Stochastic Processes
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Module - 5
Continuous-time Markov Chain
Lecture - 3
Poisson Processes

This is a module 5 continuous time Markov chain lecture 3 Poisson process. In the first two lectures, we have discussed the continuous time Markov chain definition, Kolmogorov differential equation, chapman Kolmogorov equations and infinite decimal generator matrix. Then we have discussed some properties also, in the lecture 2 we have discussed the birth death process and their properties and also we have discussed the special cases of birth death process, pure birth process and death process.

In this lecture, we are going to discuss Poisson process and its application. So, let me start with the Poisson process definition, then I give some properties in the Poisson process and I also present some examples.

Poisson process is a very important stochastic process; and never something happens in some random way occurrence of some event, and if it satisfies a few properties, then we can module using Poisson process. And Poisson process has some important properties whereas the other stochastic processes would not be satisfied with those properties therefore the Poisson process is a very important stochastic process for the many modeling’s in a applications like, telecommunication or wireless networks or any computer systems or anything any dynamical system, in which the arrival comes in some pattern and satisfies few properties.

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Example 1

Consider the car insurance claims reported to the insurer. Assume, that the average rate of occurrence of claims is 10 per day. Also assume that the rate is constant throughout the year and at different times of the day. Further assume that in a sufficiently short time interval, there can be atmost one claim. What is the probability that there are less than 2 claims reported on a given day ? What is the probability that the time until the next reported claim is less than 2 hours ?

So, before moving into the actual definition of Poisson process, I am going to give one simple example; and through this example I am going to relate the Poisson process definition, then later I am going to solve the same example also. Say example number 2 example 1, I have something else consider a car insurance claims reported to insurer; it need not be car insurance, you can think of any motor car motor insurance or any particular type of vehicle or whatever it is.

Assume that the average rate of occurrence of claims 10 per day, it is a average rate per day therefore, it is a rate per day; the average rate is a 10 also assume that, this rate is a constant throughout the year and at the different times of a day.

So, even though these quantities; average quantity there is a possibility, some day there is no claim reported at all or there are some day more than some 30 40 claims reported and all the possibilities are there. But, we make the assumption the average rate is a constant throughout the year at the different times of a day, also further assume, that in a sufficiently short time interval there can be at most one claim.

Suppose you think of very small interval of like, 1 minute or 5 minutes or whatever very small quantity comparing to the because here I have given the average rate is a 10 per day. Therefore, whatever the time you think of very negligible in that the probability of are, it is sufficiently small interval of time, there is a possibility of only maximum one claim can be reported.

The question is, what is the probability that there are less than 2 claims reported on a given thing? What is the probability that less than two claims reported means? What is the probability that in a given day either no claim or one claim also. We are asking the second question, what is the probability that time until the next reported claims less than 2 hours? Suppose some time one claim is reported, what is the probability that the next time is going to be reported before 2 hours?

We started with these problem in the car insurance claims reported therefore, the claims is nothing but, some event and these events are occurring over the time, suppose you make the assumption of sufficiently smaller interval of time, at most one claim can happened and the average rate of occurrence of claim is a constant throughout the time.

So, with this assumption one we can think of sort of arrival process, pure birth process satisfying some condition and that may need into poison process, so this same example we are going to consider it to again also.

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Derivation

Let *N(t)* denote the number of customers arriving during the interval *[0,t]*. Assume:

(i)X(0)=0;

(ii)Probability of an arrival in *(x, x+Δt)* is $\lambda\Delta t + o(\Delta t)$

(iii)Probability of more than one arrival in *(x, x +Δt)* is *o(Δt)*.

(iv)Arrivals in non-overlapping intervals are independent.

Now I am going for derivation of a Poisson process, how one can drive the Poisson process? Poisson process is a stochastic process with some conditions; so how one can derived the Poisson process for that let me start with the random variable n of t, that denotes the number of customers arriving during the interval 0 to time t.

That means, how many arrivals takes place in the interval 0 to t? that means, for fixed t n of t is a random variable over the time, this n of t collection that is the stochastic process making.

Some 4 assumption with these assumptions, I am going to be conclude the n of t is going to be a stochastic process. The first assumption not x of 0 n of 0 is equal to 0, at time 0 the number of customers is 0, n of 0 is equal to 0 is wrong n of 0. second one, the probability of arrival in a interval x to x plus delta t; that is the lambda times delta t, where lambda is strictly greater than 0.

That means, probability that a only one arrival is going to takes place in the interval of delta t; that probability is lambda time delta t, for as very very small interval delta t its independent of x, that means it is a increments are stationary; that property I am going to introducing in these assumption.

The probability of more than one arrival, in the interval x to x plus delta t is negligible, that means at most maximum one arrival can occur in a very small interval of time, that is the assumption; that I am specifying in third one.

The fourth assumption, arrivals in non over lapping intervals are independent, that means if the arrivals occurs in a some interval, and another some non overlapping interval then those arrivals are going to be form a independent.

That means there is no dependency, over the non over lapping intervals arrivals going to occur or not; so with these four assumptions n of 0 is equal to 0, and probability of one arrival is lambda times delta t in a small interval more than one arrival occurrence in a interval delta t. Where delta t is a very small; that is that probability is negligible and non over lapping intervals arrival are independent.

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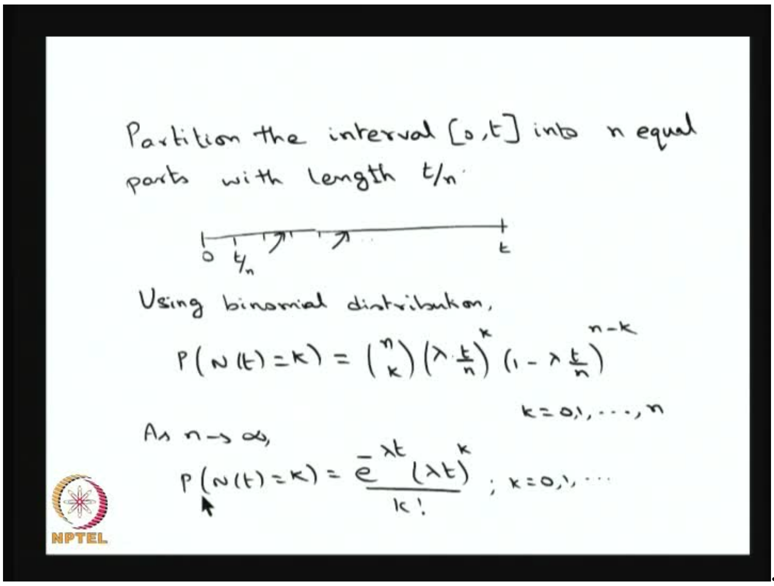

So with these derivation, I am going to find out the distribution of n of t; To find the distribution of n of t, first I am doing I am partitioning the interval 0 to t into n, equal parts with the length t divided by n, the way I use the the way I partitioned the interval 0 to t into n pieces. Such that, p by n is going to be a very small interval, so that means I have to partition that interval 0 to t in such a way, that the t by n is going to be as small as therefore, I can use those assumption of a probability of occurring one arrival in that interval of length t by n, that probability is lambda times t by n.

