Stochastic Processes
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Module - 1
Probability Theory Refresher
Lecture - 1
Introduction to Stochastic Processes

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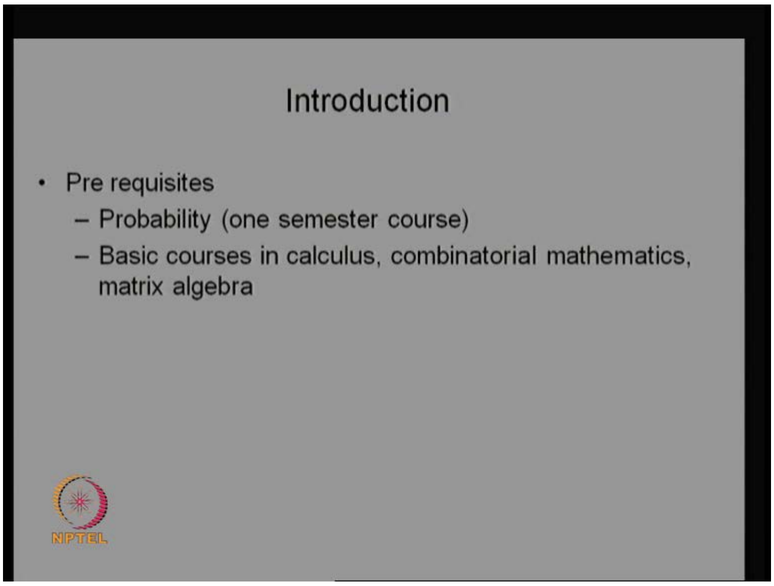

Our lecture is stochastic processes. This course needs a pre requisite of a probability as a full one semester course. So, most of the universities they have a course probability theory along with the stochastic processes or random processes or probability and statistics. So, whatever the courses we have at least some 30 lectures of probability theories needed for this stochastic processes course as a pre requisite. Other than probability course we need basic course in calculus and some mathematical background over the combinatorial problems and also the matrix algebra. So, these courses would have been covered in the maths 1 or mathematics 2 courses. So, that is enough for to understand the stochastic process course. So, what we are saying is we need pre requisite as the probability theory as well as the maths, mathematics 1 and mathematics 2 course is enough to do the or to understand the stochastic process course.

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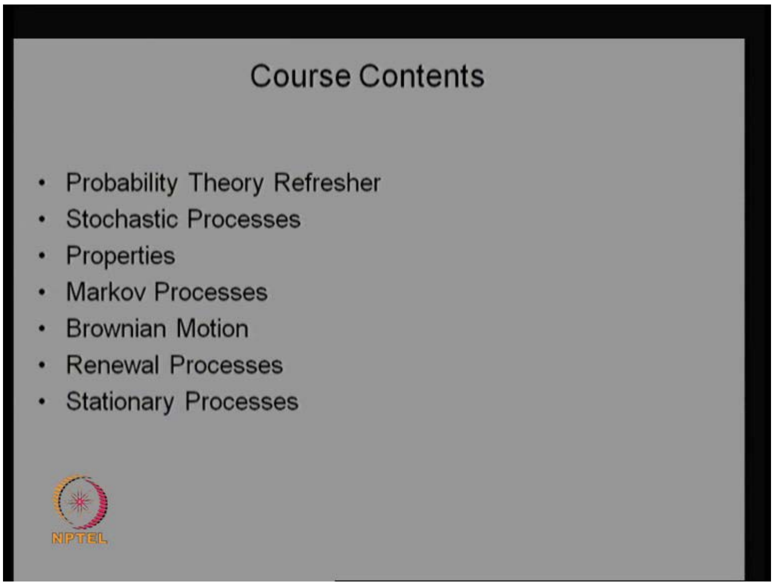

The whole lecture series, I am planning to give for 40 lectures. In the 40 lectures are covered by this course content briefly and I am planning to give a lecture for 2 hours a probability theory refresher. And then I will go for defining stochastic processes, and giving many or few a simple stochastic processes, then followed by that properties of the stochastic processes like Markov property, Martingale property, and so on. After that we will discuss a very important stochastic processes, Markov processes. In that we are going to study that two types of Markov processes have a discrete time Markov chain as well as continuous time Markov chain. After that, we are going to study Brownian motion, and finally the renewal processes and the stationary processes. So, with that we are going to complete the stochastic processes of about 40 lectures.

