Stochastic Processes
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Module - 7
Brownian Motion and its Applications
Lecture - 4
Ito Integrals

This is a stochastic processes module 7 Brownian motion and its applications lecture 4 Ito calculus, Ito integrals. In the last three lectures, we have discussed the definition and properties of a Brownian motion in the first lecture. In the second lecture, we have discussed geometric Brownian motion and the other process derived from the Brownian motion and their properties also. In the third lecture, we have discussed stochastic differential equations. In this lecture, we are going to discuss the Ito integrals.

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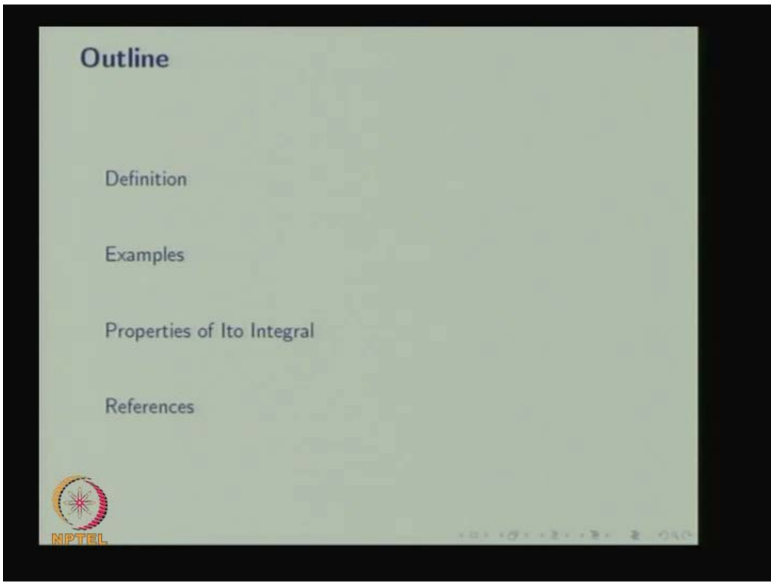

First we are going to start with the definition of Ito integral, when we say the stochastic process is Ito intergrable, then we are going to discuss what is Ito process? Then followed by these definitions, we are going to discuss few examples followed by the examples, we are going to discuss the properties of Ito integrals, so with that, this lecture will be completed.

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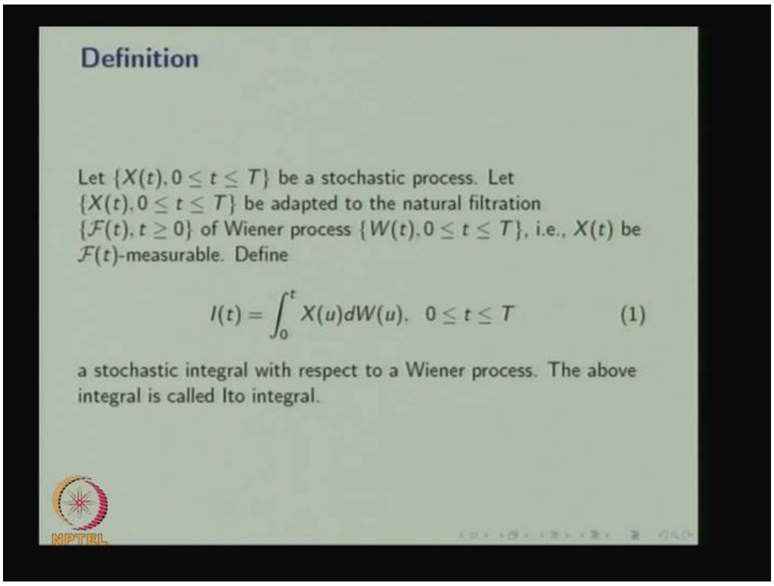

This is the definition of Ito integral. Let X(t) be a stochastic process, which is adapted to the natural filtration F(t) of Weiner process W(t); that is X(t) be a F(t) measurable. Define I(t)? I(t) that is nothing but integration between the limit 0 to t X(u) integration with respect to W(u), where t is lies between 0 to T, where T is a positive constant. I(t) is a stochastic integral with respect to a wiener process, the above integral is called the Ito integral. A detailed interpretation and the motivation of this can be found in the reference books.

So, in the Ito integral I(t) is defined in the form of integration 0 to t and the integrant with respect to the integral with respect to the wiener process W(t). The wiener process already discussed the wiener process or Brownian motion is discussed in lecture 1.You can find in lecture 1. What are all the properties of wiener process and so on? So, the wiener process W(t) is starting with the standard one W(0) is equal to 0 and it has a increment or stationary as well as independent.

And the increments are normally distributed with the mean 0 on the standard various processes and variance is a increment the difference for s is less than t it is the variances t minus s. So, here we have a stochastic process X(t) which is defined between the intervals 0 to T. The same stochastic process is a adapted to the natural filtration F(t), already we know that the Wiener process is adapted to the W(t) and here X is also adapted to the W(t) that means a for fixed t X(t) is a F(t) that means in the sigma field generated by the X(t) for fixed t we will be contained in F(t) that means the information accumulated at time t.

That is sufficient to point out the value of X(t) for fixed t. So, the X(t) is F(t) measurable for all t between the interval 0 to T then we can define this integral will be call it has an Ito integral. So, the X(t) has to be F(t) measurable and F(t) is a natural filtration or the Wiener process W(t) then the for all values of t between the interval 0 to T. I(t) is the called stochastic integral because this is not a usual integral this is a stochastic integral with respect to the wiener process W(t) is a definition of Ito integral.

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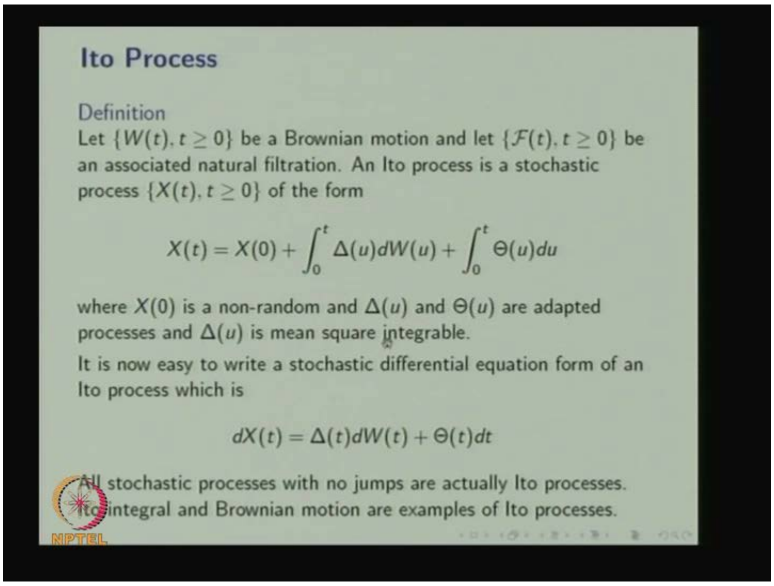

Now we are going to discuss the Ito process. Let W(t) be the Brownian motion and F(t) be the associated natural filtration a inter process is a stochastic processes X(t) is of the form X(t) is equal to X(0) plus 0 to t integration delta u dW(u) plus integration 0 to t theta u du. So, here we have two types of integration the 1 type is usual Riemann integration the other one is the Ito integral where X(0) is a non-random and delta of u as well as theta of u are adapted process and delta of u is the mean square integral.

If this condition is satisfied then we can say this X(t) is going to be a Ito process for all t we are able to write X(t) is of the form the one Ito integral and Riemann integral and more the integral these are all the adapted process that means X(u) is F(u) measurable as well as theta of u is F(u) measurable as well as delta of u is a mean square integral.