And the probability of not occurring a event, in that interval t by n is 1 minus lambda time t by n, so I can use those concepts for that, I have to partitioned interval 0 to t into n parts with the sufficient larger n therefore, t by n is going to be smaller. Now since, I partitioned this interval to n pieces, n parts. I can think of yet each parts I can think of a binomial or Bernoulli distribution at each pieces therefore, all the non-overlapping intervals occurrence are independent. therefore, I can think of it is accumulation of a n independent Bernoulli trials.

Since, it is an n independent Bernoulli trials for each intervals t by n of n th t by n therefore, the total number of event occur in the interval 0 to t, by portioning into n equal parts, this is a sort of what is the probability that k events occurs in the interval 0 to in the time duration 0 to t as a n portion?

So, out of n equal parts what is the probability that k events occur in the interval 0 to t? that is nothing but, since it is each interval is going to form a Bernoulli distribution with the probability p is lambda times t by n therefore, the total number is going to be binomial distribution with the parameters n and p where p is a lambda times (()) n therefore, this is the probability mass function of k, event occurs out of n equal parts therefore, n c k lambda times t by n power k 1 minus lambda times t by n power n minus k.

Now, the running index for k goes for 0 to n; that means there is possibilities no event takes place in the interval 0 to t, or maximum of n interval, n event takes place in all n intervals.

So, this is for a sufficiently large n such that, the t by n is smaller, we take n tends to infinity to understand the limiting behavior of this scenario, as the partition becomes finer. now I can go for n tends to infinity what will happen has n tends to infinity.

If you do the simplification, here as n tends to infinity that simplification, I am not doing in this presentation, has a limits n tends to infinity, the whole thing will land up, the e power minus lambda t lambda t power k by k factorial.

Now, the k running indexes is 0, 1, 2 and so on. This you can use the concept the binomial distribution, as n tends to infinity and p tends to 0. you are n into p becomes lambda, so that will give the Poisson distribution. The limiting case of a binomial distribution is the Poisson distribution.

So, using that logic this binomial distribution mass has n tends to infinity, this becomes a Poisson distribution mass function. So, this is nothing but, the right hand side is the probability mass function for a Poisson distribution with the parameter lambda times t, and this is the random variable for n of t for fixed t.

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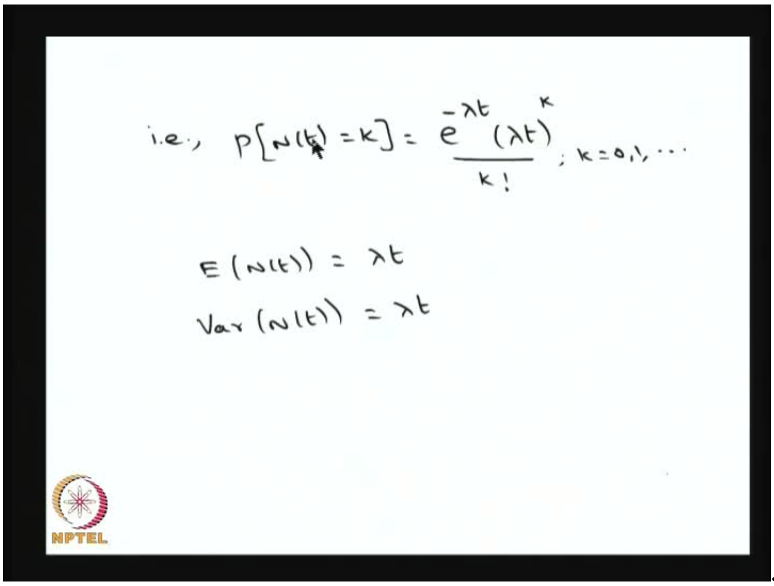

Therefore for fixed t, n of t is a Poisson distributed random variable, with the parameter lambda times t. where lambda, is greater than 0 therefore, we can conclude the stochastic processes related to the n of t for fixed t, n of t is Poisson distribution therefore, the stochastic process n of t over the t greater then or equal to 0, that is nothing but, Poisson process.

So from the Poisson distribution, we are getting Poisson process because each random variable is a Poisson distributed with the parameter lambda times t therefore, the collection of random variable is a Poisson process with the parameter lambda t. Since, it is a Poisson distributed random variable for fixed t, you can get the mean and variance and all other moments also, by using the probability mass function of n of t.

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Formal Definition

A stochastic process {N(t), t ≥ 0} is said to be a Poisson process with intensity or rate $\lambda > 0$ if the following conditions are satisfied:

(i) It starts from 0, i.e. N(0)=0

(ii) It has stationary and independent increments. Stationarity means that for time points s and t, s> t, the probability distribution of any increment $X_s - X_t$ depends only on the length s – t of the time interval and that the increments on equally long time intervals are identically distributed. Independent increments mean that for non-overlapping intervals [t, s] and [u, v] the random variables $X_s - X_t$ and $X_v - X_u$ are independent.

(iii) For every t > 0, N(t) has a Poisson distribution with parameter λt

Formally, we define Poisson process has follows a stochastic process n of t, t greater than or equal to 0; is said to be a Poisson process with the intensity or rate lambda, greater than or 0 with the following conditions are satisfied; first condition, it start from 0 that is n of 0 is equal to 0.

Second condition, the increments are stationary and independent stationarity means, that for time points s and t s greater than 0, the probability distribution of any increment n of s minus n of t depends only on the length s minus t of the time interval, and that the increments on equally long time intervals are identically distributed.

Independent increments means, that for any non overlapping intervals t comma s and u comma v, the random variables n of s minus n of t and n of v minus n of u are independent. For t greater than 0 n of t has the Poisson distributed random variable with the parameter lambda t.

And the difference of the random variables defined, over non overlapping intervals are independent. lambda t is the cumulative rate t time t, the exercise are independent and identically distributed random variables, with sum distribution function g independent of the Poisson process n of t, t greater than or equal to 0.

It is Markov in nature, because the two q is act independently, and are themselves m m one queuing system, which satisfies the Markov property, assuming that each queue behaves as the m m one queue that is all.

(( )) I do not know, I change the word Poisson distribution into Poisson distributed random variable, you have to say this sentence, because queue i j were obtained by differentiating the queue i j. we need said that but, (( )) it was not with the full stop it is continuation [FL].

Yeah, the way I was and similarly, that n of t I can tell again that Poisson [conversation between student and professor] the details of the proof can be found in the references books, because k i j's are obtained by differentiating the p i j's for every t greater than 0 n of t has Poisson distribution, with the parameter lambda t like that you can go for many more increments also.

for illustration, I have made it with the two increments, that means the occurrence of arrival during, this non overlapping intervals are independent and stationary means, it is a time in the variant only the length maters not the actual time.

Third one, for every t n of t as a Poisson distribution with parameter lambda t, so the Poisson logic is coming into the fourth condition only. the first condition is start at 0 implements are stationary and increments are independent. the third condition for fixed t n of t is the Poisson distribution random variable with the parameter lambda t therefore, this stochastic process is called a Poisson process.

Now, we can relate the way you have done the derivation, we have taken care this three assumptions starting at a time 0 increments are stationary, that we have taken and increments are independent; that is non overlapping intervals are independent, then when we are derived.

We are getting the distribution of the random variable n of t is the Poisson distributed random variable therefore; this is the Poisson process. the another way of defining the Poisson process, we can start with the birth-death process.

