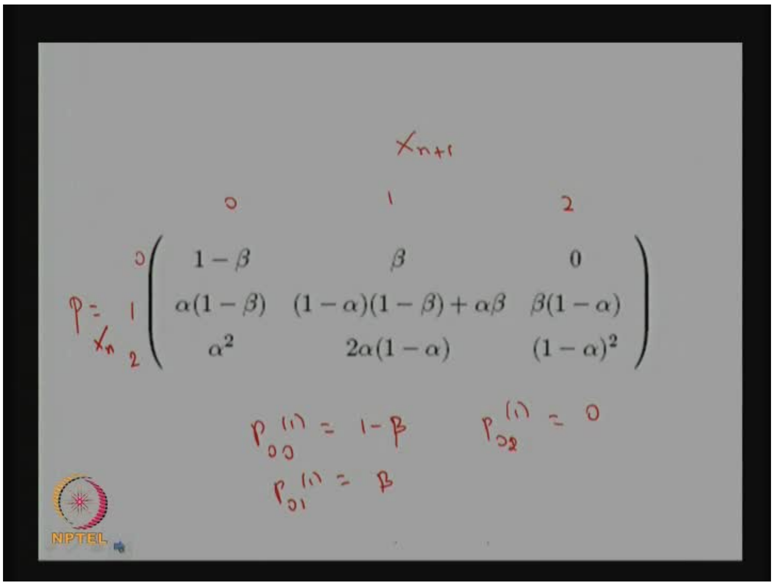

Therefore, 0 to 0 that probability is 1 minus beta and 0 to 1, it is going to be beta and there is no arc from 0 to 2 therefore, that probability is 0 and similarly, now we can go for filling the second row. So, 1 to 0 is alpha times 1 minus beta, 1 to 1 is 1 minus alpha times 1 minus beta plus alpha beta, and 1 to 2.

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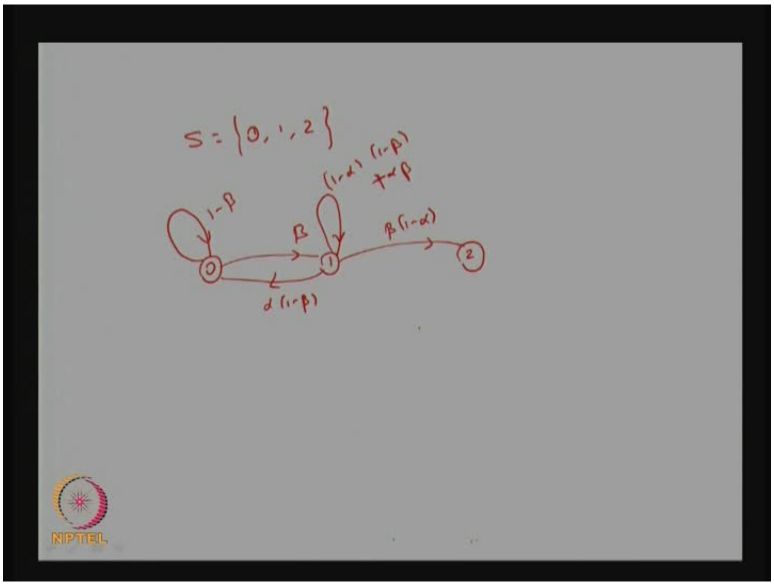

So, you have all the three probabilities are greater than 0 therefore, 1 to 0 that arc is alpha times 1 minus beta, and 1 to 1 is 1 minus alpha times 1 minus beta plus alpha beta, 1 to 2 is beta times one minus alpha.