That means a integration 0 to t expectation of delta square of u t u is a finite then we say it is mean square integrable if these three conditions are satisfied by the X naught delta of u theta u and we can say the X(t) is going to be a Ito process. Once it is a Ito process we can write down in the differential form that is a stochastic differential equation because we have dW(u) in the differential form it will be because X(t) is equal to X(0) plus integration.

Therefore in a stochastic definition form it is a dX(t) is equal to delta t times dW(t) plus theta of t times dt. So, this is the stochastic differential form of Ito process that means whenever delta of t as well as theta of t both are adapted process. And X(0) is a non random as well as delta of t is mean square integrable then the stochastic process written in the stochastic definition form dX(t) is equal to delta of t dW(t) plus theta of t dt we may call it has a Ito process.

So, the Ito process is nothing, but a stochastic process is of this form as well as satisfying this conditions the way you use we write the stochastic differential equation, dX(t) is equal to delta of t times dW(t) plus theta of t times dt. Therefore is it the sample path this is going to be a continuous function therefore, all the stochastic process with no jumps are actually a Ito process either and we see the Ito process we would not find the jumps.

A jump Ito process is nothing, but Ito process in which the moments are discrete rather than continuous so, all the stochastic process with new jumps are actually a Ito process. Because it has the term of dt as well as dWt for the increment of dX(t) for all t.Ito integrals as well as the Brownian motion are the examples of Ito process the way the previously written I(t) has a Brownian motion both are called the Ito.

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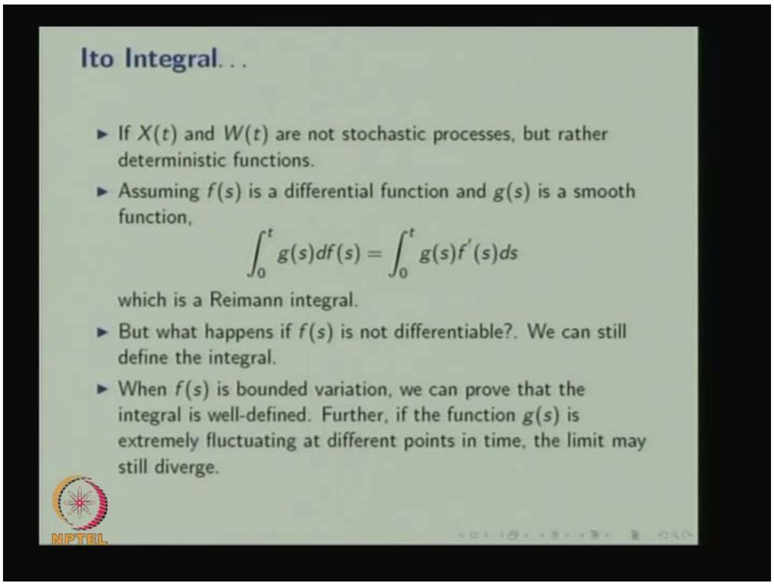

Now we are going into discuss the Ito integrals. If X(t) and W(t) are non stochastic process, but rather deterministic functions then the situation is different assuming f(s) that means we discuss the deterministic situation we written integration zero to t, g(s) df(s) where f(s) is a differential function and the g(s) is a smooth function. Since, it is a differential function you can write df(s) is the f dash of s ds which has nothing but the Riemann integral.

Whenever the X(t) and W(t) are non stochastic processes rather the deterministic functions then write down the integration has a g(s) df(s) that is nothing but Riemann integral, but in our situation the f(s) is not a differentiable we can still define the integral remember that the W(t), which is a lower differentiable as well as unbounded variation. So, here we are going to discuss when f(s) is a non differential as well as when f is bounded variation then the still integral is well defined further if the function g(s) is extremely fluctuating at different points in time the limits may still diverge. When f(s) is a bounded variation you can prove that the integrable is well defined further if the function g(s) is a extremely fluctuating at different points in time the limit may still diverge as s tends to infinity.

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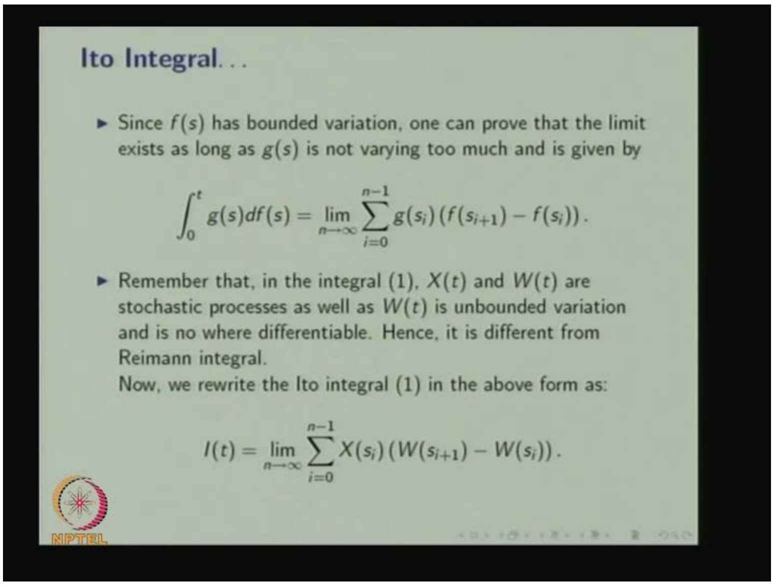

Since f(s) is a bounded variation, one can prove that limit exists as long as g(s) is not varying too much and it is given by integration g(s) df(s) is nothing but limit n tends to infinity summation of i is equal to 0 to n minus 1 g(s i) integration with the difference f(s i) -plus 1 minus f(s i) that means since the integration is with respect to df(s). So, you can find the difference f(s) plus i minus i s i and manipulate with the g(s i) and the that means you have a partition in the interval 0 to t into n parts and has a n tends to infinity that summation limit n tends to infinity will be the integration. Since f(s) is a bounded variation.

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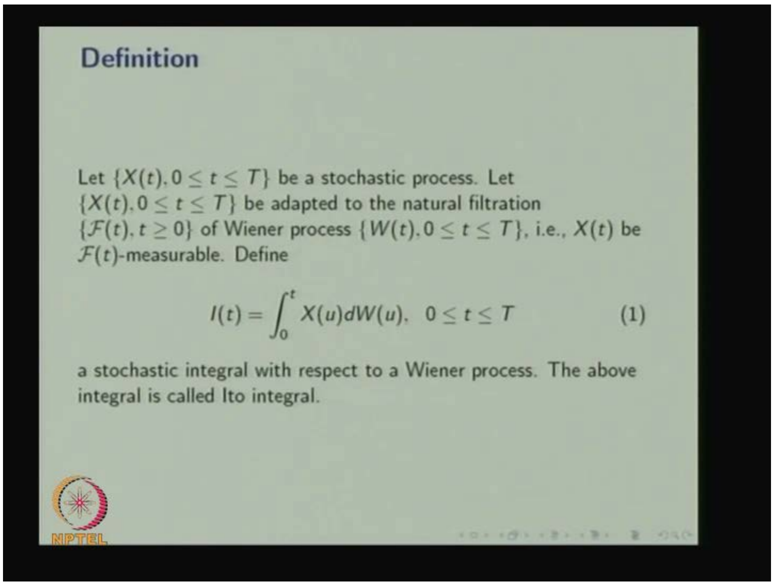

Then get the you can prove that the limit x tends to remember that in the integral 1 that is the is the integration the Ito integral. I(t) X(u) is a random variables for fixed u, W is a random variable for fixed u.