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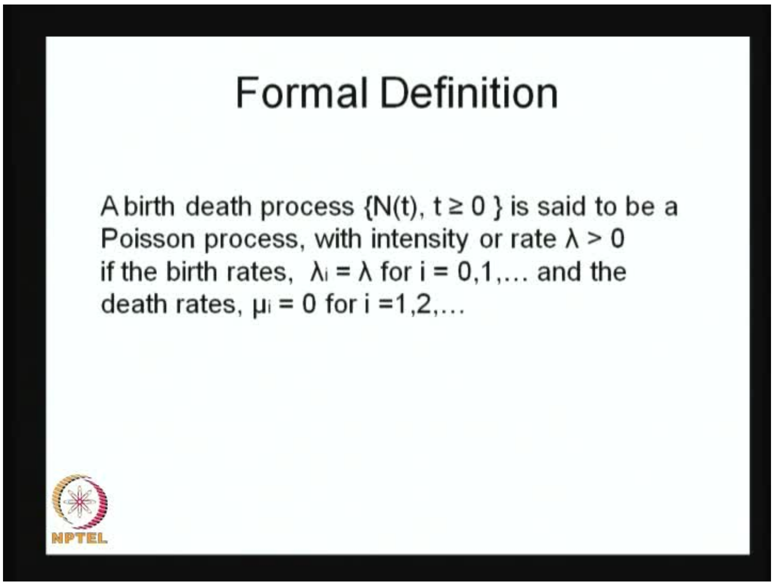

You know that, birth-death process is a special case of a continuous time Markov chain also it is a special case of a sorry it is special case of Markov process also, you can think of stochastic process, then the special cases of Markov process then the special case are continuous time Markov chain, then you have a special case, that is the birth death process.

So, you can define the Poisson process from the birth-death process, also a birth-death process n of t is said to be, a Poisson process with the intensity or rate lambda. if birth rates are constant for all i, and the death rates are 0 we start from the birth-death process, with all the birth rates are same; that means it is a special case of pure birth process, in which birth rates are constant for all the states and the death rates are 0.

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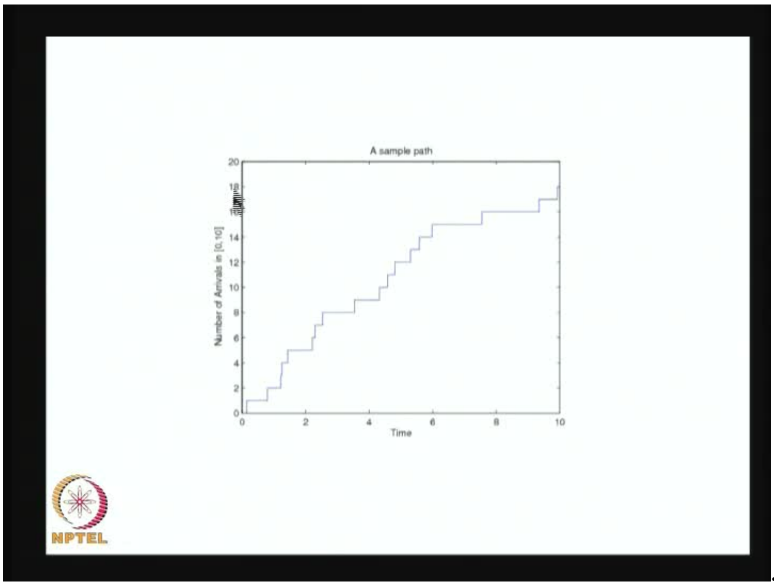

Then also, you will get the Poisson process. Here I am giving a sample path for the Poisson process; so this is the created using the mat lab, write the simple code of Poisson process, then you develop the sample path. That means at time 0, this system at 0 at some time one arrival takes place therefore, the system land up one therefore, the y axis is nothing but, the n of t so at this time one arrival takes place therefore, the number of customers in the system number of arrivals till this time that is one.

So, it is the right continuous function. the value at that point and the right limit is same as both are same features, different from the left limit of the arrival a poke arrival time of a poke. So, the system was in the state one till the next arrival takes place, so suppose the arrival takes place here, then the n of t values is 2 at this time point, in which the arrival a poke and the right limit and so on.

So, this is the way therefore, the system at any time, it will be the same value are it will incremented by only 1 unit. the Poisson process sample path will be with the one units step in increment at any time, there is no way the two steps the system can move forward at even in very small interval of time. The system will move into the only one step, that you can visualize here therefore, you can go back to the assumption, which we have started with the derivation in absolute is equal to 0 in a very small interval of time at most one event can take place.

And the difference of the random variables, defined over non overlapping intervals or independent and increments are also stationeries are those things, you cannot be visualize in the sample path, so this is the just one sample path over the time and n of t.

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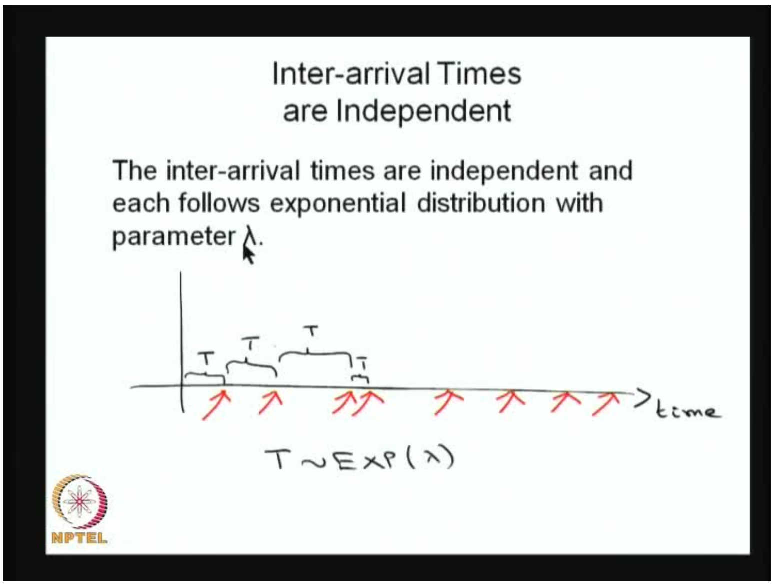

The second one, inter arrival times are independent as well as, we can conclude the inter arrival times are exponentially distributed also, the inter arrival times are independent, and each follow exponential distribution with the parameter lambda, what is the meaning of inter arrival times at time 0? This system is in the state 0. First arrival occurs, at this time point second arrival occurs this time point, and fourth third forth and so on. The inter arrival time means, what is the time taken for the first arrival? then what is the interval of time taken for the first arrival to the second arrival, and second to the third and so on.

So, that is the inter arrival time. So, whenever you have a Poisson process that means the arrival of event occur over the time, that follows Poisson process then this inter arrival time suppose, I make it as a random variable capital T and those random variables going to follow exponential distribution with the same parameter lambda. And all the inter arrival times also independent, that means these are all identically distributed random variable. I can go for different random variable label, also x 1 x 2 x 3 x 4 and so on. So, all those random variables are iid random variables and each follows exponential distribution with the parameter lambda.

