Stochastic Processes
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Module - 8
Renewal Processes
Lecture - 6
Application of Markov Regenerative Processes

This is the stochastic processses module 8, renewal processes. In the lecture 1, we have discussed the renewal function and renewal equation. In the lecture 2, we have discussed the generalized renewal processes and renewal limit theorems. Markov renewal and regenerative processes are discussed in lecture 3. Non- markovian queues such as M G 1 M G 1 N M G C C are discussed in lecture 4. G M 1 G M 1 G M C G G 1 non-Markovian queues are discussed in lecture 5. In this lecture, we are going to discuss application of Markov regenerative processes.

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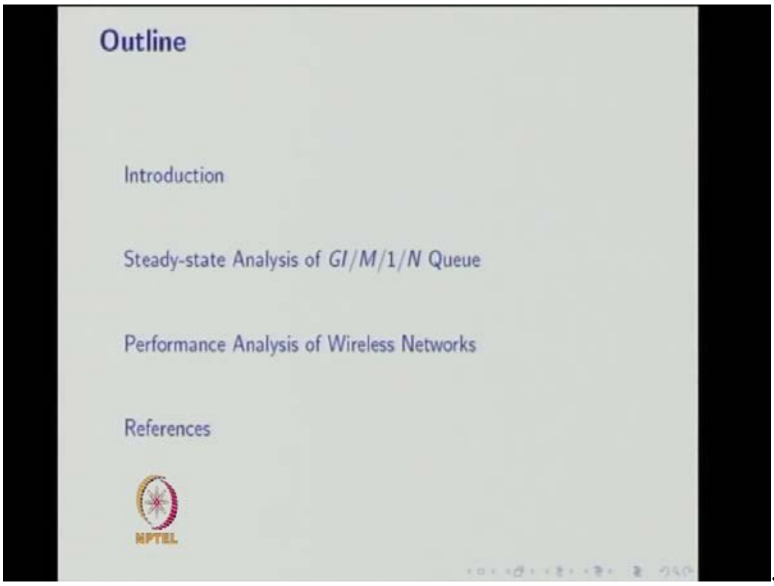

In the first half, we are going to discuss steady state analysis of G I M 1 N queue; in the second half we are going to discuss the application of Markov regenerative process in performance analysis of wireless networks.

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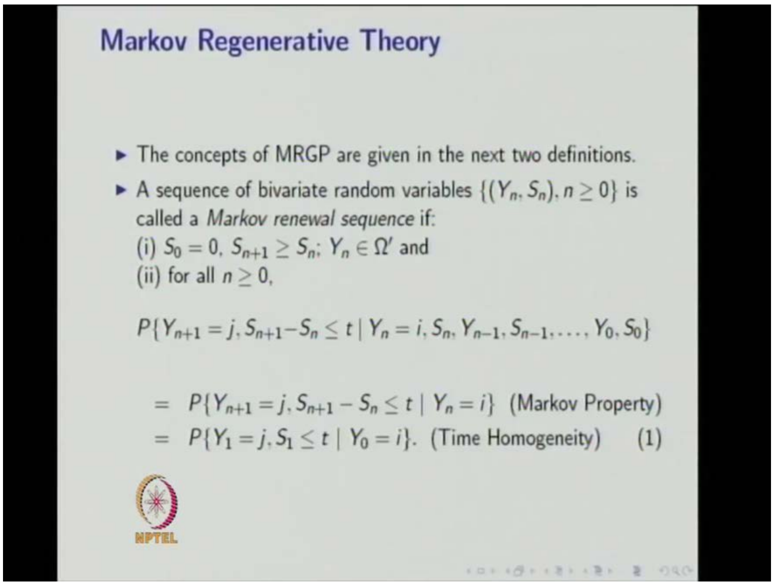

Already we have discussed the Markov regenerative theory in the lecture 3. It involves the Markov renewal sequence and embedded Markov chain.

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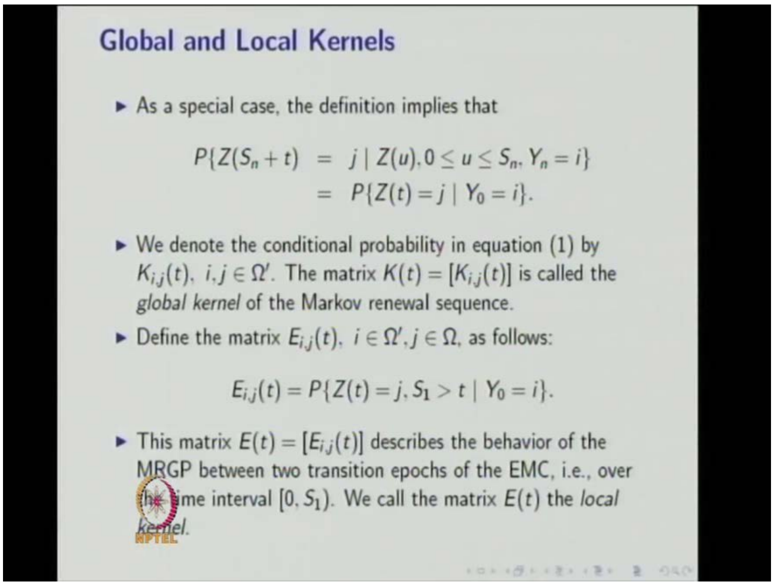

To study the limiting behavior first, we need the global kernel k of t it is consist of a matrix elements K i j of t and the another matrix local kernel that is the matrix E of t that consist of elements E i j of t, where i is belonging to omega dash, j belonging to omega.

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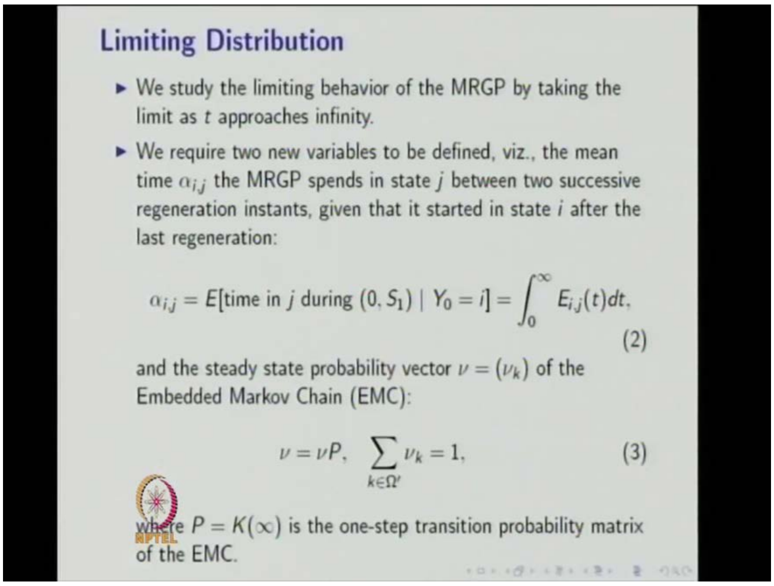

And also you need a mean time alpha i j. Also, the steady state probability vector v, which can be obtained by solving v is equal to v p and summation of v i is equal to one where i is belonging to omega dash.

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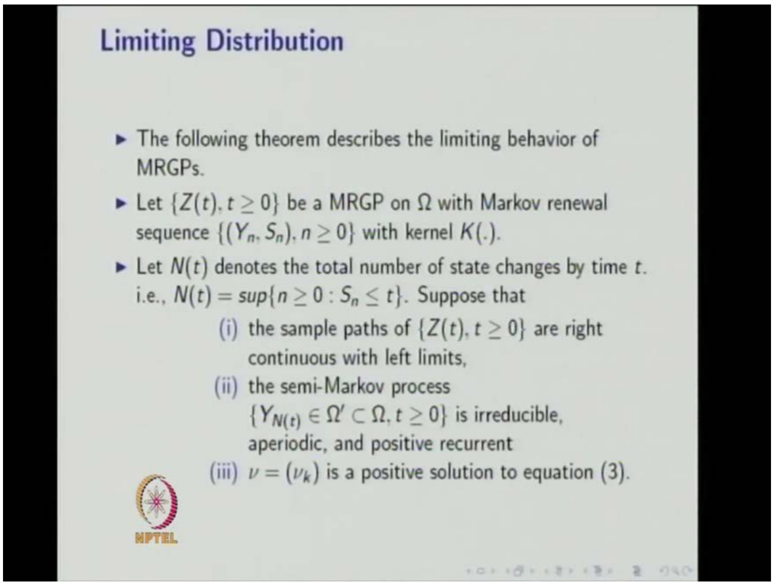

So, we have explained the limiting, how to find the limiting distribution in the lecture 3.