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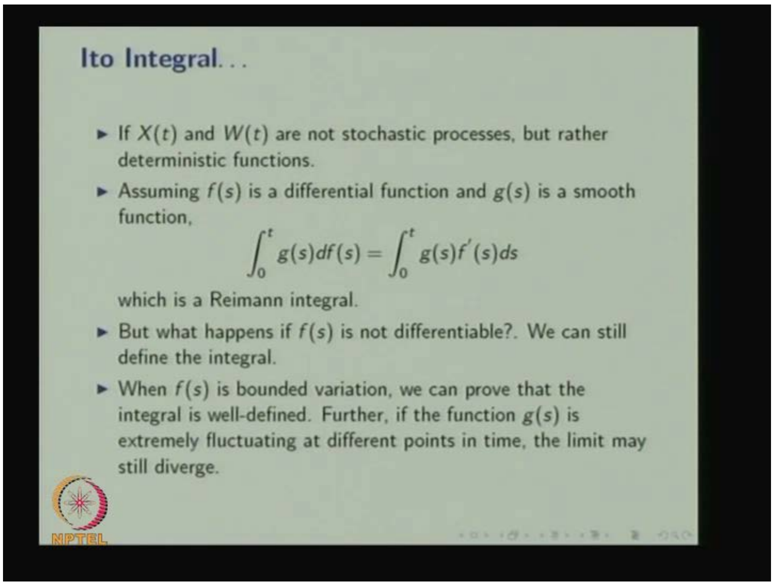

And Wu is the lower differentiable as well as inbounded variation. Now we have discussed what the situation is if what is the situation f(s) is a bounded variation the integration with respect to f(s) where f(s) is bounded variation.

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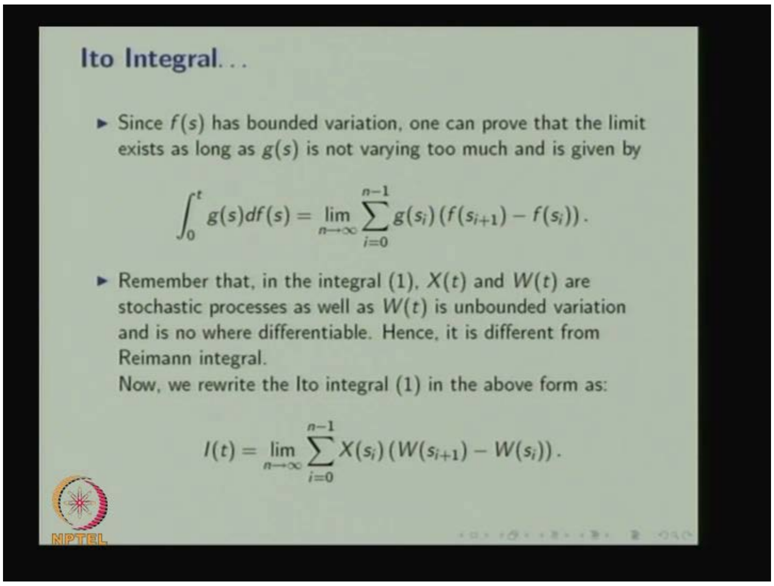

Where has our Ito integral X(t) and W(t) are stochastic processes as well as W(t) is a unbounded variation and no where differentiable. So, that is the difference between the Riemann integral and the integral which we have discussing now that is Ito integral. The integrand is X(t) the integration with respect to the W(t) where W(t) is bounded variation as well as nowhere differentiable where as the Riemann integral which we have discussed the f(s) is a differentiable as well as later we discuss f(s) is a bounded variation.

Where has the Ito integral W(t) is a unbounded variation as well as W(t) is a no where differentiable. Hence it is different from the Riemann integral. So, in the Riemann integral the integration with respect to the function which is a bounded variation and it is a differentiable where has here the Ito integral it is unbounded variation as well as nowhere differentiable.

Now, we can rewrite the Ito integral 1 in the above form in the following form. I(t) that is nothing, but the way we have written in the above form Riemann integral integration with respect 0 to t g(s) with respect to f(s) is limit n tends to infinity the same way we can write I(t) is limit n tends to infinity summation i is equal to 0 to n minus 1 because the integration is 0 to t X(u) dW(u).Therefore it is X(s i) and the W difference that is the W(si) plus 1 that means si the way of written the Riemann integral in the same way we writing the Ito integral where X(t) is a adapted process and W(t) is the stochastic W(t) is a Wiener process or Brownian motion.

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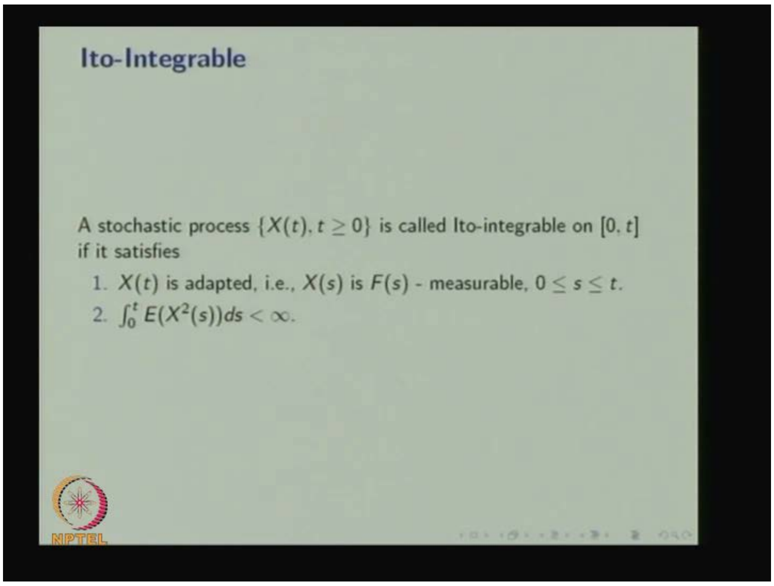

And the X(t) is adapted to the F(t) and the Ito integral I(t) we can write it has a limit n tends to infinity the summation. Here since the expression on the right hand side the some of n random variables. Therefore the limit here refers to one of the mode of the convergence of sequence of random variables that is limit in the sense of probability in distribution almost surely r in r eth mean then we can say the Ito-integral. Whenever we have a stochastic process is Ito integrable which has to be adapted that is a measurable as well as it should be a mean square integral. If these 2 conditions are satisfied then we can say the stochastic processes Ito integrable.

And the Ito integral which we have discussed in the positive line I(t) is equal to integration 0 to t X(u) dW(t). So, a stochastic process is called the Ito-integrable on the interval 0 to t. If it satisfies the first condition which is adapted that means X(s) is a F(s) measurable and it is mean square integral that is integration 0 to t expectation of X square of s ds must be a finite.

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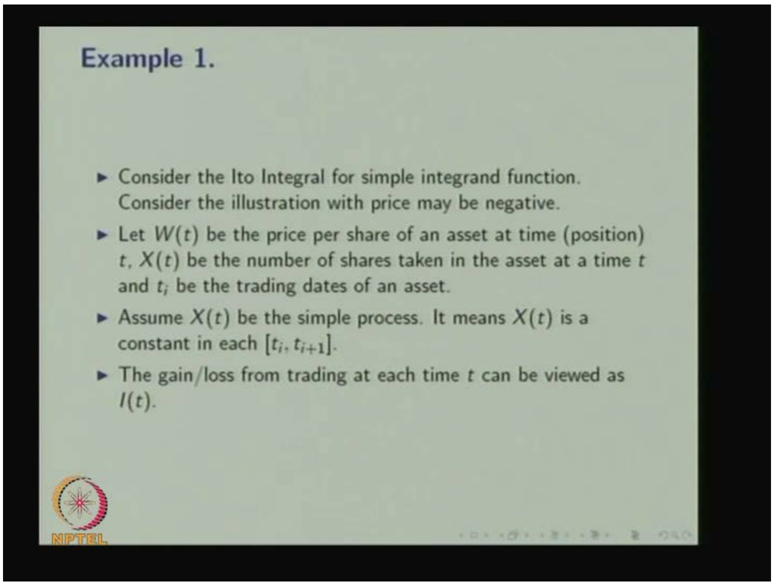

Now we are going to discuss the few examples from that we are going to study the Ito integral. First we can consider the simple situation considers the Ito integral and simple integrand function that means for example, you can see the illustration with the price may be negative. Let W(t) be the price per share of an asset at time t and X(t) be the number of shares taken in the assert at a time t and ti be the trading dates of an asset.