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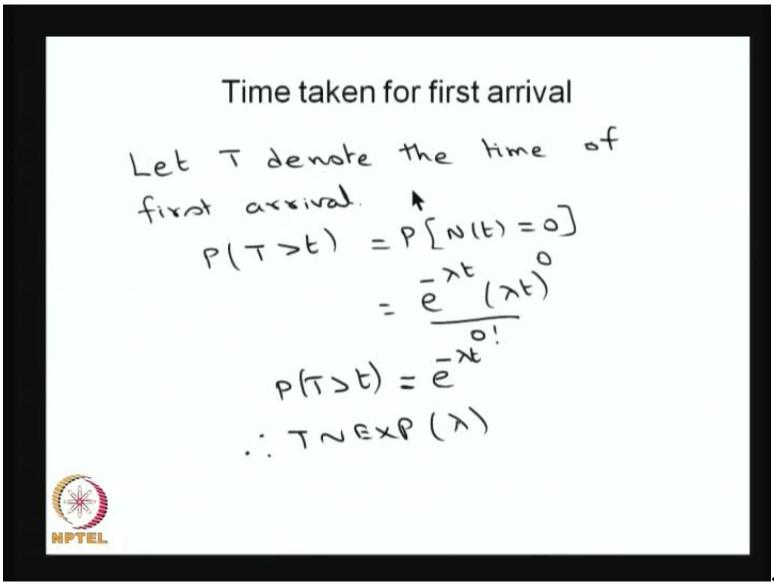

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So, this can be proved easy. let we start giving the proof for the first arrival time that means, the first one from 0 to the first arrival like, that you can go for the other arrivals also using the other properties are you can use, the multi dimensional random variable distribution concept and use the function of random variable and you can get the distribution also.

But, here I am finding the distribution for the first arrival, so let t denote the time of first arrival. my interest is to find out, what is the distribution of capital T? I know that, this is going to be a continuous random variable; because it is a time. so any time the first arrival can occur. So, to find since it is a continuous random variable, I can find out the cdf of the random variable or complements cdf.

So, here I am finding the first complements cdf using, that I am going to be find out the distribution, let me start with the probability, that the first arrival is going to takes place after time t, what is the meaning of that? The first arrival is going to occur, after time small t that means till time t, there is no arrival. So, both the events are equivalent events, the probability of t greater than small t, that is same as the probability of n of t is equal to 0.

That means no event takes place till time t, because the n of t denotes the number of arrival of customers during the interval 0 to small t; both are closed 0 to 1 0 to t therefore, n of t equal to 0, that means till time t nobody turned up; that is equivalent of the first arrival is going to takes place after t. I do not, what is that i do not the distribution of the capital t? but, I know the what is the probability n of t? that is equal to 0 therefore, I am writing this relation, so once I substitute the probability mass at 0 for the random variable n of t.

Just now, we have proof that n of t for fixed t is a Poisson distribution random variable with parameter lambda times t therefore, I know what is the probability mass at 0? so substitute the probability mass function with the 0. I will get e power minus lambda t that is complement cdf of the random variable in capital T.

Once you know the complements cdf, I can find out the cdf from the cdf, I can compare the cdf of some standard continuous random variable, I can conclude this is nothing but, exponential distribution with the parameter lambda, because this is the complement cdf at time t therefore, it is a lambda times t.

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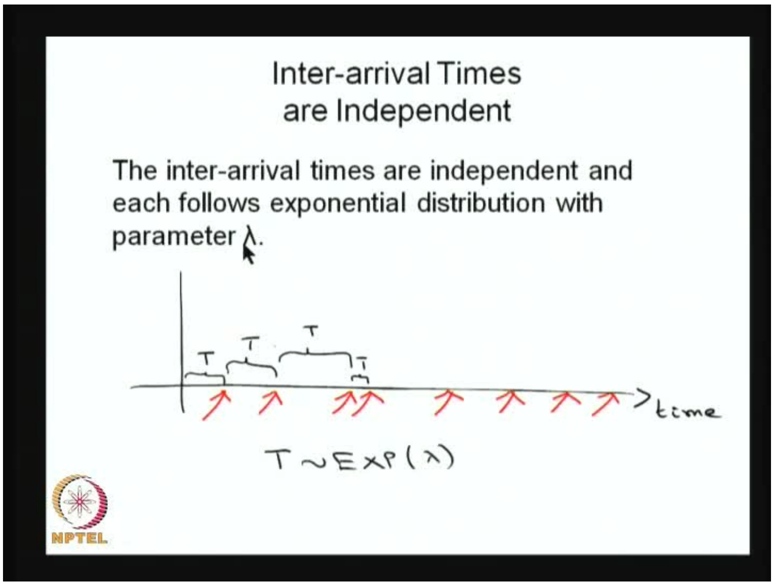

So, I conclude the distribution of capital T is exponential distribution with the parameter lambda, that means the first time of arrival this random variable, that is the continuous random variable, and the continuous random variable follows the exponential distribution with the parameter lambda.

Since, I know the increments are independent increments are stationary and so on. I can use the similar logic for inter arrival time of the this time, also then that is also going to follow independent exponential distribution since increments are independent. So, this is the first time and this is the second time therefore, the inter arrival times also going to be independent; that means whenever you have a Poisson process, that means the arrival occurs over the time in a very small interval maximum one arrival takes place, and the probability of one arrival in that small interval is lambda times delta t from that you will get the lambda.

So you can conclude, that is Poisson distribution Poisson process, so once the arrival follows is the Poisson process, the inter arrival times are exponential and independent. so from the poison process, one can get the inter arrivals are exponential distribution and independent the converse also, true that means if some arrival follows with the inter arrival times, exponential and exponential distribution and all the inter arrival times are independent.

Then, you can conclude the arrival process is going to form a Poisson process, that means arrival process and Poisson process implies, the inter arrival times are exponentially distributed and are independent similarly, inter arrival time are independent as well as exponentially distributed with the parameter lambda.

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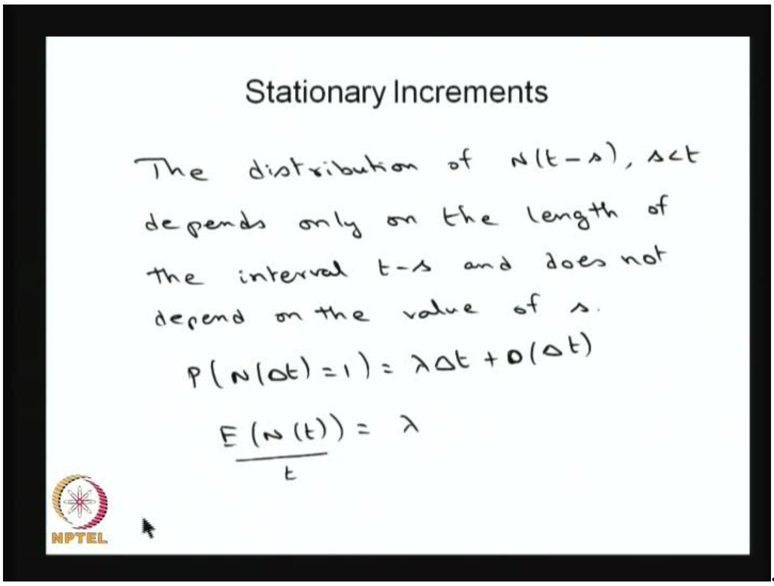

Then, the arrival process is the Poisson process with the parameter lambda with the intensity or rate, now I am going for the stationary increment the distribution of n of t minus s depends, only the length of the interval t minus s and does not depend on the value of s; that means during the interval delta t the one arrival is going to be lambda times delta t order of delta t, that will tends to 0 as n tends to has delta t tends to 0.