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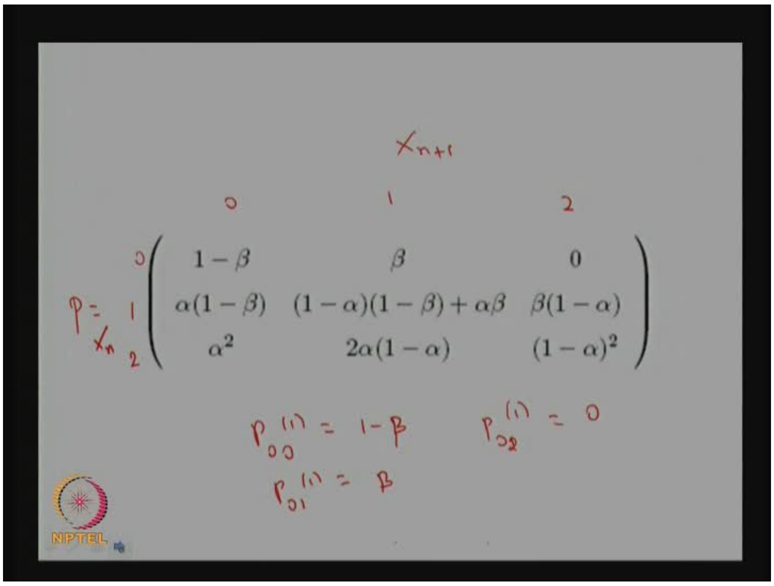

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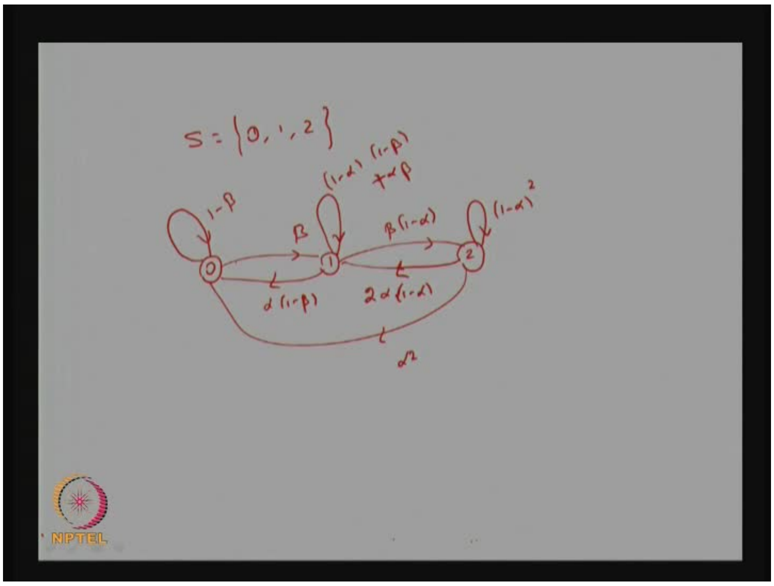

Similarly, you can draw arc for 2 to 2 to 0 that is alpha square and 2 to 1 and so on.. therefore, 2 to 0 that is alpha square and 2 to 1 2 to 1 is 2 alpha times 2 alpha times 1 minus alpha, and 2 to 2 and 1 minus alpha whole square. So, the state transition diagram is a pictorial view of the 1 step transition probability matrix. This is nothing to do with the initial probability distribution.

It gives only the information about the whenever the DTMC is a times homogeneous suppose, the system start from 1 particular state what is the probability that the system will move into the another states with probability. And it will not give it will not give more than that information but, this much information is useful when you are going to study the properties of the discrete time Markov chain, as well as when you are finding the limiting distribution that is the distribution of X n as n trends to infinity, the diagram will be very useful to conclude whether the limiting distribution will be exist or not; if it is exist, whether it is going to be unique or not and so on.. So, those things can be visualize easy by seeing state transition diagram.