Assume that X(t) be a simple process that means it takes a constant value in the interval ti to ti plus 1.The gain or loss that is possibility the price may go down and so on you may be negative also moving that the assumptions gain or loss from training at each time can be viewed as I(t), because I(t) is nothing but the integration 0 to t X(t) dW(t).

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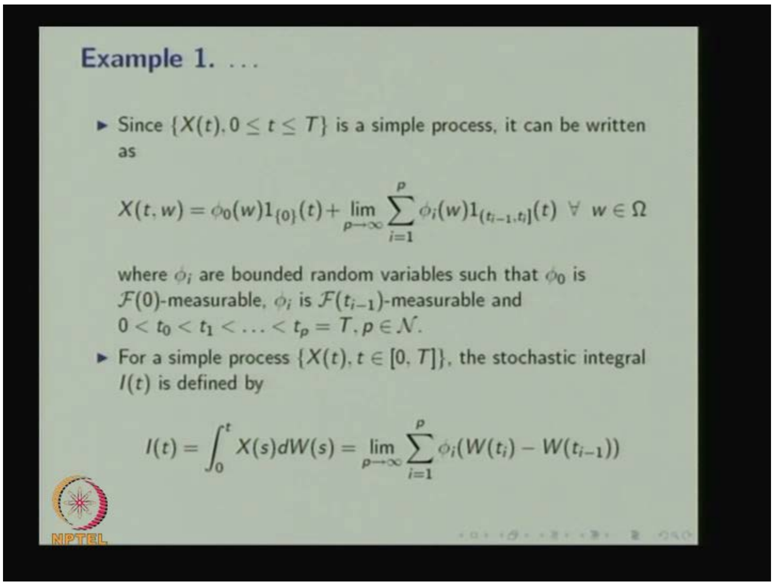

Therefore the gain or loss from trading at each time can be viewed as the I(t) so, since it is a simple process you can write it X(t), W where W is belonging to omega you can write down in the simple process in this fashion. Where psi i be the bounded random variables such that psi naught is f naught measurable and psi s are f ti minus one measurable because it is a taking a constant taking a value between the interval t, t is a constant value between the interval ti to ti plus one.

Therefore you can write down the simple process X(t) in this form. So, for a simple process X(t) is a stochastic integral I(t) that is nothing, but the gain or loss that I(t) is nothing, but the integration 0 to t X(s) dW(s). That is nothing, but the limit t tends to infinity of i is equal to 1 to t psi i multiplied by the difference of W(s) in this limit the term the first term of the above equation that is psi naught pi naught y indicator function of t does not contribute anything. Hence W naught is equal to 0.

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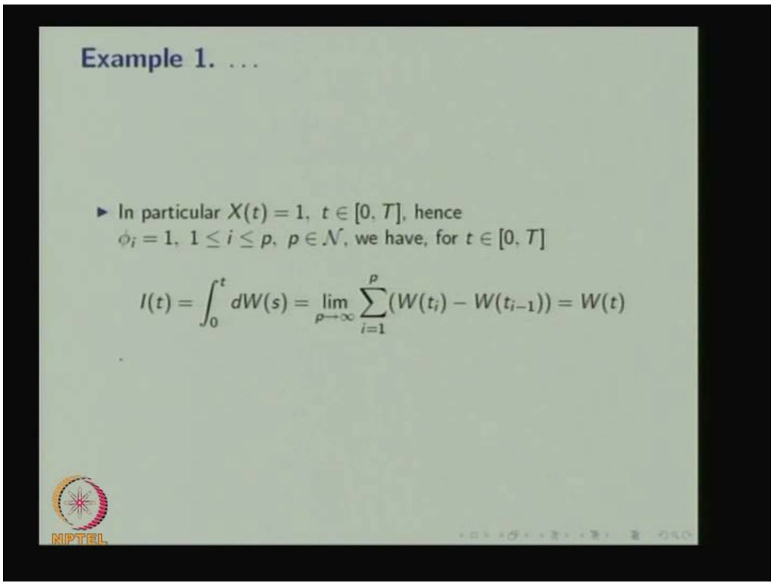

In particular if you take X(t) is equal to one for all interval then the psi i s is equal to 1 for all the interval therefore, the I(t) nothing but since X(t) is equal to 1 you are just integrating 1 with respect to the W(s) where you will get W(t) we calculated Ito integral for a simple integrand has a general integrand can be a expressed has a limit of simple integrands. Therefore the Ito integral of general integrands can be obtain by taking a limit of sequence of Ito integral of the simple integrands this is the simple situation in which the integrand is a simple function.

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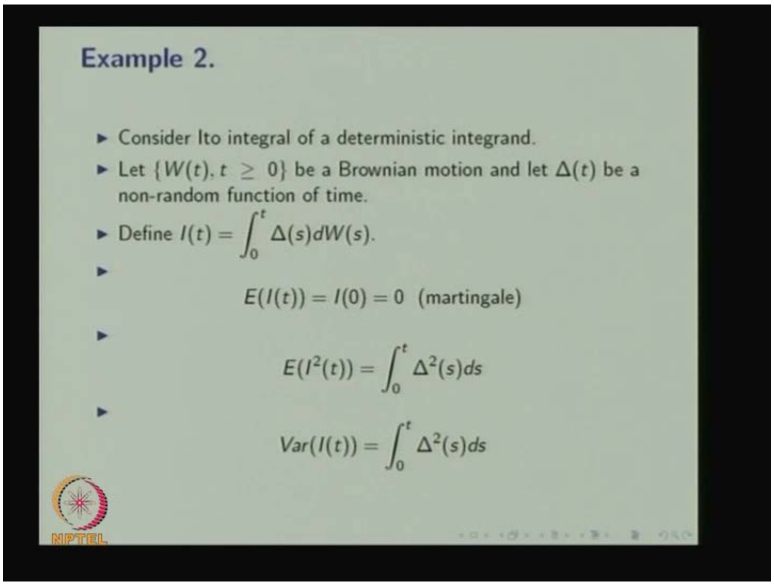

Suppose the integrand is deterministic function at a random the delta t is not a random now we are defining the Ito integral I(t) that is 0 to t delta of s dw(s) since expectation of I(t) is nothing, but I(0) that is equal to 0. Because the Ito integrals are satisfied is the martingale property this we are going to discuss at end of this lecture the Ito integral is going to be a martingale so, this property is used to here.

Otherwise you can directly find out expectation of I(t) that you land up 0 or by using martingale property it will be I of zero, but I(0) is equal to you can find out the variance of I(t) also. For that we have to find expectation of I square of t that is nothing, but expectation of 0 to t delta u of dW(s) then you can find out the expectation of I square t so, that land up delta square of s of ds.

So, using these we can find out the variance. Variance is equal to expectation of I square t minus expectation I(t) whole square since expectation of I(t) is equal to 0 therefore, variance of I(t) is same as expectation of I square of t that is zero to t delta square of s ds or you can use the isometric property for finding the expectation of I square that is same as 0 to t expectation of delta square of s ds. Since delta t is a non random function the expectation of delta square s is same as delta square x. So, one can use the definition and find out this expectation or you can find out we can use the isometric property and use the expectation of a constant is same as a constant therefore, 0 to t delta square of s dX.