That means the stationary increments means, if you find out the rate that means you find out the average per unit of time, then that is going to be a constant. So, this is the assumption, we have taken it in the the car insurance a problem the average rate per day. that is going to be constant, and that is the assumption we are taking at going to be constant throughout the year and also the different times of day.

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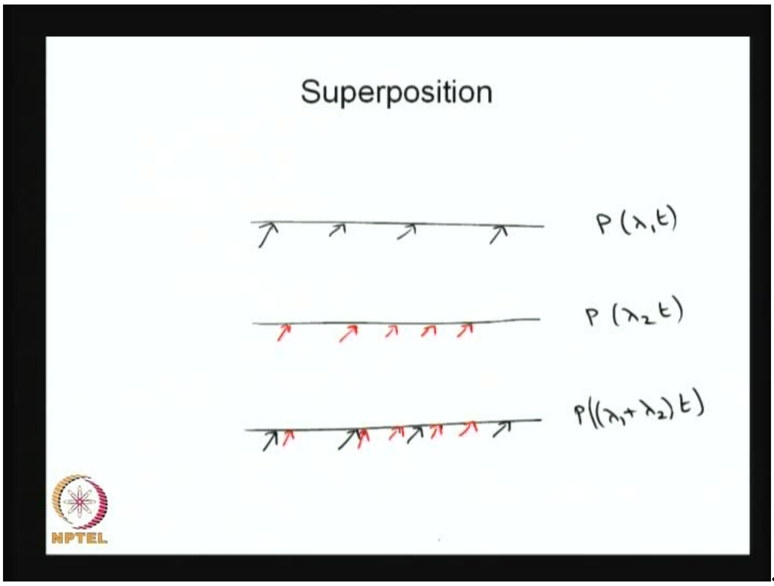

So, here also we will get, whenever you have Poisson process, then the average rate is going to be a constant; because of the stationary increment, the next property suppose you have Poisson process of a one arrival and you have Poisson process of the other arrival.

That means, the one type of arrival is the Poisson process with the parameter lambda one and another type of arrival two, that is also Poisson process with the parameter lambda 2, as long as both are independent, the arrivals are independent then the together super position. That, is going to be a again Poisson process with the parameters lam parameter lambda 1 plus lambda 2. you can add the parameter, that means for fixed t that is going to be a Poisson distributed random variable with the parameter lambda 1 plus lambda 2 times t.

Whenever, you have two independent or more than one independent Poisson process arrival, then the merging are the superposition, will be again Poisson processes as long as, they are mutually independent with the parameter is nothing but, the some of those parameters, that is you can is you can combine.

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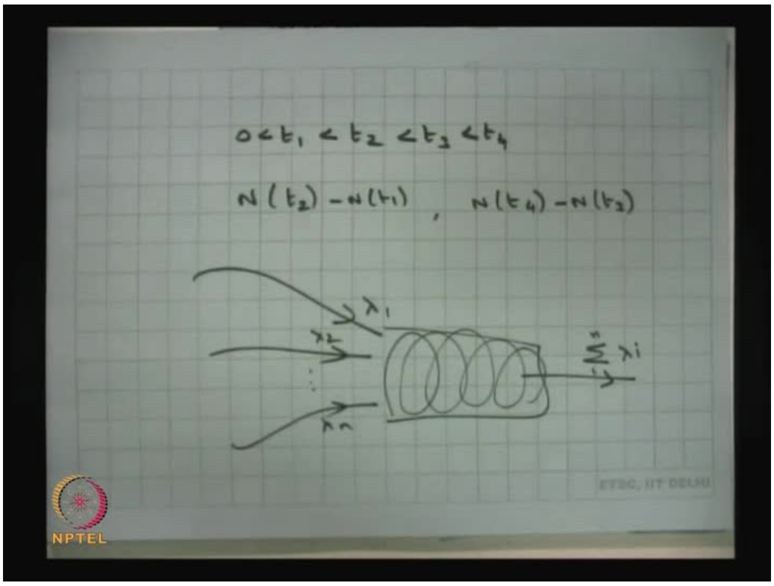

You can combine many Poisson process is stream into one stream, and that is going to be a Poisson stream with the parameter sum of a parameters lambda 1 to lambda n; so this is possible, this is used in many telecommunication of vocation. That means suppose, you have a Poisson arrival of a packet from different streams, and all the streams are mutually independent, that arrival are independent, then the total number of packets arriving into the particular switch or router.

Whatever it is then, the multiplexed one that is going be always be a Poisson process, that arrival follows the Poisson process, the parameter are some of parameter is nothing but, the some of these parameters as long as they are Poisson as well as the independent.

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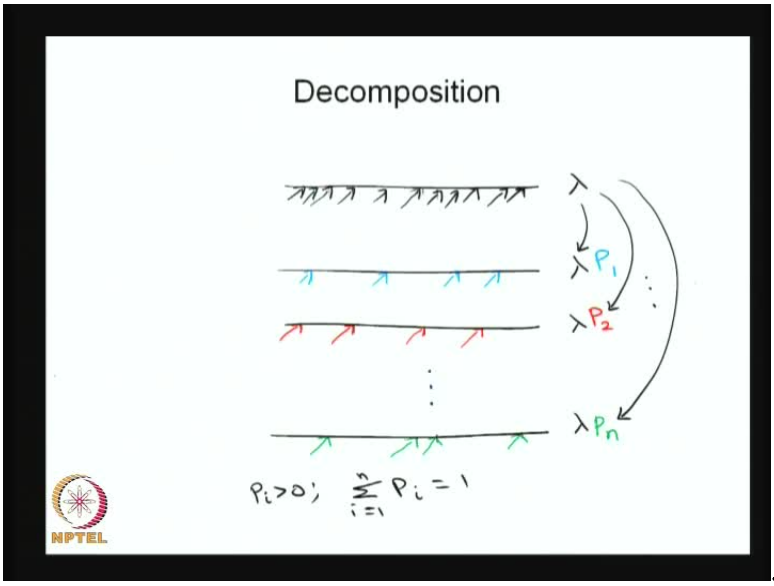

The next property decomposition, suppose if you have a one Poisson stream, you can decompose into many Poisson streams with the some proportion, so that proportion are the p 1 p 2 and p n's. So, one Poisson stream can be split into n Poisson streams with the parameter lambda times p, 1 lambda times p 2 per each p i's are greater than 0, the summation of p i's are as to be 1, that means these are all are properties with this probabilities, you can split one Poisson stream into many Poisson stream.

So, here I made a n Poisson streams that means, the same arrival is with some probability p 1 it land up here, with some probability p 1 this put up here with some probability, p n it is put up here, so this split of one Poisson stream into n Poisson streams is allowed that means the same example.