And before we move into the stochastic processes; I am going to give what is a motivation behind the stochastic processes. When you see the few last decade's problems, more of the probability models are not the deterministic; that means, you need more probability theory to understand the stochastic to understand the system then only you can study the dynamics of the model. If you see the, if you want to study the dynamics of the system then you need a more probability theory.

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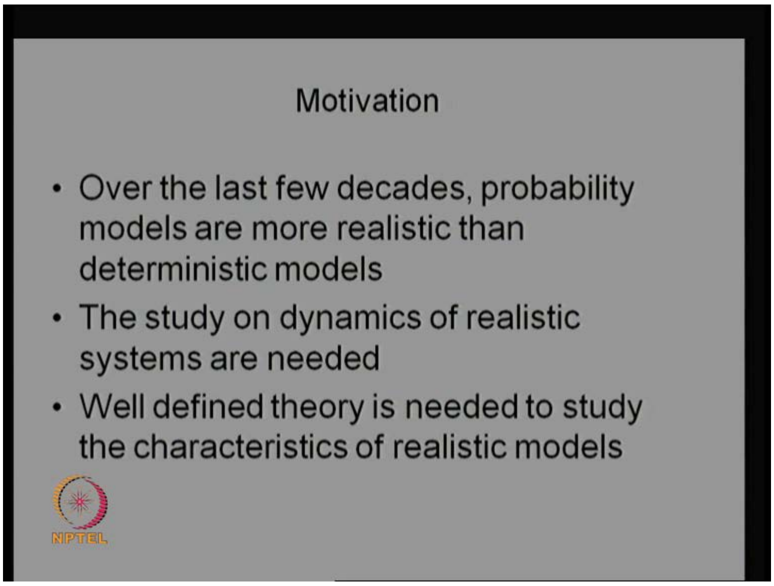

So, the simple probability theory may not be enough to study the more, more study on the realistic systems. The way the realistic system behaves in a very dynamical way. It is not easy to capture everything through the probabilistic or usual probability models. That means, you need more than the probability models probability theory to understand the systems or to study the system in a well behaved way for that the one of the important thing is stochastic processes. It deals about the collection of a random variables.

So, that you can study the dynamics of the system in a better way. Even though I am giving very light way of saying the collection of random variables. First we should know how the random variable can be defined. So, that we can study the collection of random variable in a better way. So, for that we are going to spend few examples through that how the more realistic models needs a more probability theory other than usual probability theory. So, that the stochastic processes definition and those things I am going to cover it later part.

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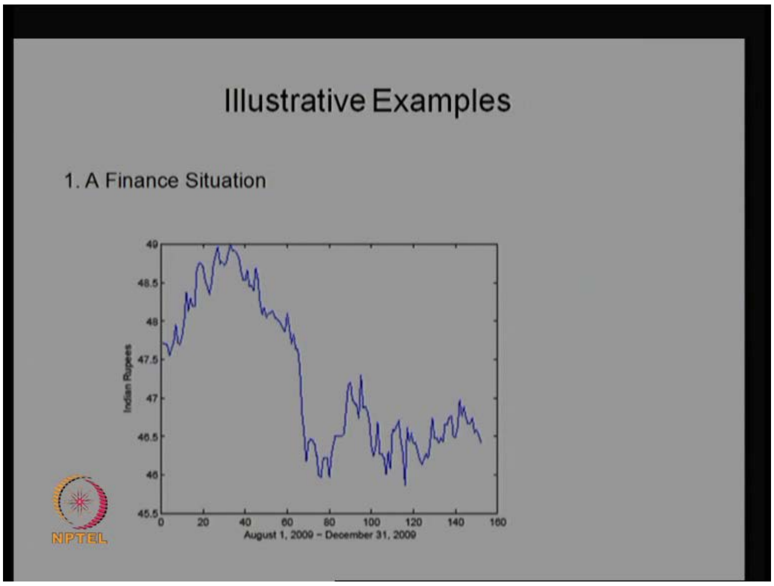

First let us see, the first example that comes in the finance situation. This is the actual data which captured over the period of time from August 1, 2009 to December 31, 2009 of what is the current price of the 1 US dollars in Indian rupees. So, if you see the graph you can make out August 1, 2009, the price of 1 US dollar was 47 rupees 57 or 58 paisa. And if you see the dynamics over the years, over the days from August 1, 2009 to till December 31, 2009, it keep on changing and it takes some values higher and after that it goes down and it fluctuates and so on.