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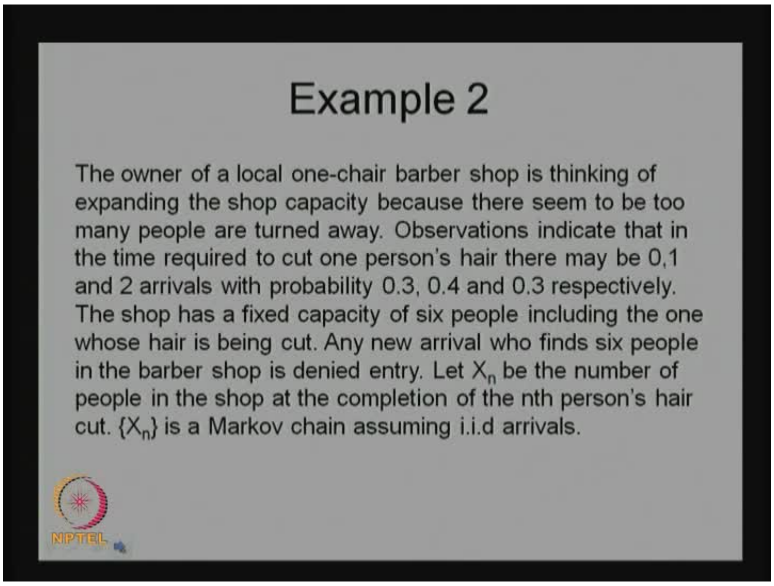

Now, I am moving into the second example. In this example I have taken the barber shop example which I have discuss in the module 1 also the same example. So, the owner of the local one-chair barber shop is thinking of expanding the shop capacity because there seems to be too many people are turned away. Observation indicate that the time required to cut one person's hair there may be 0, 1 and 2 arrivals with probability point 3, point 4 and point 3 respectively. So, this information is very important that means during one person hair cut, what is the probability that no people turned up with the probability point 3, and 1 people turned up with the probability point 4. And there is a two possibilities of arriving two persons hair cut with the probability point 3.

Therefore, the summation of the probability is going to be 1 so, during 1 persons hair cut these are the only three possibilities are possible with the 0 arrival, or 1 arrival or 2 arrival. The shop has a fixed capacity of six people including the one whose hair is being cut. That means maximum six people can be allowed in the system so, five people can wait maximum, and 1 people can be under the service. Any new arrival who find the six people in the barber shop is denied entry, that is the meaning of the capacity of the system is finite with the size six.

Now, I am going to define the random variable. Let, X n be the number of people in the shop at the completion of the nth person hair cut, this is very different random variable or this is very different stochastic process, usually the parameter space is time but, here the parameter space is the number of people in the shop. The n is the parameter space is the persons who leaves after the hair cut. So, it is the nth person who leaves the system that becomes the parameter space whereas, the random variable is how many people in the system when the nth person leave the system that means, you should not count that person when you are finding the values of X that means, this number is counted at the departure time point. So, when the nth person leaves how many peoples in the system the system is the maximum six people allowed therefore, he cannot see more than five people in the system when he leaves.

So, because of this constrain because of during the 1 person arrival either 0 or 1 or 2 arrivals can takes place and so on.. based on this information the stochastic process X n is going to be a discrete time discrete state stochastic process, as well as the Markov property satisfies. That means the probability of X n plus 1 takes some value given that all pervious values are known, that is same as the conditional probability distribution of X n plus 1 takes some value, given that X n was some value. So all So, the future distribution given that the present as well as the past information is same as the future distribution given the present not the whole past information.

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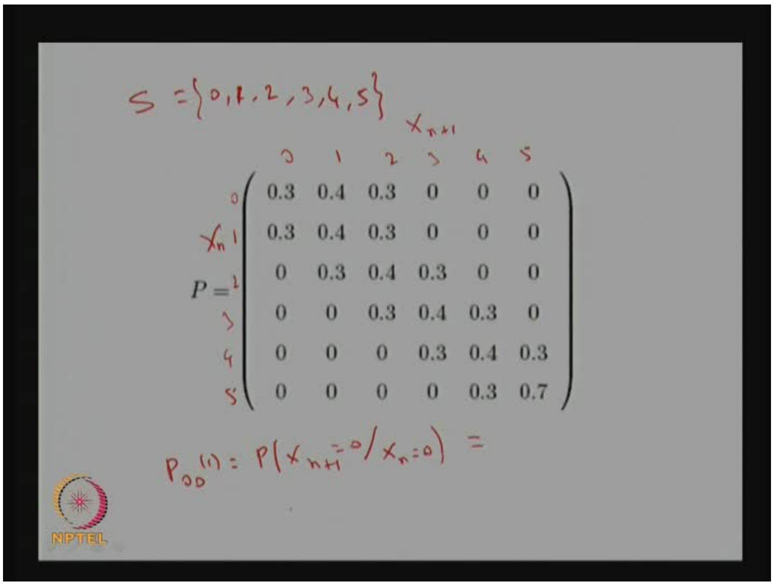