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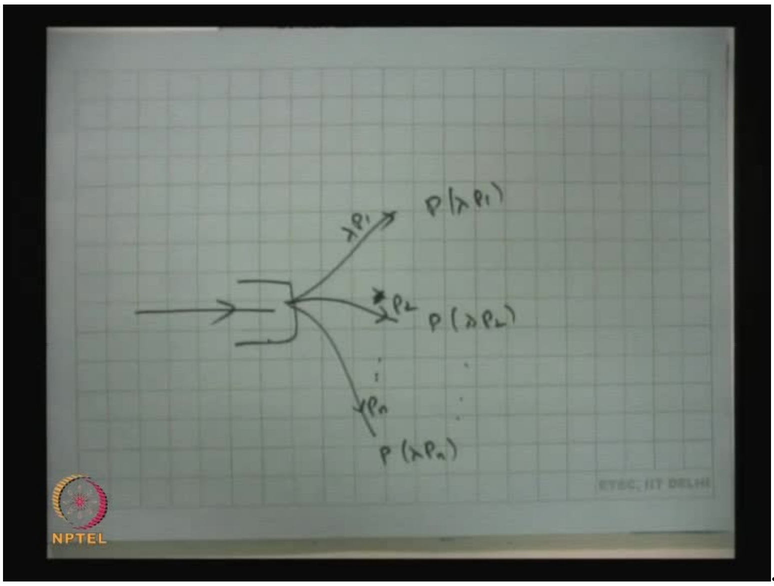

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If you have a one router from the router, if the arrivals is splitted into many streams, with probability p 1 it goes to the first stream, with the probability p 2 is goes to the second stream and with the probability p n is goes to the last stream, then each one is going to be a Poisson process, each one is going to be a Poisson process with the parameter lambda times p 1 and lambda times p 2 and so on lambda times p n.

So, the split is possible as well as, the superposition is also possible from the Poisson process. So, this also has the many more applications in the telecommunication networks one packet one type of packet arrival can be splitted into n proportion, so p 1 p 2 p n's and each one is going to be a Poisson process.

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Simple Problem

Consider the situation of waiting for a bus in a bus stand. Assume that the bus arrivals (in minutes) follow Poisson process with parameter 5. Suppose you come to the bus stand at some time. What is the average waiting time to get the bus ?

Now, I am going to be give the first example, to illustrate the Poisson process consider; the situation of waiting for a bus, in a bus stand in a bus stand assume, that the bus arrivals in minutes, follow a Poisson process with the parameter five with the rate, the parameter here that is nothing but, the intensity or rate suppose you come to the bus stand at some time, what is the average waiting time to get the bus?

When you land up with the bus stand, there is the possibilities the bus would have come before sometime, the time in which the bus the next bus is about to come, you are going to take that bus and till that time you are going to wait in the bus stand, that is the waiting time.

So, the waiting time is a random variable; that is a continuous random variable. the question is what is the average waiting time? one can find out the distribution of the waiting time also, here the question is what is the average waiting time? so the what I can do I can use the Poisson concept here.

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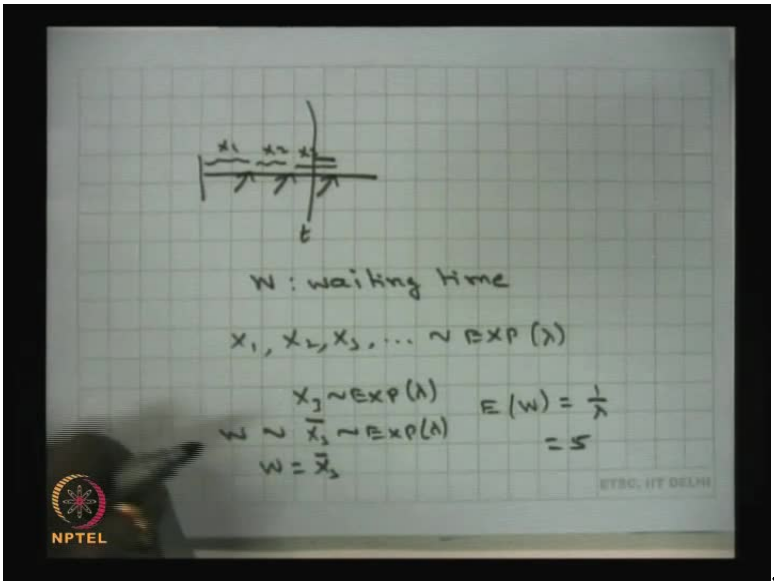

The arrival follows, the arrival of a bus follows a Poisson process, suppose at some time move to the bus stand, and the suppose the bus is going to come, at this time your waiting time is this much, so suppose you make a w is going to be your waiting time w is going to be your waiting time.

The question is, what is the average waiting time? just now, I have explain the Poisson process as the property, the inter arrival time are exponential distribution and the inter arrival times are exponential distribution, and all the times are all the inter arrival times are independent all therefore, this x 1 and this is x 2 and this is x 3.

So, x 1 comma x 2 , x 3 like that so many all the inter arrival times, that is going to follow exponential distribution with the parameter lambda, since the waiting time is going to be the remaining time of arrival of the third bus. So, the w waiting for the time is same as the remaining or residual time of the third bus to coming into the bus stand, so x 3 is a exponential distribution with the parameter lambda the residual life time of x 3.

Suppose, I make it us the notation x 3 bar, the residual life time residual life time of arrival not life time residual arrival time of a the third bus coming to the bus stand, that is also going to be exponential distribution, this is because of the memory less property. there is residual time also, whenever some time is exponentially distributed some random variable time is exponentially distributed, then the residual time is also going to be exponentially distributed using the memory less property.

Therefore, residual arrival time of bus to come to the bus stand, that is also exponentially distribution with the parameter same lambda. so this is same as the w the waiting time w is same as the residual time therefore, the w is also going to be exponentially distributed with the parameter lambda. That means, the waiting time for the bus to come to the bus stand, to catch the to cache, so the w is exponentially distributed therefore, the question is what is the average waiting time? So average waiting time is nothing but, one divided by the parameter.

One divided by the parameter so, here it is says the Poisson process with the intensity 5, that rate is lambda; that is the mean inter arrival time between the buses is 5 minutes, that means in the mean inter arrival time between the buses is 5 minutes is nothing but, its exponentially distributed with the parameter, that is the average 5 minutes therefore, that is the same thing therefore, that is equal to 5 minutes .

Because, the way I have given the clue the mean inter interval between the buses is 5 minutes, that means the ra the average of x i's that is equal to 5 minutes, so that is the same as your waiting time, because it is exponentially distribution therefore, the residual is also exponential distribution. Therefore, you can use the same value therefore, the average is going to be 5 minutes, so using Poisson process one can find out the different results related to the number of arrivals. next I am going to give some more process related to the Poisson process.

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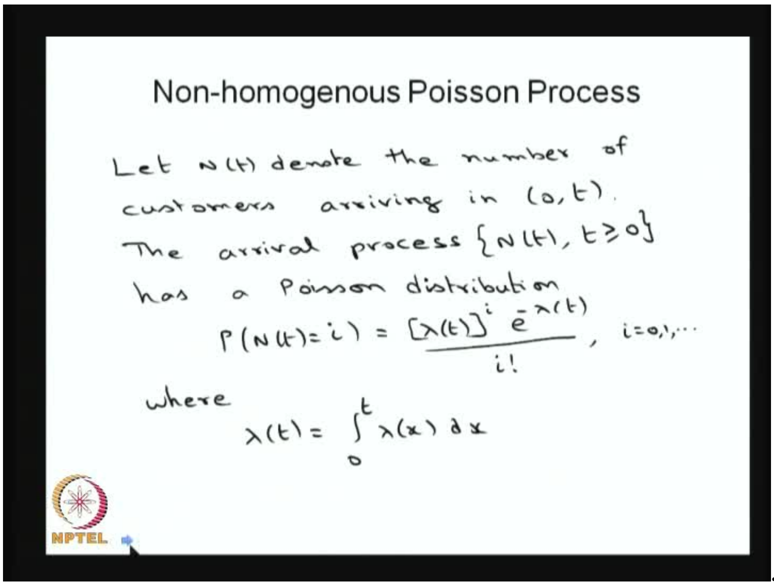

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He first one is the non homogenous Poisson process, let n t denote the number of customers arriving in the interval 0 to t, the arrival process as the Poisson distribution but, here the change instead of, the mean arrival rate is a constant mean arrival rate; is a constant lambda but, here it is a function of t lambda t ;is the cumulative rate at time t. That is the change from the Poisson process, then this stochastic process is called a non homogenous Poisson process, instead of mean arrival rate is a constant, here the lambda t is a function of t therefore, this stochastic process is called a non homogenous Poisson process.