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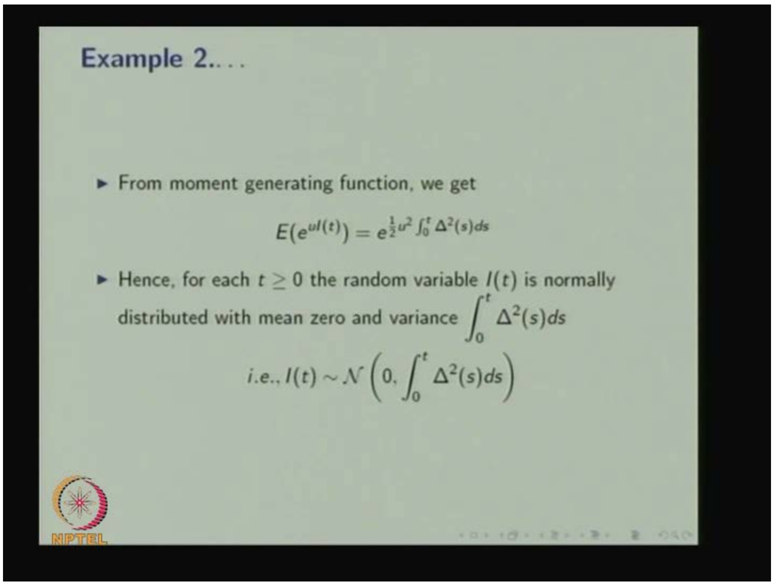

In this example we are finding the expectation and the variance of I(t) you can find out the distribution of I(t) also. You can find out the distribution using so, find out the moment generating functions of I(t). You know that moment generating function for any random variable is nothing, but the expectation of e power u times x so, here we finding the distribution of the random variable I(t) for fixed t.

Therefore it is a expectation of e power u times I(t) you can find out these values e power of times u square integration 0 to t delta square of s ds. So, once you are able to find out the moment generating function for fixed t. You can compare the moment generating function of some standard distribution then you can conclude what is the distribution of I(t) here the moment generating function e power half integration 0 to t delta square of s ds this is same as the moment generating this of the form function of normal distribution.

Therefore you can conclude for each t I(t) is normally distributed with the mean now we have to compare is moment generating function with the moment generating function for function of normally distributed normal distributed random variable. If the mean u and variance sigma square by comparing the m g f s we can conclude what is the mean and the variance of f(t) so, here we come to the conclusion the I(t) is normal distributed random variable with the mean and variance is integration 0 to t delta square of s ds for fixed t it is a normally distribute normal distribution.

So, in this example we are finding the mean and variance of I(t) as well as what is the distribution of I(t) also we are finding the mean and variance as well as the distribution of I(t) when the integral is non random function, when the integrand is non random function of time.

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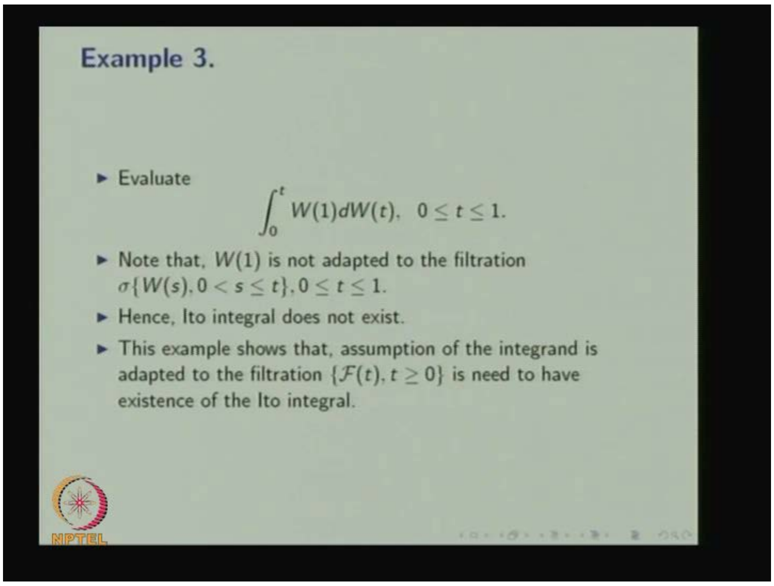

Third example here we are evaluating the integration 0 to t integrand is W(1) dW(t). Note that W(1) is not adapted to the filtration that is F(t) sigma of W(s) are the interval 0 to 1 has the t increases from 0 to 1 W(1) is not adapted in the filtration F(t) where F(t) is a natural filtration of Wiener process. So, since it is not satisfying the condition of I integrand is adapted therefore, this is Ito integral does not exists.

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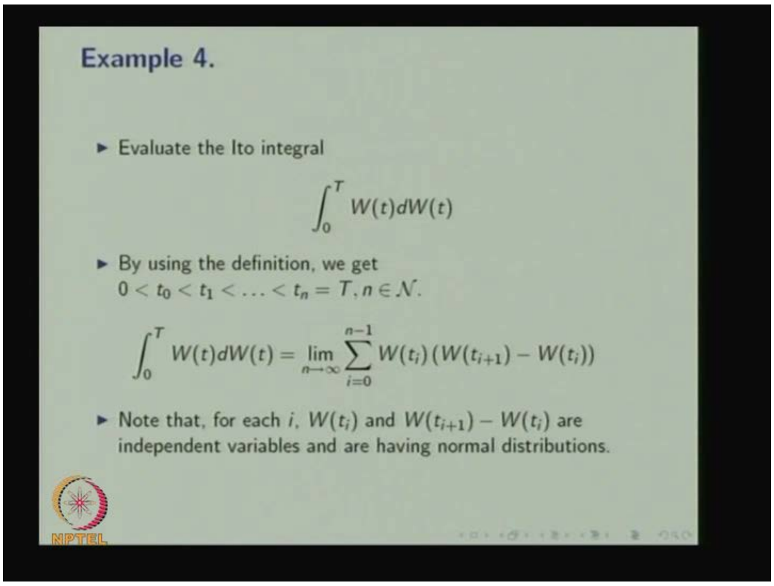

So, this examples shows that the assumptions of integrity are adapted to the filtration whose limit have the existence of Ito integral so, here the Ito integral does not exists.

Here moving into the next example evaluate the Ito integral integration between 0 to t W(t) dW(t). That means the integrand is W(t) integration with respect to W(t) so, you can use the partition in the interval 0 to T into n pieces n parts then has a limit n tends to infinity this is the integration is nothing, but limit n tends to infinity of i is equal to 0 to n minus 1 W(t i) and the difference of W(s).

So, the same definition we have given in the Ito integral also, here the Ito integral is well defined because W(t) that is the integral is adapted and also the mean square integrable. Therefore this is Ito integral so, the I(t) is equal to 0 to T W(t) dW(t) integrable is the Ito integral, but here we are going for the upper limit is the T not the variable limit. So, for each i we have written the limit n tends to infinity summation with the difference of wi's and w's.

And this is a W at the point t i and were has this one is has W(t i) because one minus W(t i) therefore, these two are the non overlapping w's so, the increments are independent therefore, W(t i) and W(t i) plus one minus W i are the independent random variables.

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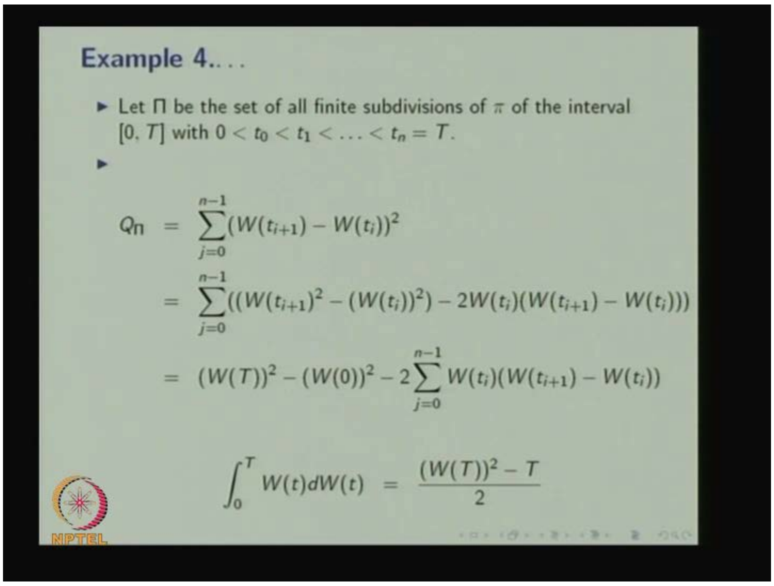

Are having the normal distributions we are using the properties of Brownian motion therefore, W(t i) and W(t i) plus one minus wi are independent random variables. So, what I am going to do here let pi be the finite set of abbreviations of pi(t) the interval zero to T therefore, first we will find out what is the q of pi.