So, this Markov property will be satisfied by this stochastic process therefore, this X n will form the discrete time Markov chain. Obviously, it is a time homogeneous discrete time Markov chain also, in this example our interest is to find out what is the 1 step transition probability matrix. This is going to be the 1 step transition probability matrix and the possible states S n s is going to be 0, 1, 2, 3, 4 or 5. Because the capacity of the system is 6, and whenever he whenever the nth person leaves either first person, second person, third person leaves, how many people are in the system? Therefore, the maximum will be 5, and there is the possibility will be when he leaves no one will the in the system also.

And this is the one step transition probability matrix, and this is also going to be a square matrix, because this is going to be countably finite number of elements, and this is a 0, 1, 2, 3, 4, 5. Now, we can discuss what is the probability that 0 comma 0 in 1 step that is nothing but, in the nth when the nth person leaves no one in the system, when the n plus 1th person leaves no 1 in the system, what is the probability for that? that is X n plus 1 is equal to 0 given that X n was 0 so, 1 step transition probability it is independents of n, because it is a time homogeneous so, 1 step transition probability matrix. So, this is possible at some person leaves whatever be the n, nobody in the system when the next person leaves no body in the system so, that is possible by when some person leaves the system was empty for some times, we do not know how much time it was empty?

Then the n plus 1th person enter into the system and during his hair cut no one turned up, or no arrival takes place during his or 1 plus nth hair is going on. Therefore, when he leaves no one in the system, so we are not bothering when he enter into the system and so on.. Our interest is how many numbers of people of the system? when the n plus 1th person leaves, and this probability is n plus 1th person leaves is 0 people in the system, and given that the n nth person also 0 person in the system. So, that is possible with the explanation I have given, no one enter into the system during the n plus 1th person hair cut.

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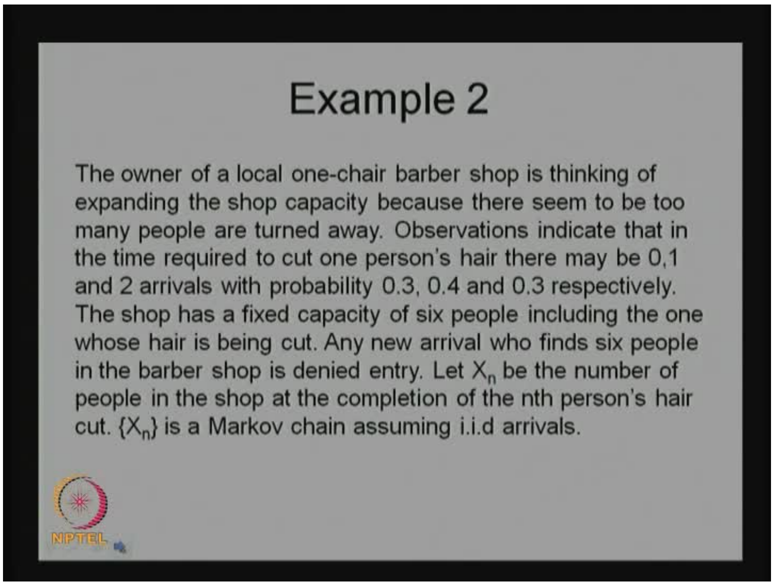