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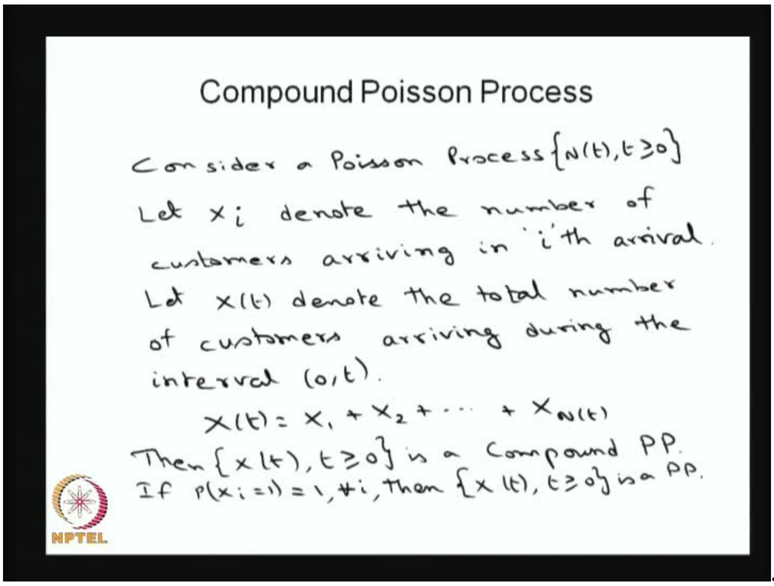

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Second one, compound Poisson process using Poisson process; one can develop a little complicated stochastic process related to the arrival that is called a compound Poisson process. Consider a Poisson process n of t, then you define a random variable x i's denote the number of customers arriving at the i th time point of arrival.

X 1 denotes the how many arrivals takes place at the time of first arrival? first arrival time point and x 2 will be, what is the second time of arrival? how many arrivals takes place? therefore, I am making a new random variable x of t; that involves t, that denotes the total number of customers arriving during the interval 0 to t.

That means, it is going to be a how many arrival takes place in the first time point x 1? how many arrival takes place at the second time of arrival that is x 2? and so on. plus x of t x of x of n t here n t is a random variable, and how many arrival takes place that is a x i's?

All together, that is going to be the total number of arrivals the x i's are independent and identically distributed random variables, with some distribution function g independent of the Poisson process n t. So this is nothing but, random some because these are all the random variable and how many random variables you are going to add? that dependence on the value of n of t over the t, this is the random sum of a x i's with n of t obviously these two are independent x i's are independent of n of t.

And since, it is a number of arriva customers arrival during, the in the i th time point therefore, x i's are discrete random variable, x i's are discrete random variable and n of t is also discrete Poisson process. Therefore, x of t is going to be a discrete state continuous time stochastic process and we are using the Poisson process, to get these stochastic process therefore, it is called a compound Poisson process. One can reduce Poisson process from the compound Poisson process by substituting, each x i's takes the value only one unit.

That means, the number of customers arriving at the i th time point is going to be only 1, that means if I make a p probability of x i takes, the value only 1; that probabilities is 1 for all i, then I will have a only 1 value possible till n of t, then this is going to be a Poisson process.

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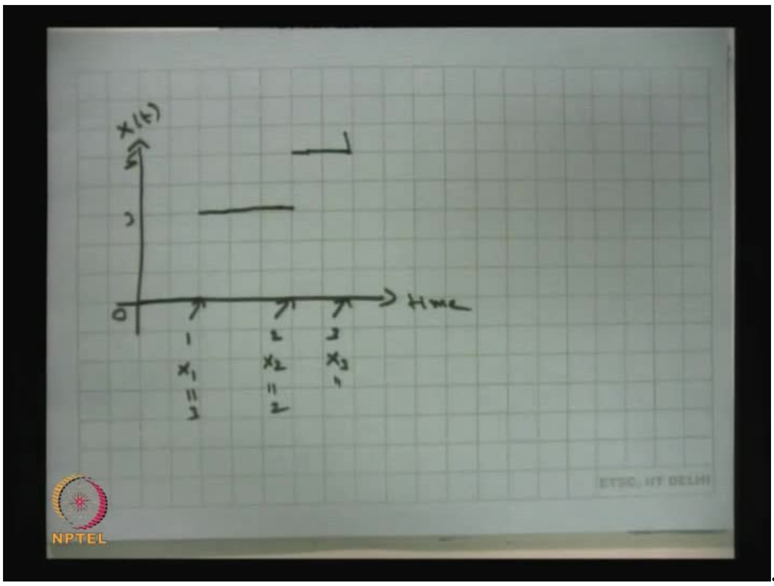

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Suppose, the probability of x i's the suppose x i's are going to be a discrete random variable with the possible values 0 1 2 and so on. Then the x of t is going to be a Poisson process. I can make a simple sample path for the compound Poisson process, this is over the time and this is over the n x of t. Suppose, these are all time points in which arrival time point, so this is the first arrival time point and this is the second arrival time point and this is the third arrival time point; it can be anywhere in the cont in the continuous times therefore, this called discrete state continuous time stochastic process .

So here, I am relating with the random variable x 1, this x 2 random variable, this is x 3 random variable, so till the first arrival till the first time of arrival the number of customers in the system is 0 at the first time of arrival the x 1. Suppose you think, you make the assumption x 1 takes the value 3, x 1 takes the value 3 therefore, this will be incremented by 3 till the second arrival, at the time of second arrival suppose, you assume that this takes the value 2 with some probability probability of x 2 takes, the value 2 is greater than 0, so you have assume the value 2 it can take any other value also

So, it is incremented by 2 till it takes the third arrival the value is a so this is 0, this is 3 then 3 plus 2 5 at this time, whatever be the number of arrival accordingly this can takes some value. So, the difference between the compound Poisson process and the Poisson process, the Poisson process increment will be only one unit increment, over the time whenever the time in which the arrival occurs arrival time a pokes.

Whereas, here wherever the time of arrival time a pokes, the number of customers entered that need not to be 1, it can be more than or equal to 1, so that is the way the job goes therefore, this is called a compound Poisson process. So we have seen two variations of a Poisson process; one is a non homogenous Poisson process and other one is compound Poisson process. So before I go to the I complete let me give the solution, for the first ex second example which i started.