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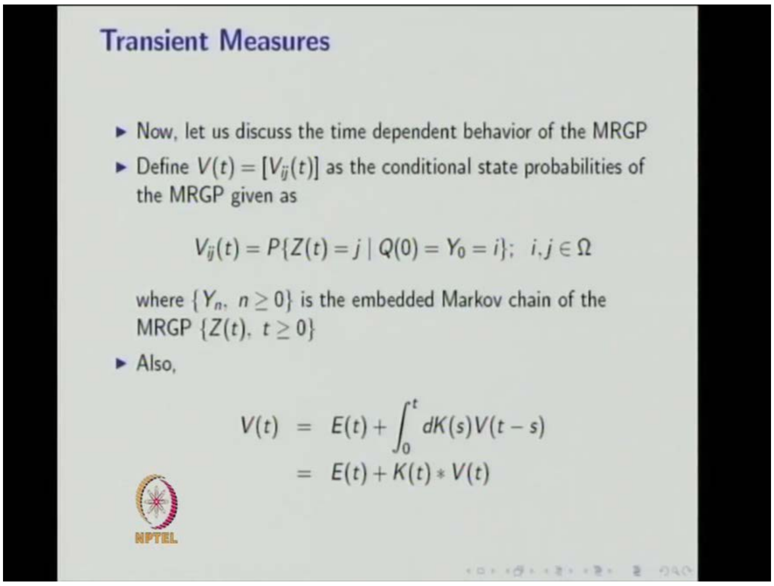

In this lecture, we are going to discuss also the transient analysis. How one can find out the transient measures of the MRGP? Define V of t is a element V i j as the conditional state probabilities of the MRGP that is the nothing but the probability that Z of t is equal to j given Z of 0 is equal to Y 0 that is equal to I, where i comma j belonging to omega. The sequence of Y n is the embedded Markov chain of the MRGP. Z of t also one can write V of t is same as E of t plus K of t convolution with V of t. So, once we solve this equation, you can get the conditional state.

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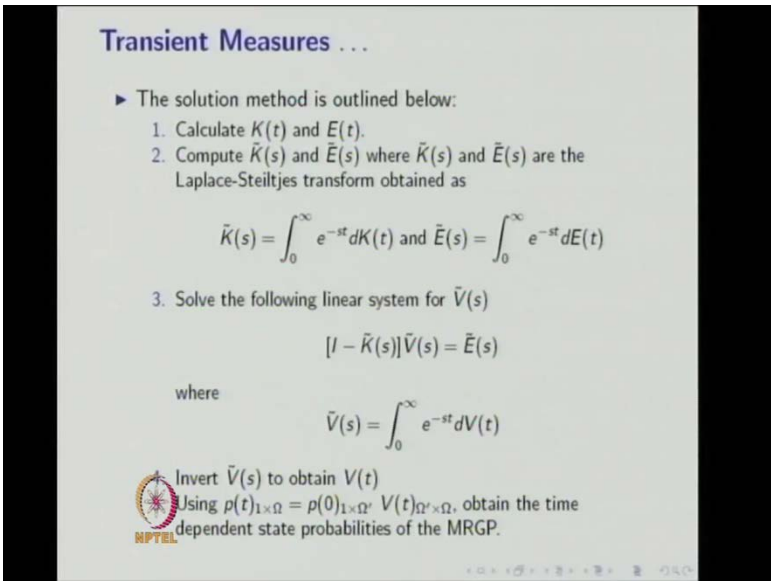

Probabilities the solution method is, outlined as follows. First you calculate the matrix global, global kernel K of t and matrix global kernel E of t. Then find out the Laplace Steiltjes transform of K of t as well as E of t, that is a second stage. In the third stage solve the linear system for V of s where V of s is the Laplace-Steiltjes a transform of V of t. The fourth stage invert V of s to obtain V of t. Once, we know the V of t that is the conditional state probability, you can find out the unconditional state probabilities using P of t is equal to P of 0 times V of t.

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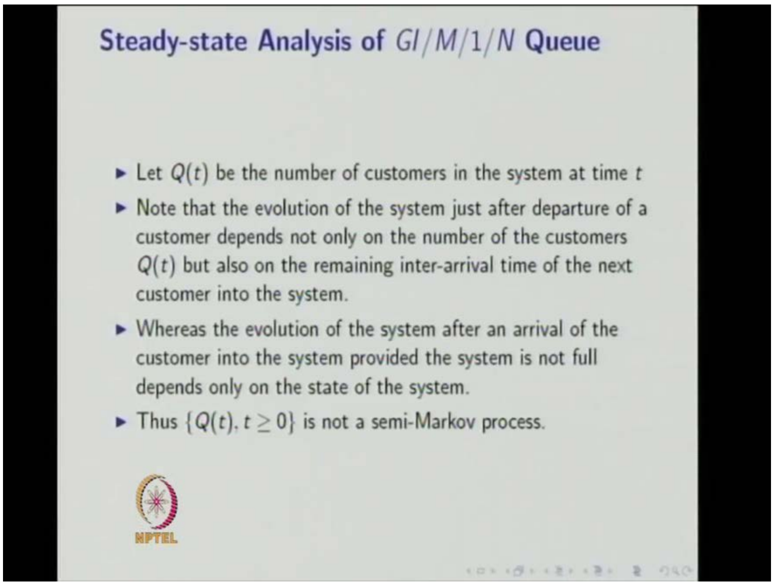

That will be the time dependent state probabilities of MRGP. The lecture 3, we have discuss only steady state probabilities of MRGP, in this lecture we are discussing the time dependent measures of the MRGP. Now, we are moving into the study state analysis of a G I M 1 N queue. Let Q t be the number of customers in the system at time t. Note that the evaluation of the system just after the departure of a customer depends not only on the number of customers Q of t, but also the remaining inter arrival time of the next customer who is entering into the system. So, the evaluation of the system after the arrival of the customer into the system provided this system is not fully depends only on the state of the system.

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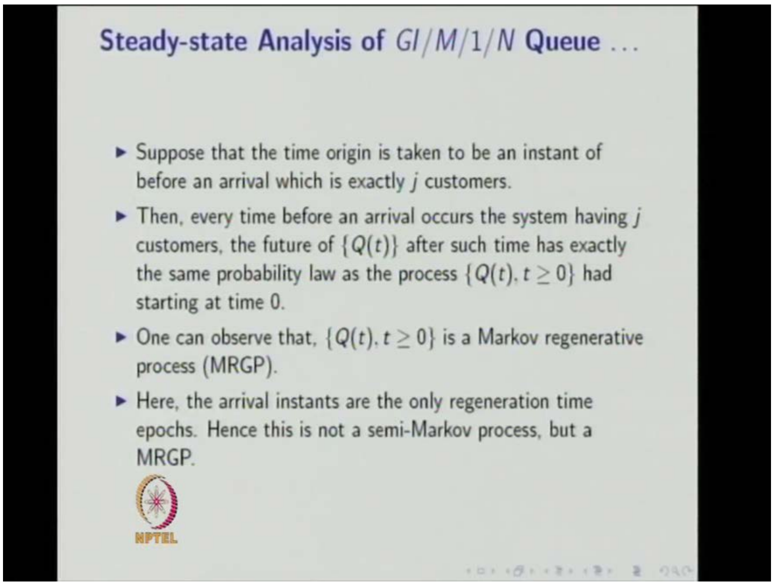

Hence Q of t is not a semi Markov process. Suppose the time origin is taken to be an instant of before arrival, which is exactly j customers, then every time before a arrival occurs, the system having a j customers, the future of Q of t after such time has exactly the same probability law as the process Q of t had starting at time 0. Hence, we can conclude the Q of t, t greater than or equal to 0 is a Markov regenerative process, MRGP. Here the arrival instants are the only regeneration time epochs, that means a and arrival epochs the memory less property is satisfied. Hence, the stochastic process Q of t is not a semi Markov process, but it is a MRGP.

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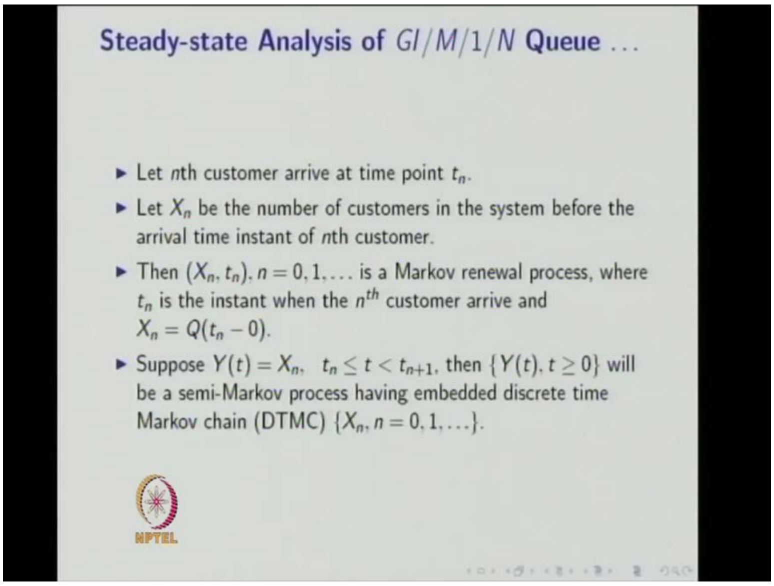

Now, we are going to study the steady state analysis. Now, we are going to now we are going to study the, study state behavior let n th be the, let n th customer arrival at time point t n. Let X n be the number of customer in the system before the arrival time instant of the nth customer that is nothing but X n is equal to X n is equal to Q of t n minus 0, number of customer in the system.

Before the arrival time instant of nth customer, Therefore, since it is a G I M 1 N queue the inter arrival follows the general distribution and the service follows exponential distribution, only one server in the system the X n comma t n for n is equal to 0 1 2 and so on common Markov renewal process or Markov renewal sequence. Suppose, we define y of t is equal to X n where t lies between t n to open interval t n plus 1, then Y t will be a semi Markov process having the embedded discrete time of Markov chain that is X n.