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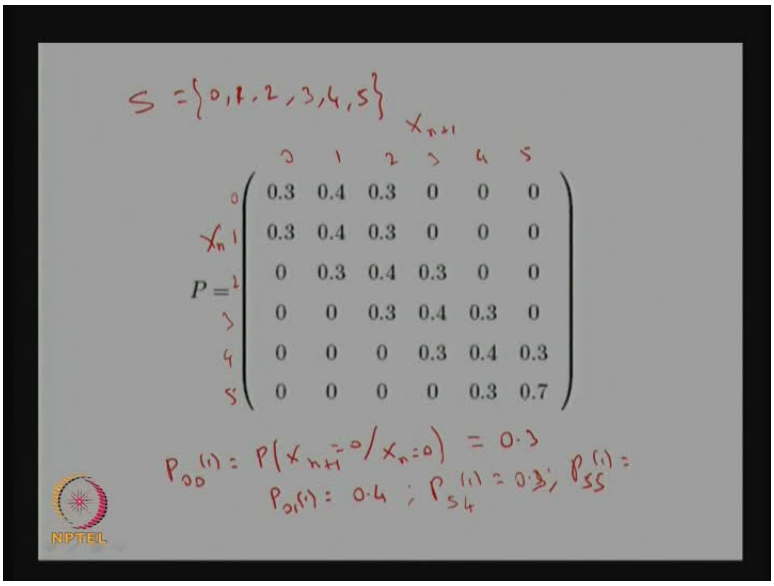

And the information provided indicated the time required to hair cut one person hair there may be a 0, 1 or 2 arrivals with the probability point 3. So, no arrival takes place during one person hair cut is point 3 therefore, this probability is possible with the probability point 3 whereas, P 0 1 of 1 step, the same way you can write the probability that X n plus 1 is equal to 1 given that X n is equal to 0, that is possible when the nth person leaves no one in the system when n plus 1th person leaves one person in the system, that means during his hair cut one person enter into the system, that is possible with the probability point 4.

Similarly, with 0 to 2 in one step that is going to be point 3 with the probability two arrivals takes place when the 1 plus nth person hair cut. Now, the second row, second row what is the probability that when nth person leaves one person in the system, when n plus 1th person leaves 0 person in the system, that is possible. During the n plus 1th person hair cut there is no one in the system, no arrival takes place therefore, that probability is point 3, and from 1 to 1 that is possible with one person arrived during the n plus 1th person hair cut therefore, that probability point 4.

And going from the state 1 to two that is possible two person arrived at n plus 1th person hair cut whereas, 2 to 0 that is not possible. Because when the nth person leaves the two person in the system therefore, n plus 1th person leaves definitely, he will see on person in the system, because of no arrival, and 1 arrival, and two arrival therefore, it will be shifted by one column and it will keep continuing till the end whereas, the last one what is the probability that, what is the probability that? the five people in the system when the nth person leave, and the four people in the system when the n plus 1th person leave, that is same as no arrival takes place during the n plus 1th arrival n plus 1th hair cut going on. So, therefore this is going to be point 3 whereas, P 5 to 5 in one step, that is possible with the combination of one person arrive the system or two person arrive the system, the system size is going to be maximum six.

Therefore, when n plus 1th persons hair cut is going on if 1 person arrives, then he will be enter, then two person arrive then he cannot be accumulated. Therefore, he will not join the system therefore, the system the number of customers in the system in X n, that is going to take the value 5 and the combination of point 4 as well as point 5.