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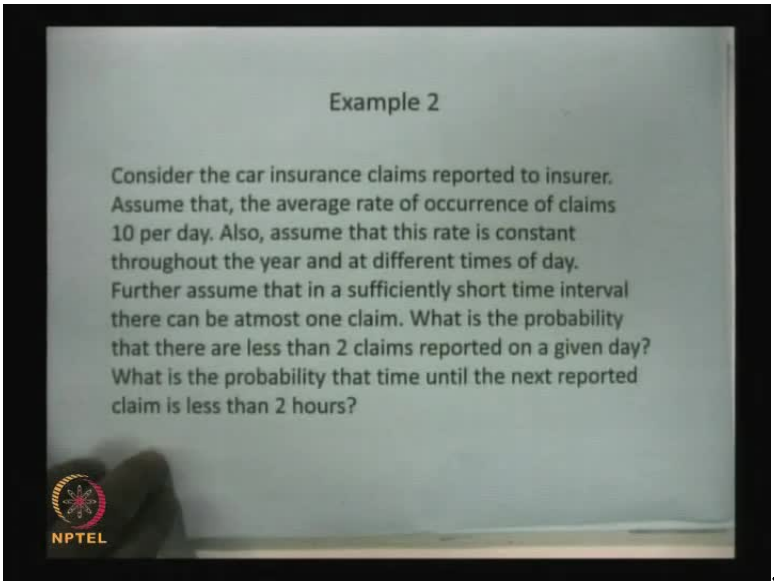

That is the question, the car insurance problem we have discuss the two problem; the first problem is related to the bus stand bus arrival issues, and this is the car insurance problem. So, in this problem we have not assume the Poisson process, but the problem is a related to the Poisson process, one can assume it is a form a Poisson process, because you see the assumption the average rate of occurrence of claims is a 10 per day, also the rate is constant and in a very small interval time, at most one claim can be happen the questions are, what is the probability that there are less then two claims reported on a given day?

Since, the increments are stationary so any day you can think, of with the only the interval what is the probability that time until the next reported claim is less than 2 hours, so this is related to use the exponential distribution, because the inter arrival times are exponentially distribution.

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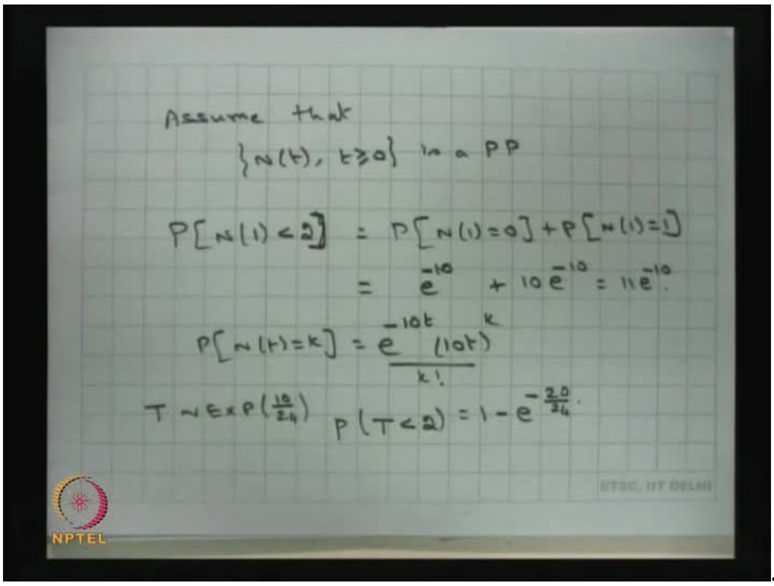

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So for the first question, you can assume that you can assume that the n of t is nothing but, the number of number of insurance car insurance claims reported to the insurer, that as the Poisson process. We can assume that n of t is the Poisson process based on the assumption given in the problem.

Once you assume, that this is the Poisson process the question is what is the probability that there 0 to 2 days itself, because of the increments are stationary so, the question is nothing but, what is probability that n of 1 is less than 2 in a given day a day, so what is probability that n of 1 is less than 2? That is nothing but… What is probability that n of 1 equal to 0 or n of 1 equal to 1? therefore, the probability is added so you substitute since, n of t is the Poisson process the probability mass function of n of t is equal to k, that is e power minus lambda. Here the lambda is the 10 per day 10 times t and 10 times t power k by k factorial.

So this is the probability mass function, for the random variable n of t for x to t therefore, n of 0 n of is equal to 0, that is nothing but, e power minus 10, here the t is one day plus n of 1 is equal to 1, you substitute here therefore, you will get 10 times e power minus 10, so the answer is the 11 times e power minus 10 numerically, you can get what is a value?

So the probability that, there are two claims a reported on a given day is a 11 times e power minus 10, the second question what is the probability that time until the next reported claims is less then 2 hours? So, this is equivalent of the next reporting claims, is less than 2 hours that means the residual time of the next claim, that is going to happen the one claim is going to happen less than 2 hours.

That means, you can use the inter arrival time that is the exponential distribution with the parameter lambda, here the lambda is 10 or 10 by 2 hours therefore, you should convert the values, it is a 10 divided by 24 claim can happen at any day throughout towards 24 hours therefore, 10 per day therefore, it is 10 divided by 24 per hour, so that is the exponentially distributed with the parameter 10 by 24.

Now the question is, what is the probability that time and the next report claim is less then 2 hours? that means what is the probability that t is the less than 2? that is nothing but, that is nothing but, since it is exponential distribution and you know the cdf of a the random variable t. So the probability of t is less than 2 is nothing but, 1 minus e power minus 2 so 2 times so 20 by 24, so the answer is 1 minus e power minus 20 by 24 , that is the probability that the next report the claim is going to be occur before 2 hours.

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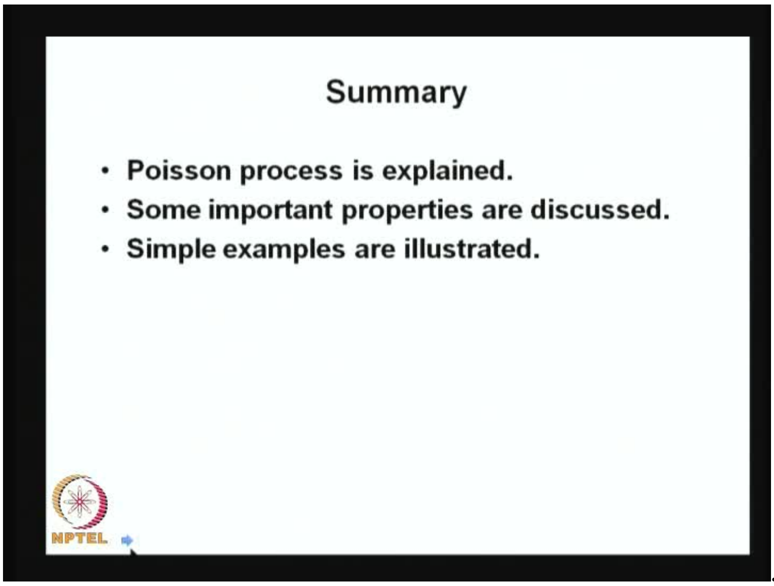

So, with this I have completed two examples also. In this lecture we have discussed Poisson process and we have illustrated two examples; for the Poisson process also, some important properties also discussed in this. The next class, I am going to discuss the applications of ctmc in queuing models. these are all the reference books. Thanks.