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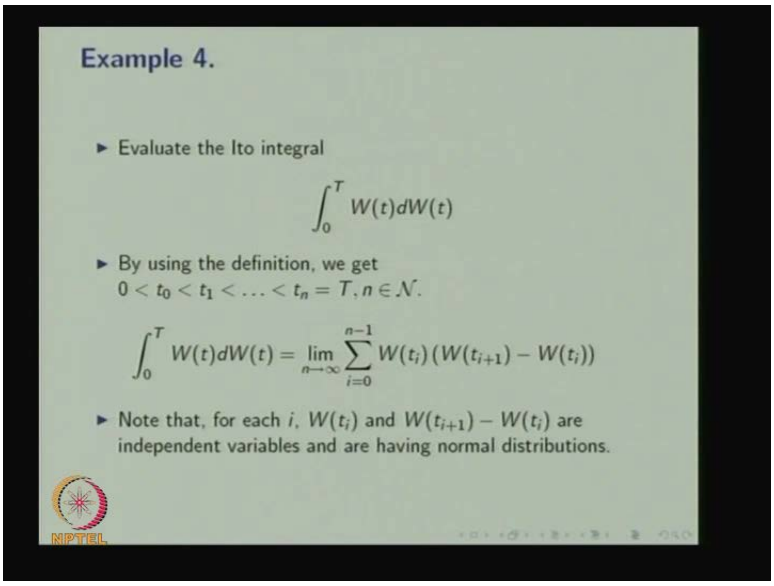

That is different from what we want what do want is different what do want is limit n tends to infinity summation i is equal to 0 to n minus 1 W(t i) the difference, but what we are defining now the pi i is nothing, but the difference whole square what you want is this W(t i) is the W(t i) plus 1 W(t i).

So, started with the q pi that is equal to the difference whole square the difference whole squared is same as suppose you treat has this has a this has the b so, a minus b the whole square were is same has a square minus b square minus 2 time a minus b, minus 2 times b a minus b so, if simplifying, but a minus b the whole square if you expand the summation now this will be the only the last term will be there only all other last terms vanishes.

Therefore you get W(t) whole square minus W(0) whole square the last term and the first term will exists and all other terms will vanishes because of minus squares where as this 1 minus of 2 summation W(t i) multiplied with W(t i) plus 1 minus W(t) that is what we want has a n tends to infinity.

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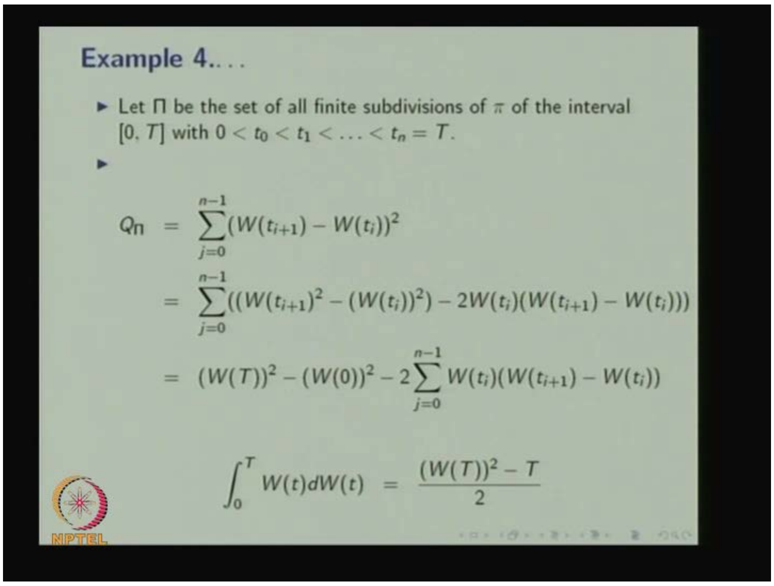

But we know the result limit norm of pi tends to 0 of q pi will be T that we have discussed the lecture 1 when we discuss the quadratic variation of Brownian motion in the lecture one we have discussed the quadratic variation of Brownian motion. In that i q pi it is define limit norm of pi tends to 0 the q pi will be T therefore, one we want is the limit n tends to infinity of this quantity that is third term in this q pi.

So, now we have play the limit n tends to infinity in this equation so, the left hand side becomes the T the right hand side becomes the W(t) whole squared minus W(0) whole square W(0) is 0 standard Brownian equation. Therefore we only get the first term second term will be 0 third term is unknown that is the integration the limit n tends to infinity of summation that is nothing, but, the I(t) therefore, 0 to T W(t) dt.

That is nothing but in the left side we got T therefore, you get W(t) whole square minus T is equal to since it is minus 2 times therefore, it will be by 2 hence the integration 0 to T W(t) dt becomes the W(t) whole squared minus T by 2. So, here we have used the quadratic variation of Brownian motion is T that we have used for the intervals 0 to T therefore, the integration 0 to the T. So, we have used the Brownian motion called quadratic variation between the intervals 0 to T that is T. Therefore the integration is going to be W(T) the whole squared minus T by 2.

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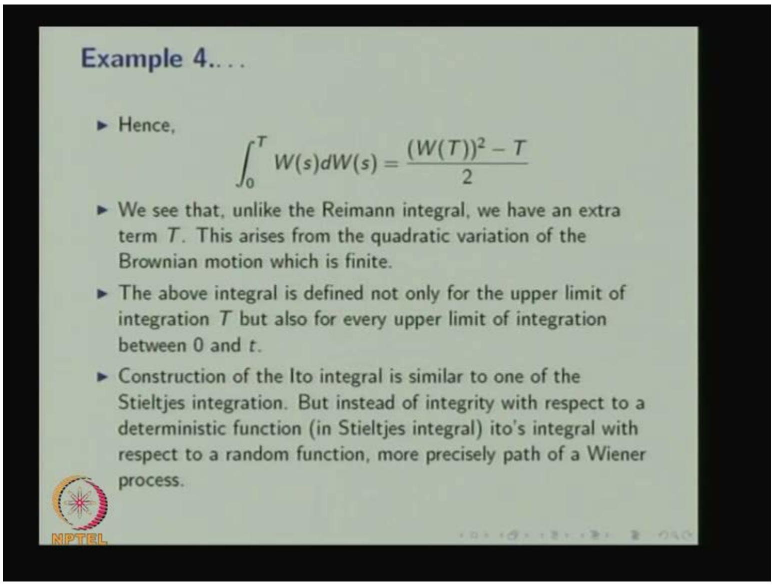

Hence this integration is the W(t) whole square minus T by 2 if see that unlike Riemann integral we have extra term T if the integration is with respect to bounded variation differential function then you would not have the term T here just it is a W(t) whole square by 2 suppose you replies W of s by x therefore, you will get 0 to T s times s d s that is nothing, but, s square by 2.

Whenever the integrand is bounded variation as well as the differential equations that is nothing, but the Riemann integral so, in the Riemann integral you would not have the extra term T by 2. But here we have the extra term minus T by 2. So, this arises from this quadratic variation of Brownian motion which is finite where as if it is a real valued function which is bounded then the quadratic variation is going to be 0.

But here the quadratic variation which is finite value therefore, you are getting minus T by 2 so, the above integral next remark the above integral is define not only for the upper limit integration T, but also a every upper limit of integration between 0 to T. This integration we have than the upper limit T therefore, we have used the quadratic variation of Brownian motion between the intervals 0 to T that is T, but instead of that we will go for variable t also.