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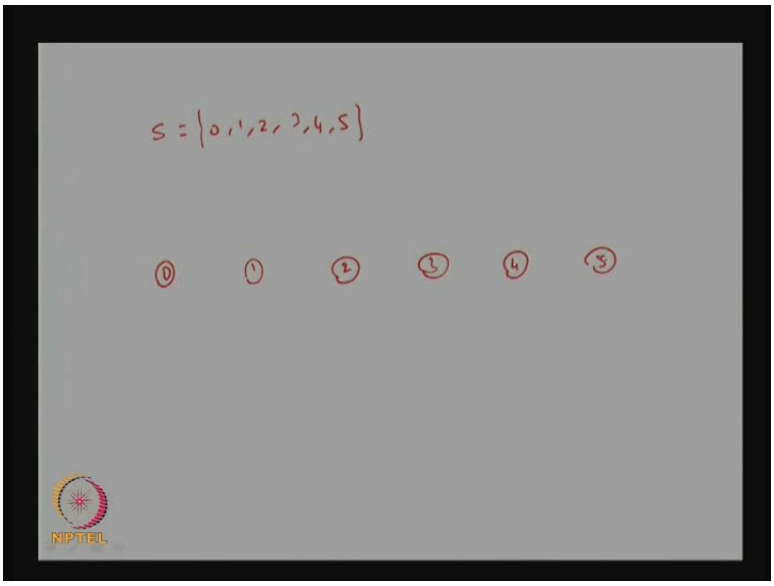

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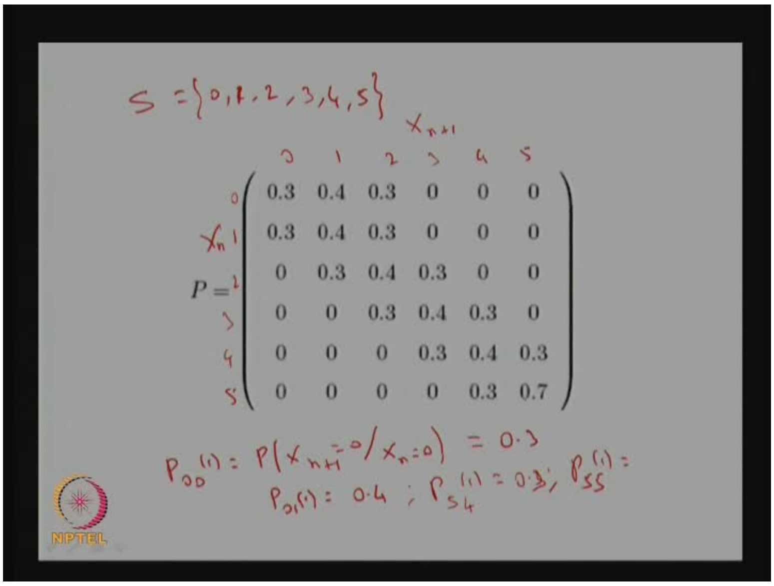

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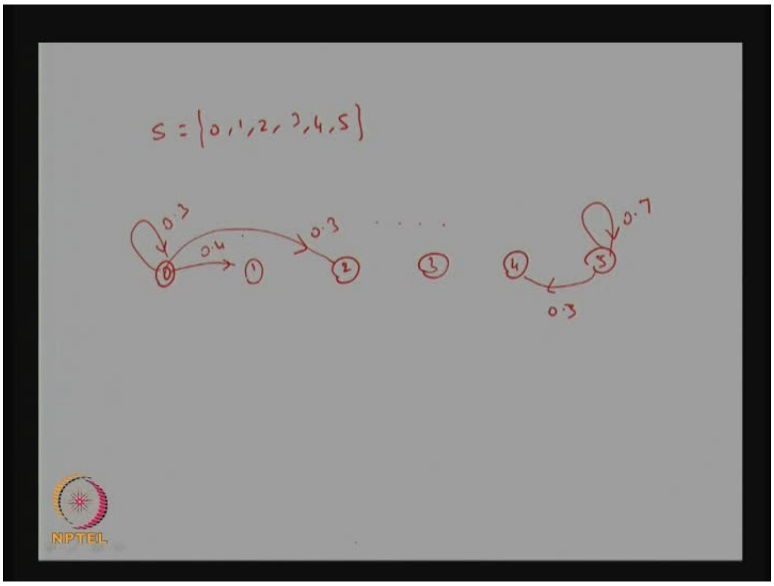

Therefore, this probability of system is moving from 5 to 5 is point 7, because of point 4 plus point 3. Now, I can give the state transition diagram. For this example, because S is going to be 0, 1, 2, 3, 4, 5 therefore, the nodes are going to be 0, 1, 2, 3, 4 and 5 and the possible values from the state transition from the one step transition probability matrix as I can make out so, 0 to 0 that probability is point 3, and 0 to 1 is point 4 and 0 to 2 is point 3. Similarly, I can fill up all other things and 5 to 5 that is very important, and 5 to 4 that is possible with the probability point 3, and 5 to 5 with the probability point 7.