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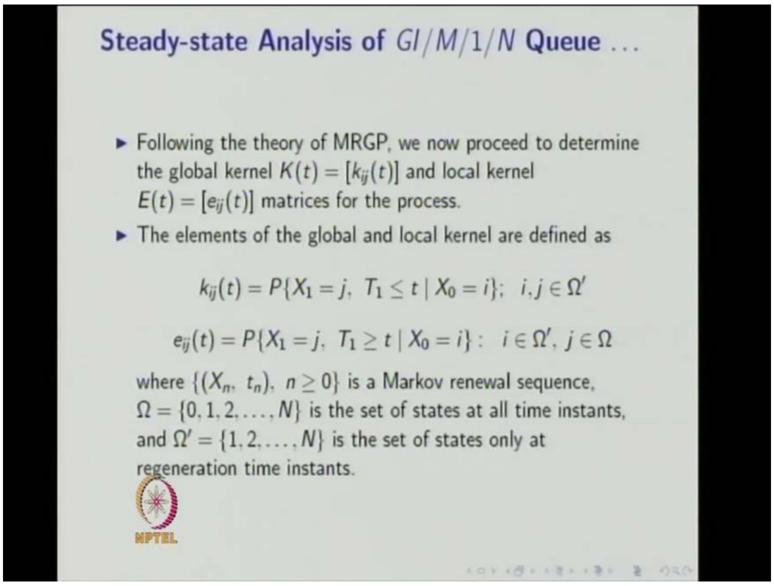

Following the theory of MRGP now we determined, the global kernel K of t and the local kernel E of t. So, the k i j of t is nothing but probability that X of 1 is equal to j with t 1 less than or equal to t given, X naught is equal to i for i and j belongs to omega dash. Whereas E of i comma j of t is nothing but probability that X 1 is equal to j if that t 1 is greater than or equal to t given X naught is equal to i here, i is belonging to omega dash and j is belonging to omega. So, the omega is a collection of set of states at all time points whereas the omega dash if the set of states only at the regeneration time points, time instants.

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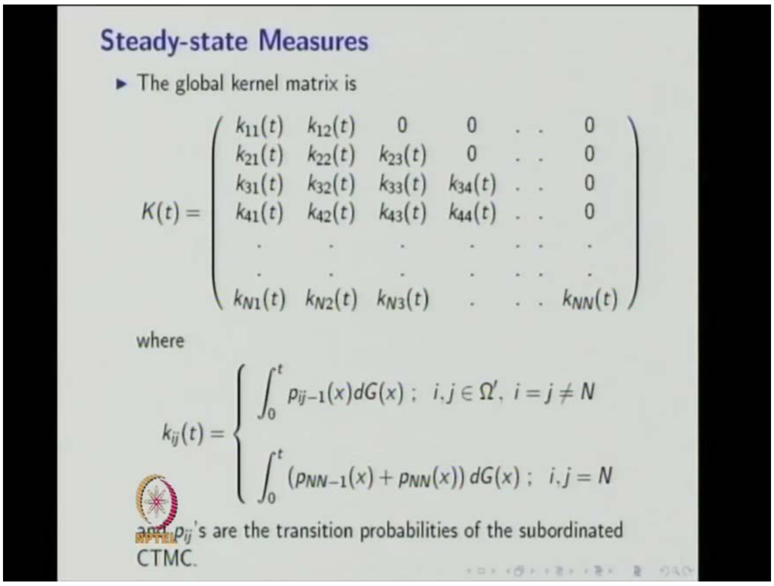

So, for this non Markovian queues we have a K of t matrix in this form with k i j of t is nothing but the integration between integration from 0 to t of p i j of p i j minus 1 of x integration with respect to G of x, where G of x is the c d f of inter arrival time. For i and j is equal to n will have a different expression. That means the system would have come from n minus 1 th the state to n th state or the system would have been stay in the same nth state retaining in the n of the state.

Therefore, you have two terms in the integrant p of n comma n minus one and p of n comma n integration between the interval 0 to t integration with respect to G of x. So, here the p i of j are the transition probability it is a one step transition probability of the subordinated CTMC. Sorry, this not the one step transition probability that p i comma j are the transition probabilities of the subordinated CTMC.

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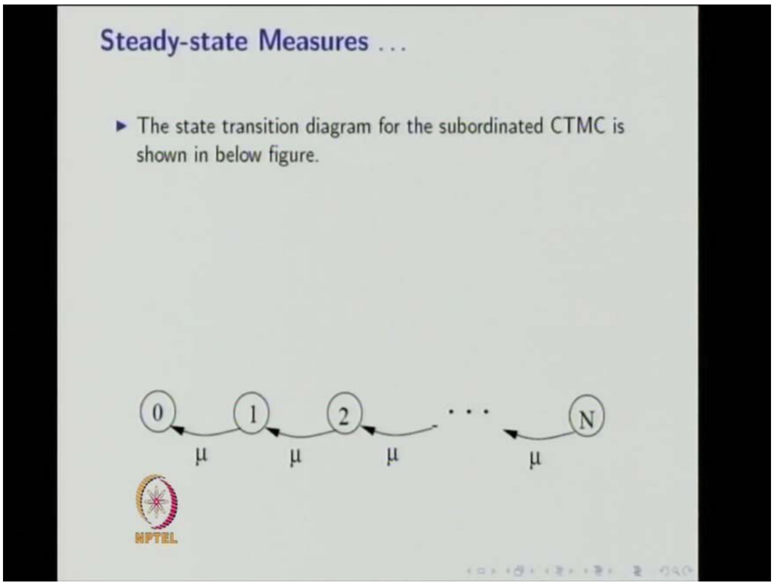

The state transition diagram of the subordinated CTMC is shown below.

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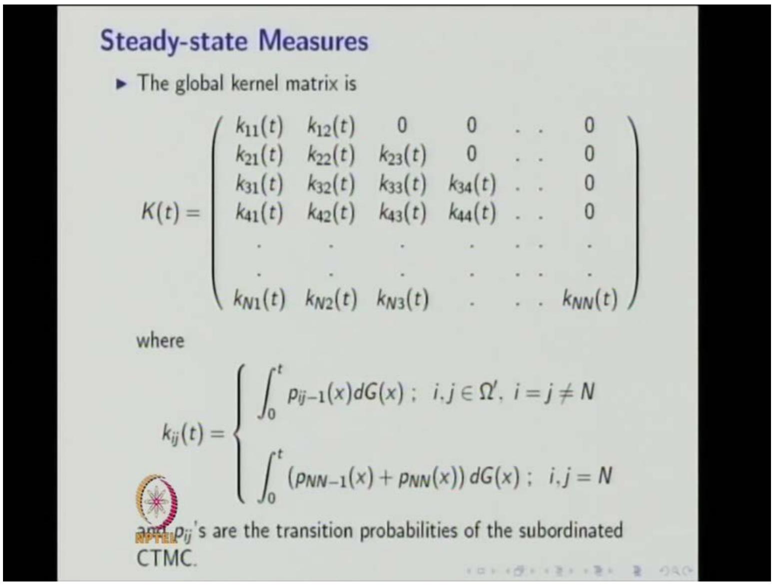

So, suppose the system start from the state n the p of n comma j of t is nothing but what is the probability that the system will be in the state j at time p given that it was in the state n at time 0. So, the previous explained the p i comma j are the transition probability of the subordinated CTMC. So, you need for all values of i and j and you have to find out the transition probability from these subordinated CTMC.

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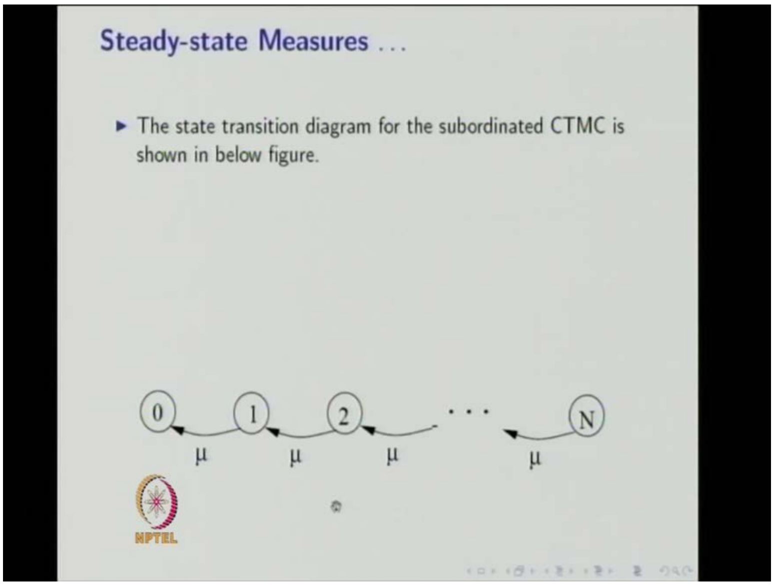

So, since the subordinated CTMC of this form you can find the you can have the close form solution for a p i comma j of t for different values of i and j.