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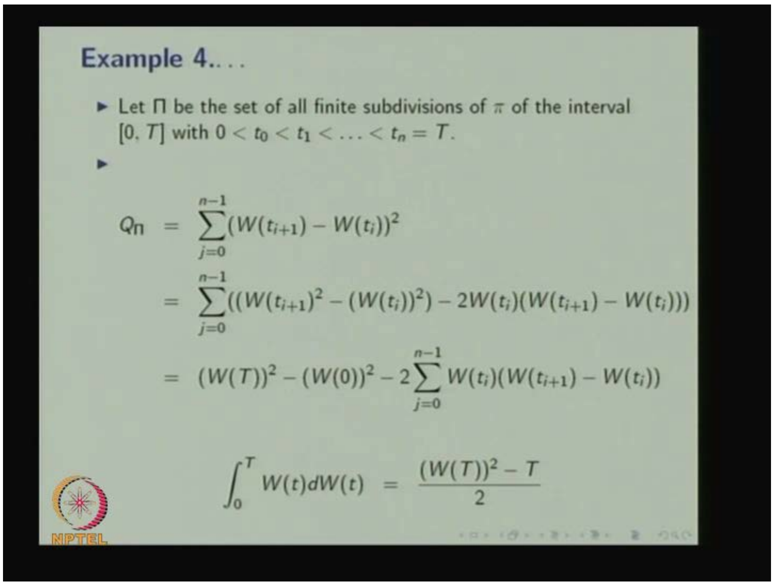

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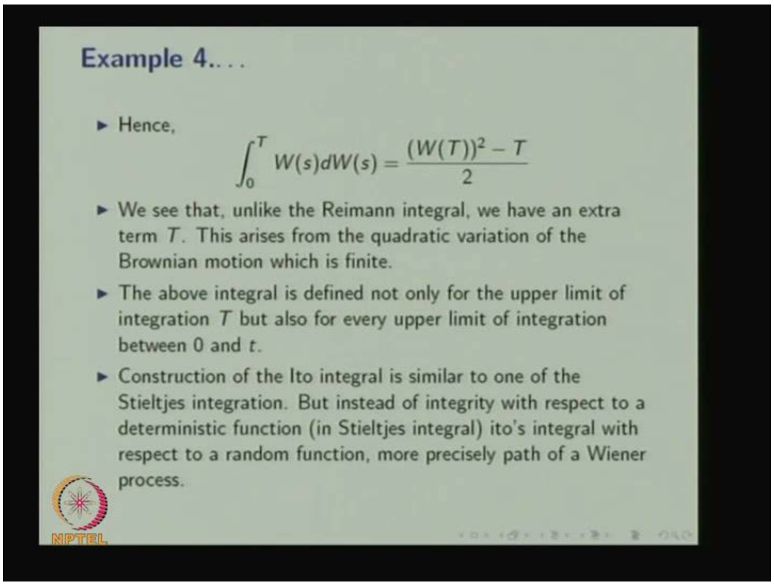

In that case here we have to use the quadratic variation of Brownian motion between the interval 0 to t and that will be the t itself with the variable t therefore, the integration 0 to t W(s) dW(s) is nothing but in the W(t) whole square minus t by 2 where both the t's are small letter variable. You see the construction of Ito integral is bring out the one of Stieltjes integration, but instead of integrating with respect to the deterministic function the Ito integral with respect to the random function more precisely path of a Wiener process.

So, that is the difference between the Ito integral and the usual integral the usual integral is with respect to the deterministic function where has the Ito integral the integration with respect to the path of Brownian motion or wiener process which is unbounded variation and no where differentiable therefore, the whole integration is different.

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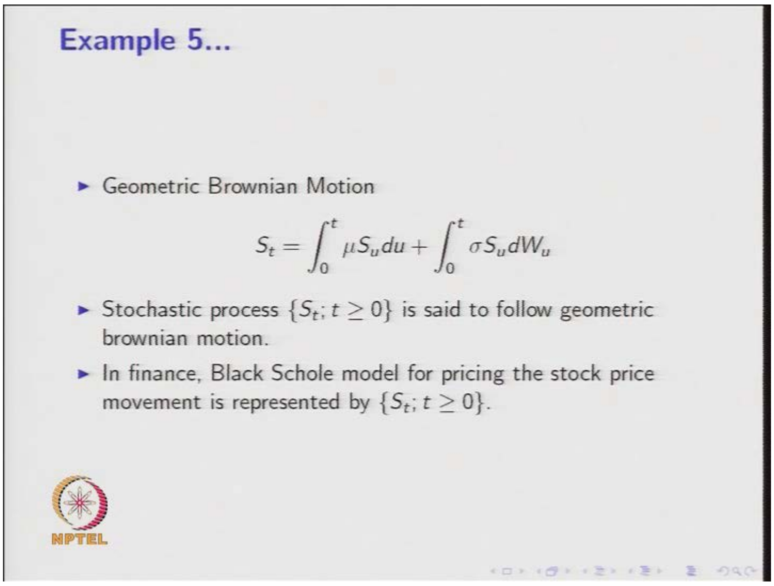

Consider this stochastic integral in case the first integral is a Riemann integral of a stochastic integrand where as the second integral is a Ito integral this equation is very useful in finance in the Black Schole model for stock price is said to follow geometric Brownian motion.

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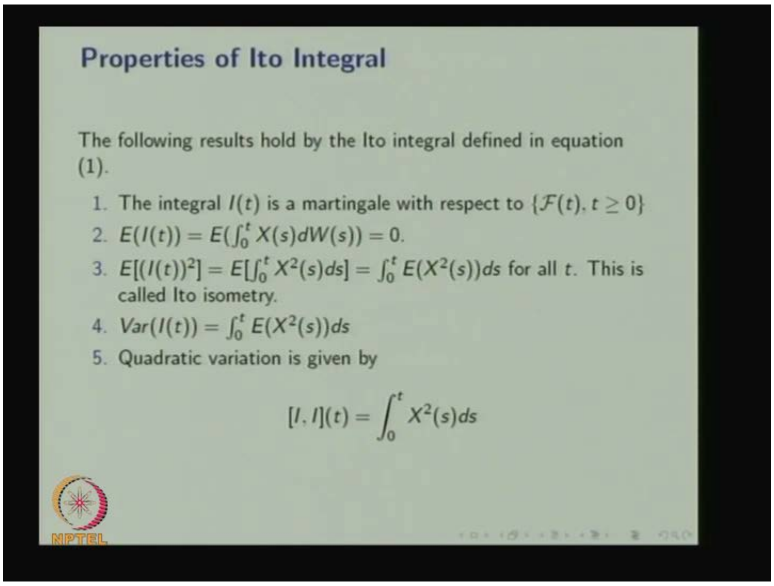

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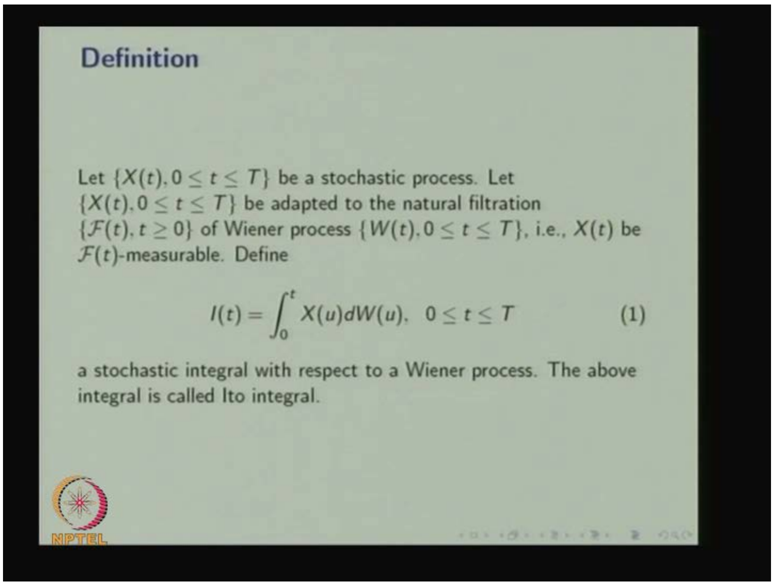

Now you are disusing few properties of Ito integrals still you have use few properties in the examples, but now we can understand how the properties will be used in the examples the following results hold by the Ito integral obtained in the equation one in the equation one we have discussed then this is the Ito integral which we discussed in the first equation.