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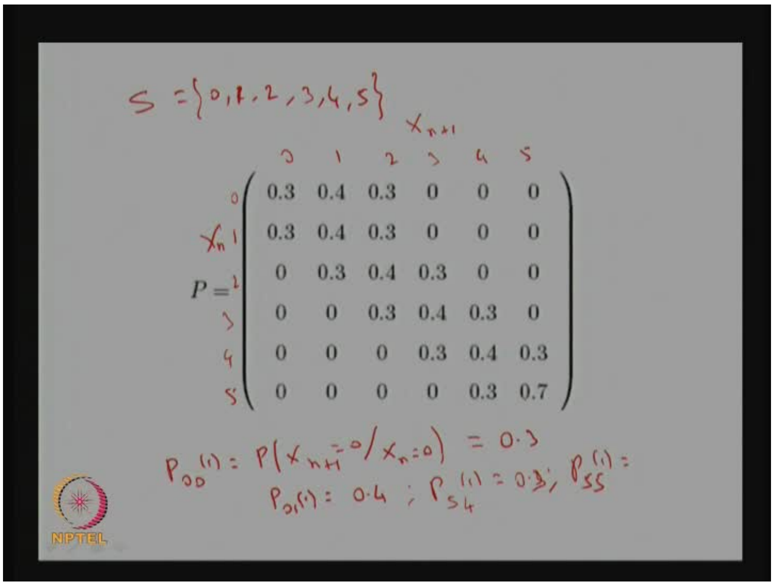

So, this is the state transition diagram I did not complete the state transition diagram. We have to fill up all the arcs with the arcs with weights going from 1 arc to other arc with the positive probability wherever, the possible probability is 0 we should not draw the arc for it.

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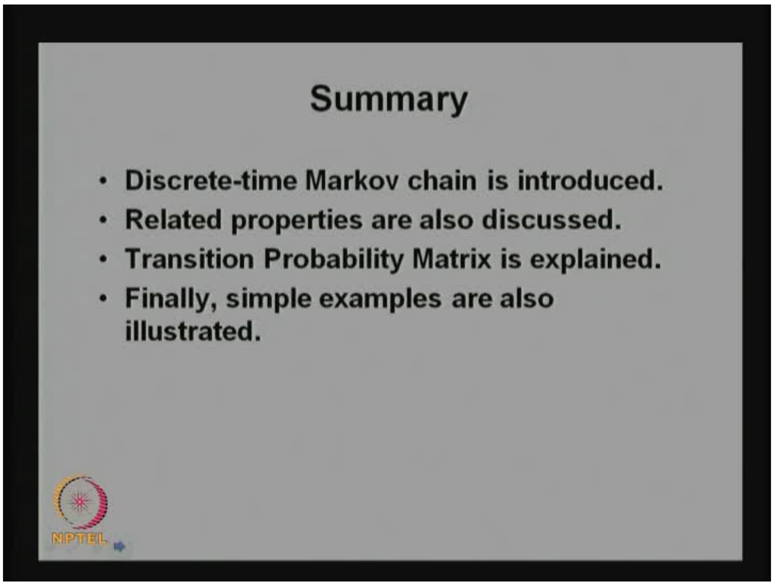

So, in this lecture we have discussed the Discrete Time Markov Chain, then we have given the few important properties, followed by we have explain the one step transition probability matrix. And also, we have given two simple examples with this the lecture one is over for the module four.

Thanks.