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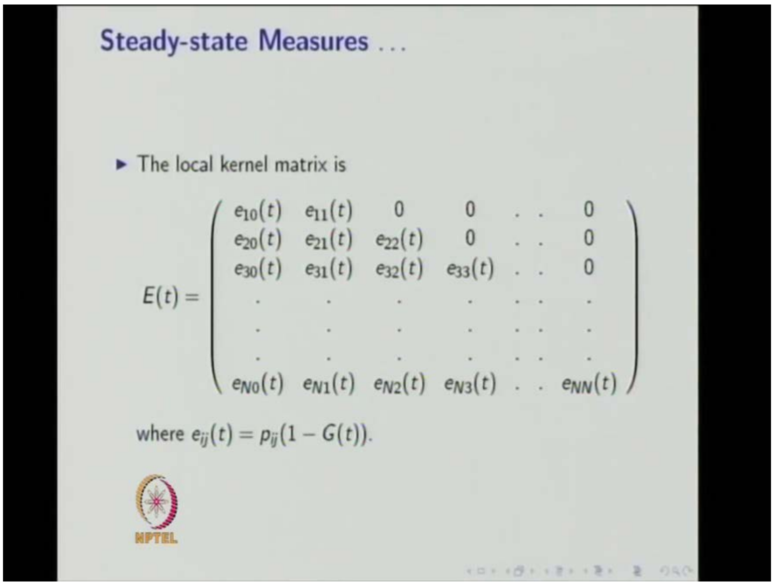

Whereas the local kernel matrix is shown like this and each entities e i j of t is nothing but what is the probability that the system goes from i to j the duration of 1 minus G of t.

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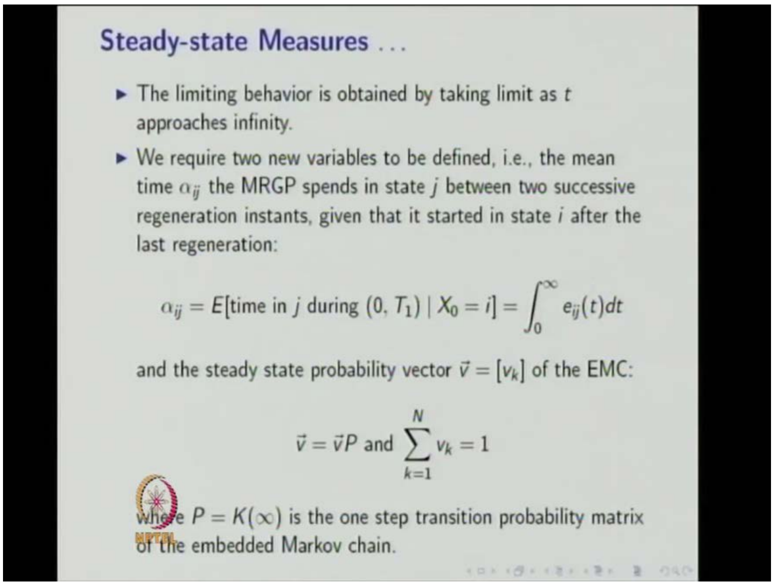

So, once we know the matrix K of t and E of t, you can find out the steady state probability vector as well as the you can find out the meantime, alpha i comma j.

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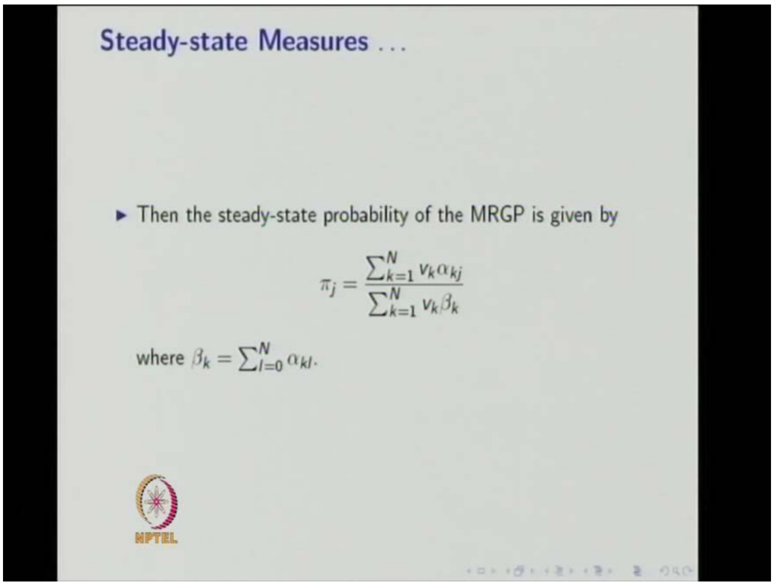

Once you know the steady sate probability vector v bar as well as the alpha j, you can use the steadies, you can find out the steady state probability using this formula that is pi j is equal to summation i summation k is equal to 1 to n V k alpha a comma j divided by summation, k is equal to 1 V k of beta k where beta k is nothing but summation over all l of alpha k comma l where l is running from 0 to capital N.

So, you get steady state probability of MRGP. First you have to find out the K of t and E of t using these two we have find out the steady probability vector V as well as alpha i j. Once you know the alpha i j as well as V using the formula you can get the steady state probability or the MRGP other state probability here. The j is running from 0 to capital N, now we are going to discuss the application of Markov regenerative process in performance analysis of wireless networks.

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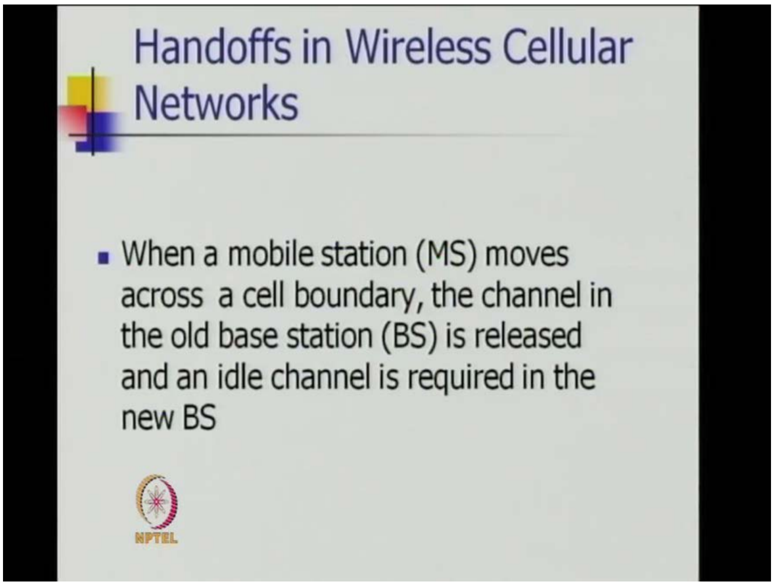

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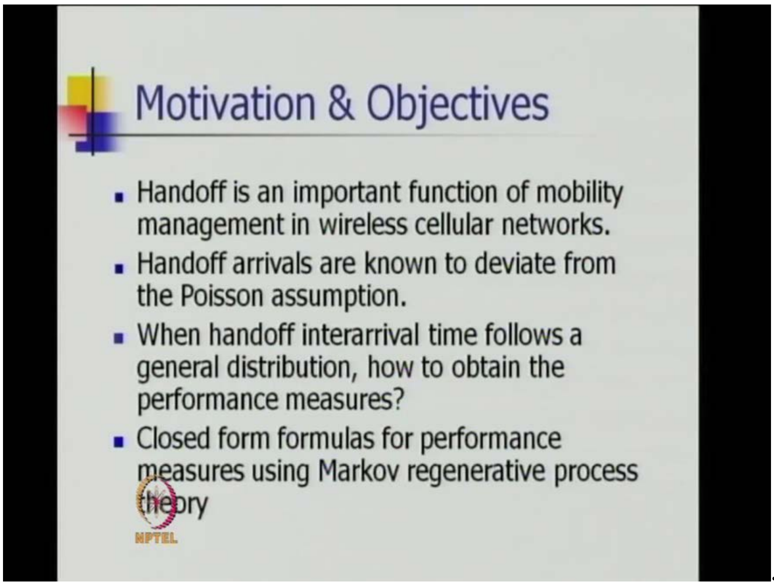

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Here we are considering the handoffs in wireless cellular networks, what is the meaning of handoffs whenever the mobile station moves across a cell boundary, the channel in the old base station is a released and idle channel is required in the new base station. This process is called the handoffs process and corresponding calls are called a handoffs calls.

Handoff is an important function of mobility management in wireless cellular networks. Handoffs arrivals are known to, known to deviate from the Poisson assumption usually the handoffs arrivals are assigned to be poison process, but usually the call arrivals are following the Poisson process. But the hand of arrivals are deviates from the Poisson assumption, when handoff inter arrival time follows a general distribution. Usually all the calls are inter arrival time follows a exponential distribution. Therefore, the call arrivals follows a poison process when the handoff inter arrival time follows a general distribution, then the question is how to obtain the performance measures?

Closed form formulas for the performance measures using Markov regenerative process theory can be applied. So, one can use a Markov regenerative process to find out the closed form formulas for the performance measures, whenever the handoff inter arrival time follows general distribution instead of exponential distribution.