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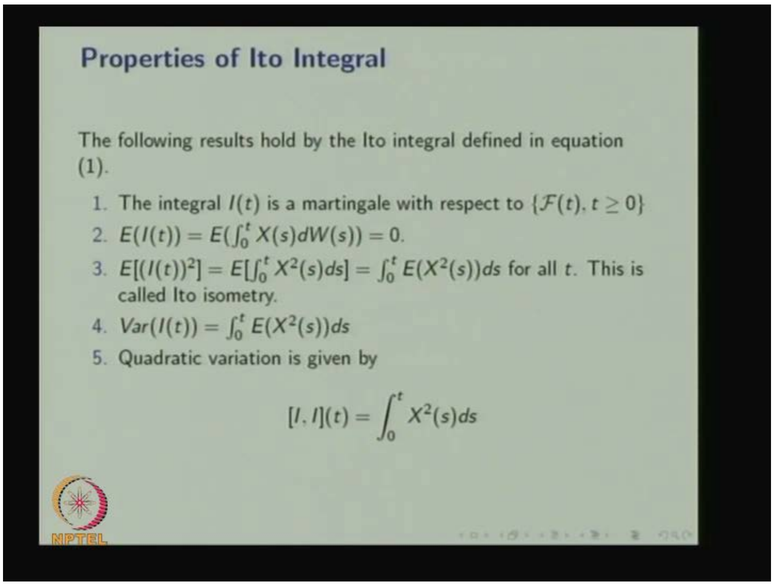

So, this we are referring yes the first property the integral I(t) is a martingale with respect to the filtration F(t) this is a very important property if we have a Ito integral that means the integrand is adapted and the mean square integral and the Ito integral is a stochastic process and that stochastic process is having the martingale property with respect to the filtration F(t) the same F(t) the integrand is adapted also with respect to the same filtration.

The integrand is the integrand is adapted also with the proof is if you want to if you verify the stochastic processes is a martingale you have to check the mean properties the first property is it has to be a first there the stochastic process for fixed t it has to be integral. Then it has to be a adapted then the third property the conditional expectation has to satisfies the equality property. So, if these three properties are satisfied by the stochastic process then we will say the stochastic process has the martingale property.

So here I(t) is the stochastic process are t greater than or equal to 0 so, first we have to verify it is integral so, you can find out the expectation of I(t) or infinity that is going to be a finite value. From the second one I(t) has to be a for fixed t one I(t) has to be a F(t) measurable that is adapted so, for fixed already the integrand is adapted the integration with respect to the W(t) and the W(t) is also adapted to the F(t) one can prove the I(t) also adapted to the filtration F(t).

The third condition expectation of I(t) given F(s) where s is less than t that is same has I(s) that prove expectation of I(t) given F(s) where s is less than t that is equal to I(s) that is prove the property for all t. Since these three properties of martingale satisfies I(t) will be a martingale with respect to the filtration F(t) here I am not discussing the proof so, here I am not giving the proof, but discuss what are all the properties has to be verified.

Second property expectation of I(t) that is same as expectation of 0 to integration the definition that will be a 0. Since it is a martingale it will have a constant mean therefore, expectation of pI(t) is same has expectation as y of 0. And you know how to evaluate the expectation of y(0) that will be 0 and the third property expectation of I(t) whole squared that is same has integration 0 to t expectation x square of s ds.

So, the x expectation and the integration inter change the price this is called the Ito isometric so, these are very important property of second order expectation that is integrand in the earlier we have the expectation of I(t) the whole square that is nothing, but integration 0 to t expectation of x squared ds. This is nothing, but the Riemann integral so, for fixed s fixed s the expectation of x square is the function of s so, you are integrating 0 to t the integrand is expectation of s square ds so, this is nothing but the Riemann integral.

Now based on the property number 2 and 3 and conclude variance of y(t) is nothing but zero to t expectation x square of s ds because the expectation is a expectation pI(t) is equal to 0 expectation of I(t) is equal to 0.

Therefore the variance is nothing but variance of I(t) is equal to the expectation of I(t) square minus expectation of I(t) whole square that is same 0 to t integration expectation x square of s ds. One can find out the quadratic variation of F(t) also because I(t) also stochastic process so, we can find out the quadratic variation between the interval 0 to t so, i comma i between the interval 0 to t that means second order variation between the interval 0 to t or the function I(t). I(t) is a Ito integral, where has the quadratic variation is nothing, but the integration 0 to t the x square of s ds, so this is the Riemann integral. So, I am not given the proof of the fifth property also we are such searching the result of Ito integral and we have used few properties in the examples.

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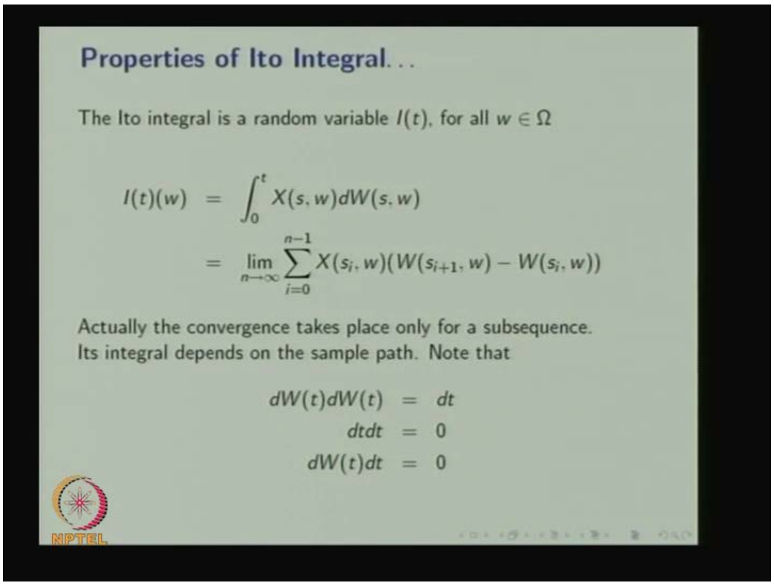

The Ito integral is a random variable for quality for all w belonging to omega therefore, you can write down I(t) of W that is nothing but the limit n tends to infinity, the value of X(s i) difference Ws. You know that x i x i x is the random variable the difference of W is also a random variable therefore, for fixed n this summation y is equal 0 to n minus 1 we will be nothing but a random variable we are finding the limit n tends to infinity of the summation.

Therefore Actually the converges takes place sub sequent that means I(t) is a random variable and that random variable is nothing but the converges of the right hand side the limit n tends to infinity of the summation. And it is a integral depends on the sample path because we are find the difference of W(s i) s plus one minus w of si therefore, this integral depends on the sample path and also we need this following properties the d(W) dW(t) will be dt.

That is nothing, but the quadratic variation of Brownian motion that is nothing, but finite value if you are finding the increment between the interval 0 to t then the quadratic variation is t. Where has if it is real value function which is a differentiable then dt the quadratic variation of the function t will be 0 and the mixed variation the cross variation dW(t) the dt will be 0. So, we find out the quadratic variation with W(t) that will be the dt where has the quadratic variation with the function will be 0 in the cross variation mean will be 0 so, this result will be used in your finding the I(t) here is the important reference important reference is for the Ito integrals. The next class we will consider the Ito formulas.