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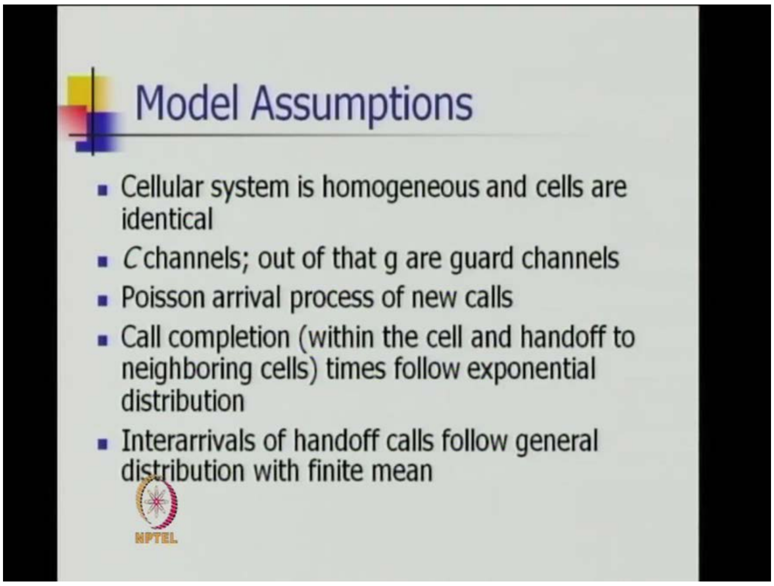

So here we make the assumptions for the cellular systems, the cellular system is homogeneous and cells are identical and assume that it has a capital C channels out of that g channels are guard channels. These guard channels are reserved for handoff calls.

Also we assume that the new calls are followed Poisson process where as a handoff calls arrival follows non Poisson. Also, we assume that call completion weather within the cell and handoff to the neighboring cells follows a exponential distribution the important assumption is a inter arrivals of handoff calls, follow general distribution with finite mean.

Therefore, we are using Markov regenerative process to find out the performance measures, if the handoff calls inter arrival also follows a exponential distribution with the proper set of one can steady the performance. Analysis of one can one can find out the performance measure using a continuous time Markov chain. Since, the inter arrival of handoff calls follow general distribution with finite mean we are going to use a Markov regenerative process.

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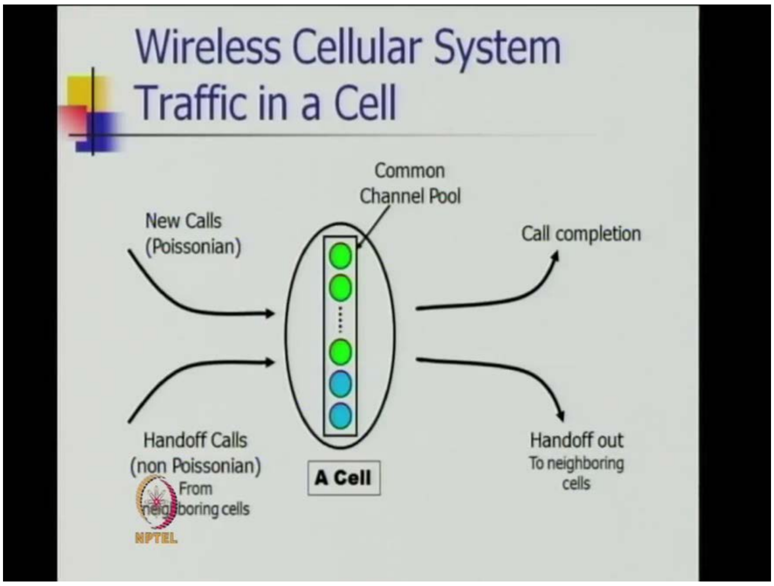

So the new calls follows the new call arrival follows Poisson process where as a the inter arrival of handoff calls follows general distribution and totally we have capital C channels out of that g channels are got channels which is reserved for.

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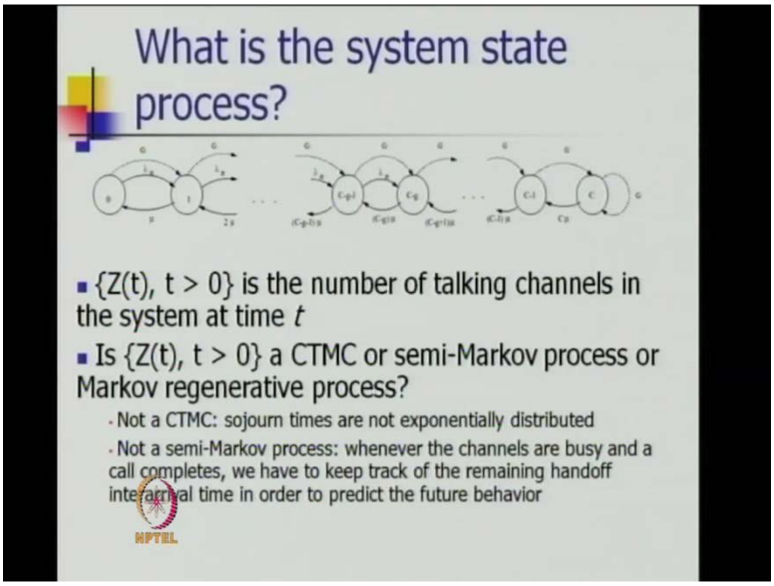

Handoff calls and call completion as well as the handoff to be the neighbor calls are exponential distribution. If you see the state transition diagram of a this process that is Z of t, that is the nothing but the number of talking channels, talking calls at any time in the system. The question is whether Z of t is a continuous time Markov chain or Z of t is a semi Markov process or it is a Markov regenerative process. The underline stochastic process Z of t. Z of t denotes the number of ongoing calls or number of talking channels in the system at time t.

It is not continuous time Markov chain because the sojourn times are not exponential distributed. It is also not a semi Markov process because whenever the channels are busy and a call completes, we have to keep track of the remaining handoff inter arrival time in order to predict the future behavior. Therefore, this is not a semi Markov process as well as this is not a continuous time Markov chain. Whereas, Z of t is a semi Markov Z of t is a Markov regenerative process. So, we are going to steady the performance measures using Markov regenerative process.

We have two performance measures, one is called call blocking probability, the other one is called call dropping probability. The call blocking probability is a related to the new calls blocked where as the dropping probability is related to the handoff calls blocked. So, the new call blocking probability is nothing but the percentage of new calls are rejected handoff calls. Dropping probability is nothing but the percentage of calls forcefully terminated while crossing cells in the cellular systems, using the same theory of Markov regenerative process.

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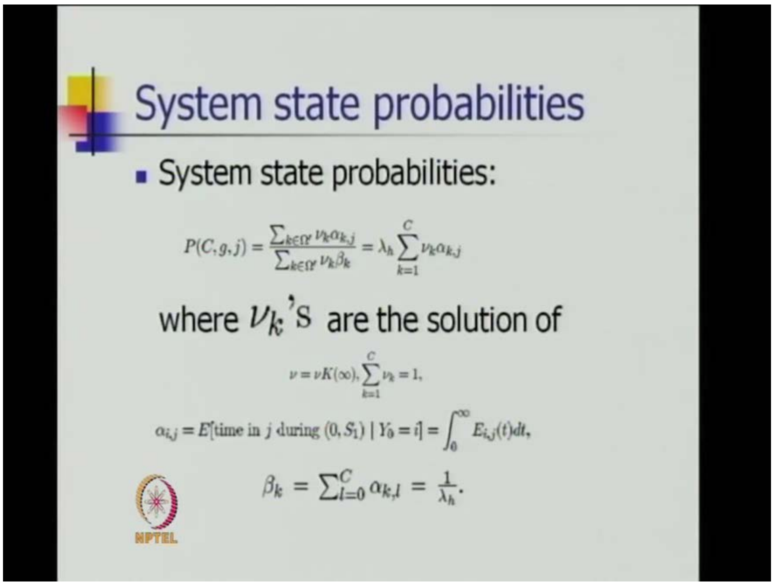

First you create the global kernel, then create the local kernel, then obtain the steady state probability vector V k's also find out the mean time alpha i comma j, using that we can find out steady state probabilities.

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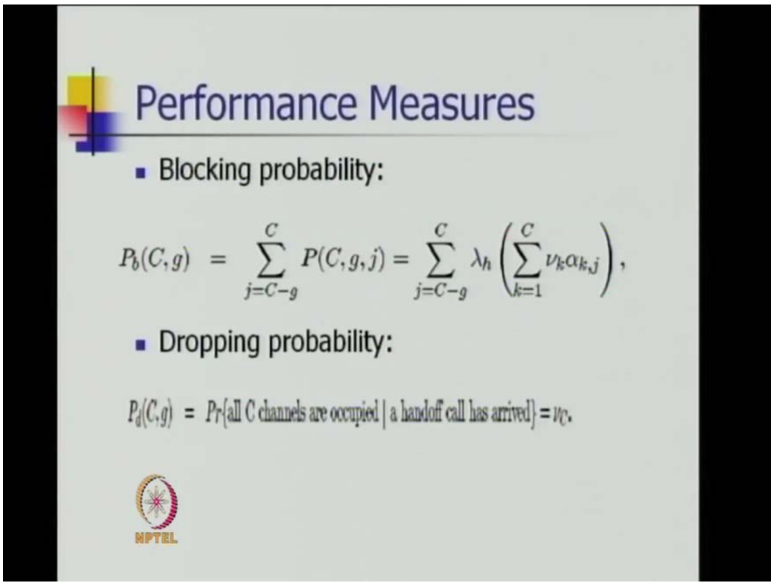

Here the lambda h is the rate of inter arrival time of involves of handoff calls. Once you know the steady state probabilities, you can find out the blocking probability as well as the dropping probability. The blocking probability is nothing but the summation of the system state from C minus g to capital C. Whereas, the dropping probabilities corresponding to the handoff calls that is nothing but the probability that all C channels are occupied and the given that the handoff calls has arrived that is nothing but the steady state probabilities of V suffix C.

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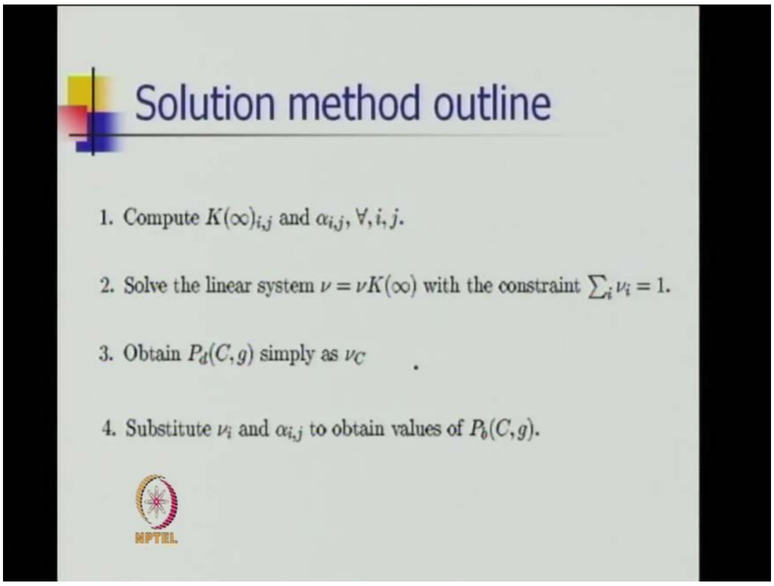

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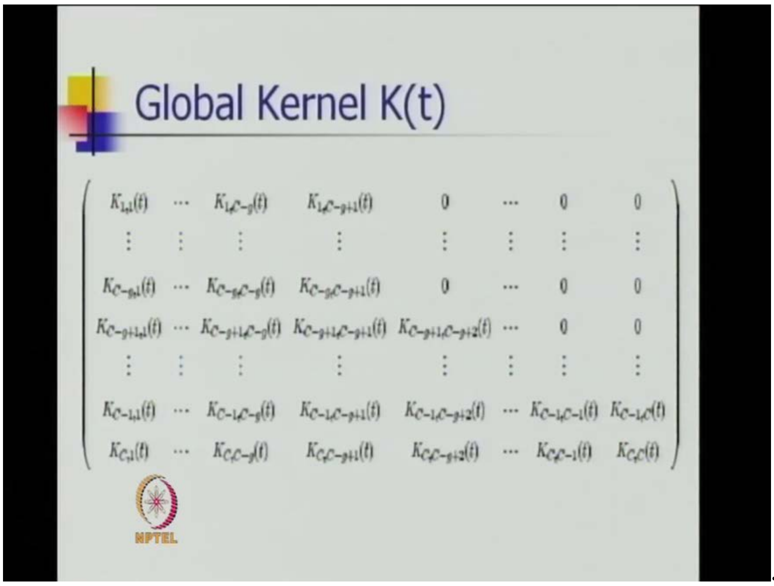

So, the solution method is compute k of infinity from the K of t by making limited t tense to infinity. So, all the linear system V is equal to V P where P is K of infinity. Then first you find out the steady state probabilities, then obtain the performance measures of a blocking probability as well as the dropping probability.

So, here the global kernel k of t will be of this form. Similarly, the local kernel E of t will be of this form with the rows C rows and C plus one columns because here the omega dash is the 1, 2, 3 and so on, t capital C where omega is 0, 1, 2 and so on till C.

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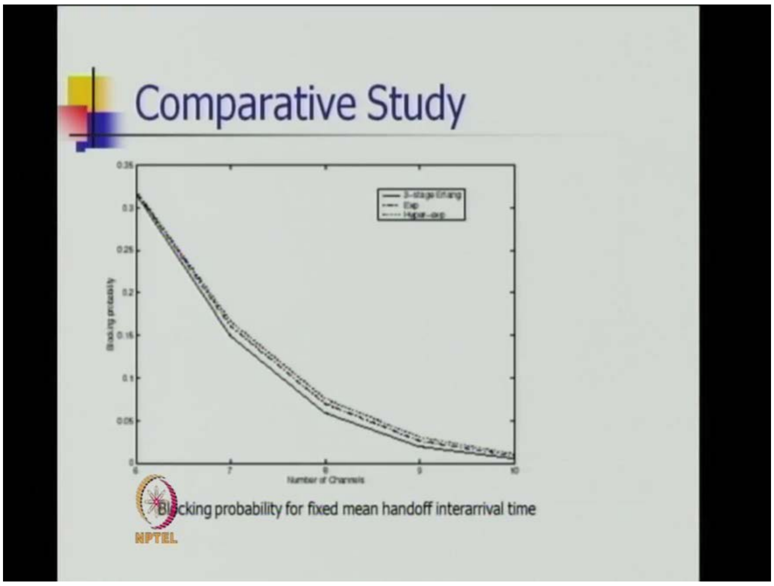

Using the proper numerical values for the inter arrival time, you can find out the blocking probabilities versus the number of channels. If you increase the number of channels capital C, then you can find out the blocking probabilities for various values of C and as number of channels C increases the blocking probability decreases. We can verify with the different distribution, you can choose three-stage airline distribution or hyper exponential distribution as well as the we can choose the exponential distribution. In underline model will be a continuous time Markov chain and you can compare the blocking probability results for the various distribution for inter arrival time of handoff calls, by fixing fixed mean handoff inter arrival time.

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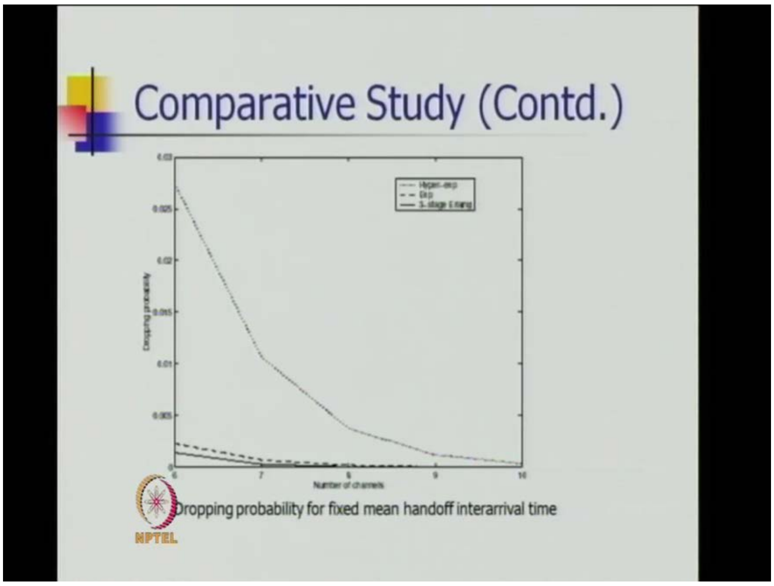

Similarly, you can numerically in the state the dropping probabilities versus number of channels are the different distributions for handoff inter arrival times with the fixed mean handoff inter arrival time.

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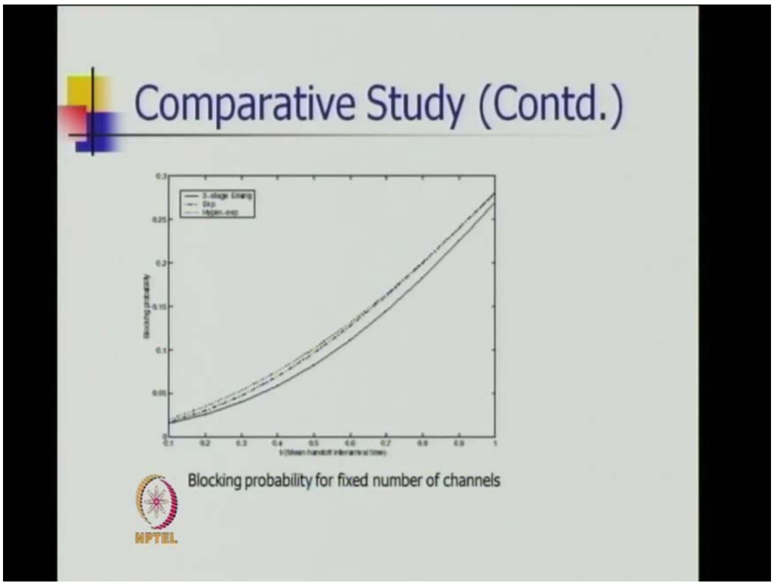

We can, we can illustrate the blocking probabilities versus the 1 divided by mean handoff inter arrival time. Also that means this the rate, so as the rate increases the blocking probabilities increases. The rate of handoff inter arrival increases the blocking probability increases for the various distributions.

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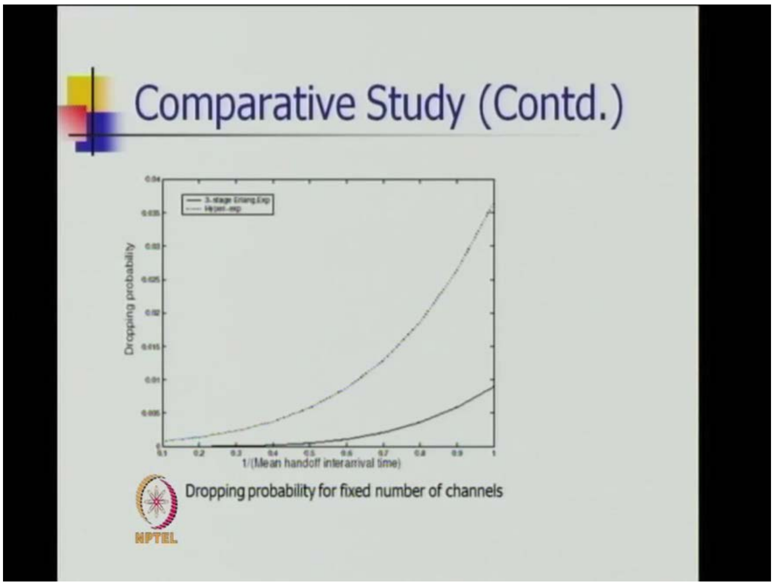

Similarly, as the rate increases because the X axis 1 divided by the mean handoff inter arrival time. Therefore, 1 divided by mean time is nothing but the rate. So, has the rate increases the dropping probability increases for the various distribution also. In this lecture, we have covered as a application of Markov regenerative process with the G M 1 N queue as well as the application of Markov regenerative process in performance analysis of wireless networks.

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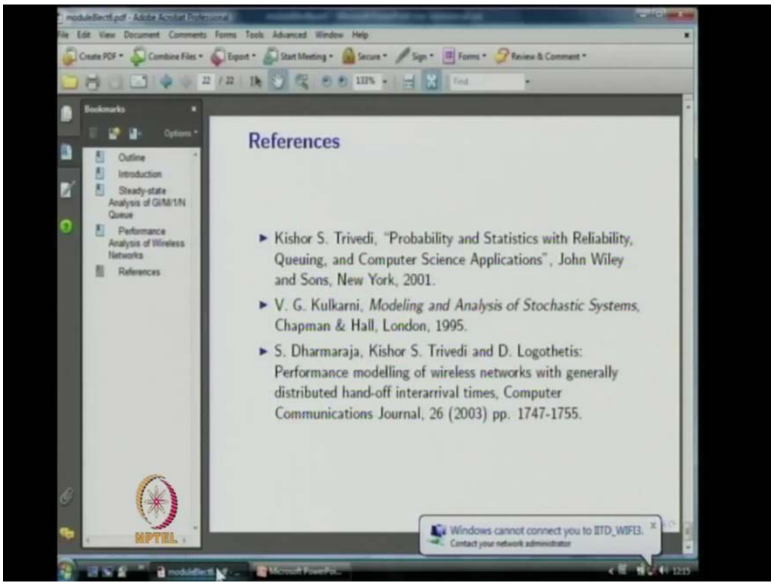

Here are the important references for today’s lecture.

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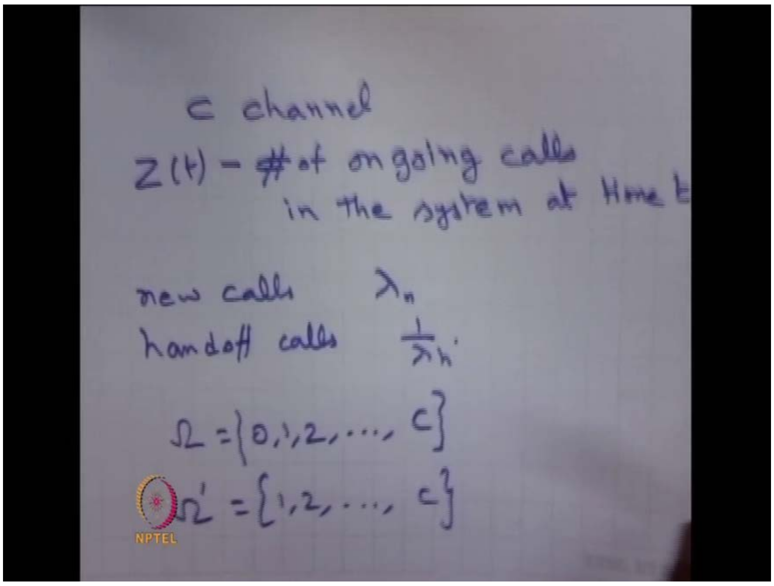

So, consider a self which he has the C channels let Z of t is nothing but number of ongoing calls in the system. Here the system is nothing but a self at time t, we have made the assumption. The new calls arrival follows poison process with parameter lambda n. Whereas, handoff calls, whereas the handoff calls arrival follows non exponential inter arrival, non exponential distribution or general distribution with the finite mean 1 divided by lambda h. Omega is the collection of possible states whereas, omega dash will form a regeneration time points. The states corresponding to the regenerative time points that is going to be 1 to C our interest is to steady the limiting behavior of the limiting behavior.

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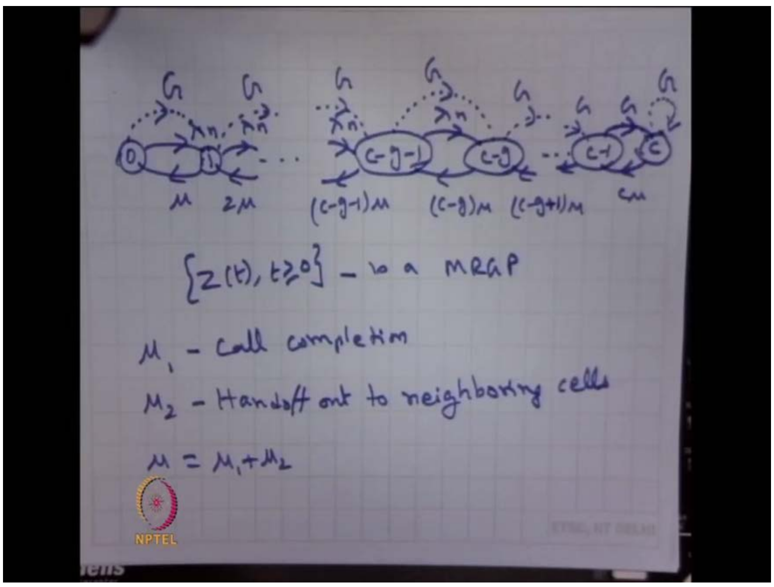

So, the state transition diagram for this module is going to be 1 C minus 1 and C. So, this are all the possible states. Suppose, the new call arrays, then the system size is one, suppose one more new call comes the system size goes to 2 and so on. So, like that the system I can be incremented by one by one. Similarly, when there are C minus g minus 1 channels are already ongoing. Then if one more new call comes, then the system goes to the size C minus g C plus g, but C is the handoff calls are having the higher priority already small g channels are reserved for handoff calls, whenever remaining g channels are available in the system.

Then those channels are reserved for the handoff calls. Therefore, no more handoff arrivals whenever the systems size is goes from C minus g onwards till C. Similarly, if the system size is 0, there is a possibility of handoff call arrival. So, that I make it as a dot at the way with the distribution capital G; whenever the system size is 1, there is the possibility of handoff. Therefore, the G comes here. Similarly, the handoff arrival can occur to lead the system sizes is g C minus G minus 1. Similarly, whenever the system size is C minus G minus 1, there is the possibility of handoff arrival capital G.

But whenever the system sizes is C minus G that means at this time C minus G calls of ongoing that includes a handoff calls as well as the new calls. If new calls arrives at this time, it will be blocked whereas if handoff calls enter into the system it will be (( )). Therefore, you have a arrow doted arrow throughout from 0 to C, because handoff calls are allowed as long as at least one channels is available in the system. Whereas the new call are allowed to enter into the system whenever more than g channels are available in the system. Therefore, the lambda n is exist from 0 to 1 and 1 2 and so on till C minus G after that there is no arc up with the lambda n.

Now, we discuss the departure part whenever the call can be completed within the cell or the call can handoff to the neighboring cell. Suppose, we make the assumption mu one is the great corresponding to the call completion time and mu two is the rate for a exponential distribution for the handoff to the neighbor cell handoff out to neighboring cells. The assumption that both are independent, then you can denote mu as a mu 1 plus mu 2. Now, suppose the systems is in the state one either 1 mu arrival can occur or one handoff arrival can occur or this call can be completed within the cell or this call can be handoff to the neighbor cells.

Since, both are independent, each one is exponential distribution with the parameter mu 1 and mu 2 respectively. Therefore, the system size both 1 to 0 is mu the system goes from the state 1 2 0 will be with the rate mu because mu is mu 1 plus mu 2. Mu 1 is nothing but and rate of call completion and which time is the exponential distribution. Similarly, the hand out to neighboring cell that is follows a exponential distribution, the rate mu 2 where mu is mu 1 plus mu 2. Similarly, the system comes from the state 2 to 1 will be 2 mu because of there are 2 calls are ongoing. Any one of the calls ongoing completed are hand out to the neighbor cells. Therefore, the system goes from the state 2 to 1 therefore, the rate will be 2 times mu.

Similarly, the system goes from C minus G minus 1 to C minus G minus 2 will be C minus G minus 1 times. Similarly, you can fill up this rate will be C minus G times mu. This will be C minus G plus 1 times and so on. The last one will be C times mu and the system moves from the state C minus 1 to C that will be, sorry that doted arc that is G if that doted arc, whenever the system size will be the state C there is a possibilities of still the hand of arrival can come. But it will draft therefore, self loop the doted arc the again the systems has it is be C.

So, this is the state transition diagram of the Z of t stochastic process. Z of t is nothing but number of ongoing calls in the system at time t. The possible states are 0, 1, 2, and so on till C. The doted arc are related to the hand off arrivals, whereas the other two arcs are corresponding to the new call arrival and call completion. So, the hand off calls are possible and enter into the system, whenever the system size is 0 to C minus g minus 1 whenever the system size is from C minus C to C minus G to C.

New calls are blocked and whenever the system size is C that handoff calls of dot. Now, the Z of t this stochastic process is discrete state continuous transition stochastic process. Now, we are going to analyst weather the Z of t is a continuous time Markov chain or is a semi Markov process or is a Markov regenerative process. First the new call inter arrival times are exponential distribution, which is independent of a handoff inter arrival time. Also it is independent of call completion, the call completion times are exponential distribution. All are independent also where as handoff call inter arrival times are general distribution the finite mu.

Therefore, the Z on times in each state of this stochastic process Z of t this not a exponential distribution because of handoff inter arrival times are general distribution. It is a non exponential general distribution. Therefore, Z of t of t greater than or equal to 0 is not a continuous time Markov chain because of not exponential distribution. Since, Markov regenerative processes Markov process, Markov properties is not satisfied at every time instant the system is entering into the all the states the Z of t is not a semi Markov process. Here the Markov regenerative property is satisfied, whenever the time instant at which they arrive arrival of handoff calls occurs not for the instant of new call arrival occurs or the call completion occurs.

The time instant in which the system moves to the different states because of new call arrival or call completion inside, it is a call completion within the cell handoff out to neighbor cells. But only the time instant at which the handoff arrival occurs, those time points the Markov property or memory less property is satisfied in this stochastic process. Therefore the Z of t is not a semi Markov process, but if you see the Markov regenerative process theory the probabilistic of exist, whenever the handoff arrival occurs. Therefore, the state 0 is not corresponding to the regenerative state. Whereas, the state 1 2 and so on till capital C are the regeneration states corresponding to the regeneration time point with respect to the handoff arrivals. So, the conclusion is Z of t is a Markov regenerative process, which is not a continuous time Markov chain.

It is not also the semi Markov process, if we assume that the handoff inter arrival times or exponential distribution, which is independent of in call arrival as well as independent of call completion. Then, the Z of t will be a continuous chain Markov chain. So, over interest which to steady the new call blocking probability because in these state the new states are blocked. Whenever the system is in the state as C minus G C minus G plus 1 and so on till C, the new calls are blocked. Whereas the handoff calls you can see the doted arc till C. Therefore, handoff calls are dropped whenever the systems in the state C. Therefore, our interest here the (( )) or the new call blocking probability as well as the handoff call blocking probabilities.

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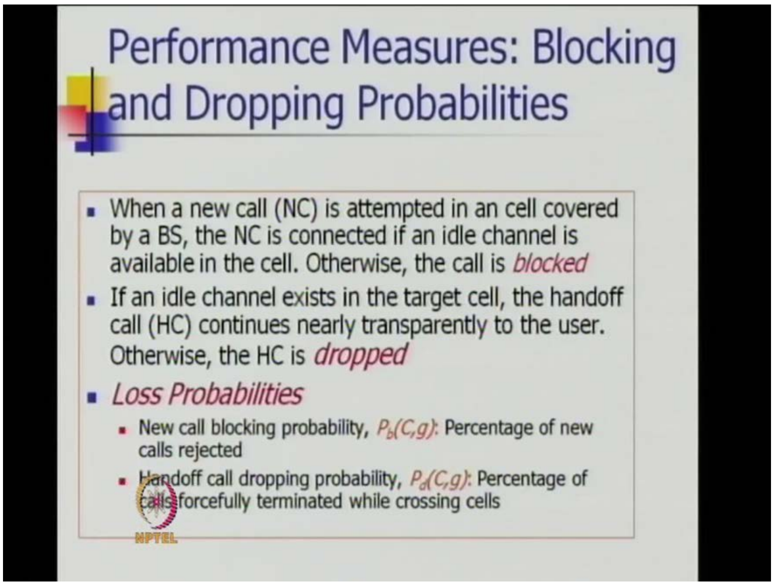

So, whenever a new call is attempted in his cell covered by a base station, the new calls are connected if in idle channel is available in the cell. Otherwise the call is blocked, the new call is blocked. If idle channel is not available in the channel because of call channel policy, whenever more than or equal to more than more than g channels are available as a idle channels. Then the new calls are accepted whereas the handoff calls continuously whenever, sorry if idle channel exist in the target cell, the handoff call continues nearly transparently to the user, otherwise the handoff is dropped.

So, here whenever the systems states in the state in C the handoff calls of dot. So, our interest is to find out the last probabilities, such as a new call blocking probability and handoff call dropping probability. The new call blocking probability is denoted by the letter p suffix p and it is a function of C comma G. That means the total number of channels as well as the total number of guard channels by fixing the total number of channels as well as the total number of guard channels, one can find the new call blocking probability for a cell. That is nothing but the percentage of new calls are reject.

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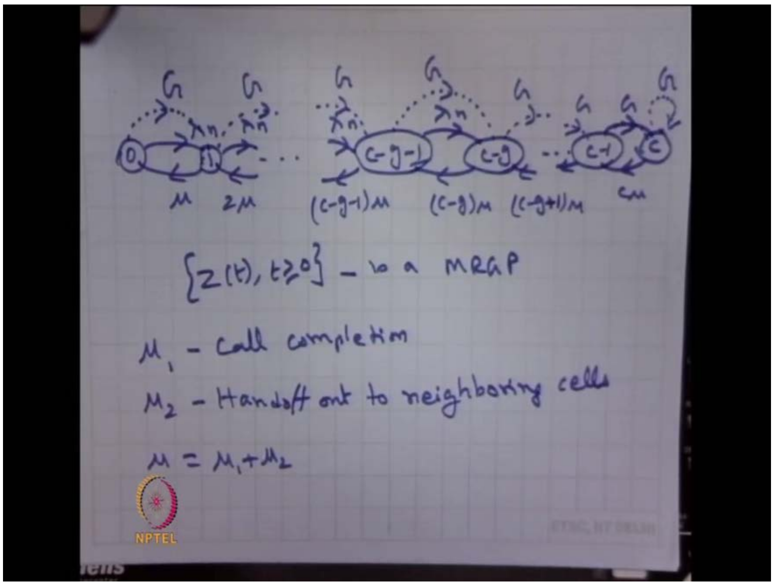

So, here is nothing but what is the probability that in a long run, the system is in the state 0 to in the state c minus g to c; that is the, that is the probability, sorry so the percentage of new calls are rejected will be the new call blocking probability.

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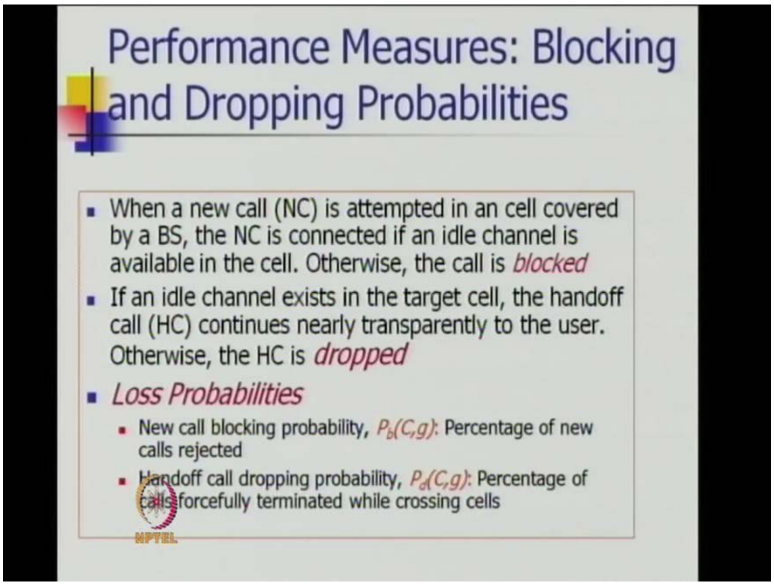

Similarly, the handoff call dropping probability in a cell, that is denoted by P suffix d. It is a function of C with the G, that is nothing but the percentage of calls forcefully terminated while crossing cells. So, here we are finding the two last probabilities, one is new call blocking probabilities and other one is handoff call dropping probability. In both are steady state probability measures. In this course, we are discussed the performance analysis of wireless networks has an application of stochastic processes by keeping various stochastic modeling techniques in the mind. Over this is the brief insight into the application of the techniques the real world problems.

Thanks.