So, this the actual data which captured from the which will we have captured, and from that our interest will be, what could be the US dollar price after some time. If I know till today what is the price? My interest will be what could be the price after 1 or 2 days or after 1 month or after 6 months; that means, I should know how the dynamics keep moving over the days, and what is the hidden probabilistic distribution is a capturing over the time. So, that I can identify what is the distribution behind that, therefore I can study the future prediction. I can study the dynamics of the this particular model in much better way.

That means, I need what is the background or what is the hidden distribution playing or hidden distribution which causes the dynamics of the system. After identifying, what is the distribution? My interest could be what could be the some other moment over the time; that means, what could be the average value or what could be the second order moment if it exists and so on, that can be obtain if I know the actual distribution the in the underlying model.

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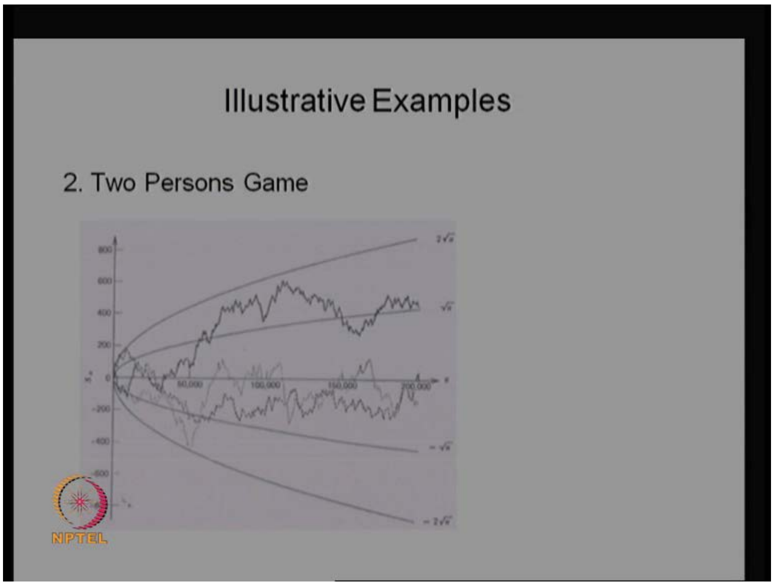

If I see the second example, I am just changing into another model in which there are two people playing game the person A and the person B.

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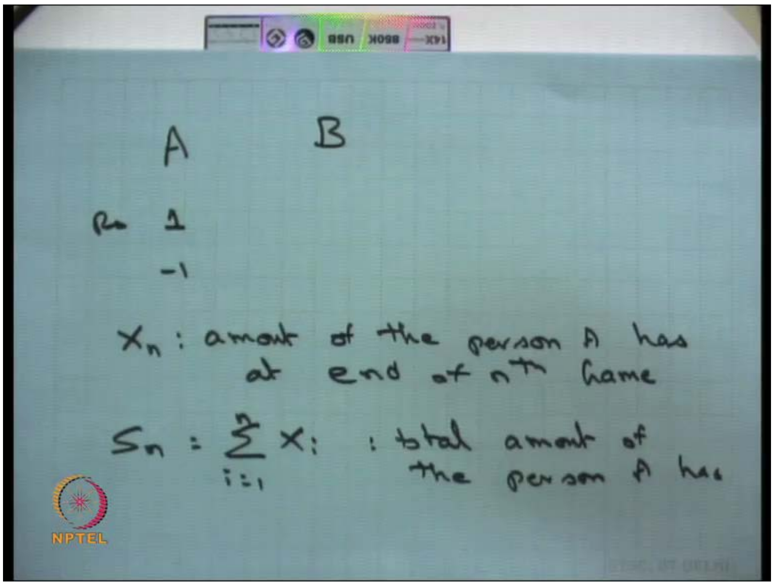

Whenever the person A wins he gets a rupee 1. Suppose the person B wins then he will get the 1 rupee and at the same time the person A loses 1 rupee the same way and the play is keep going. Suppose you make the random variable as x n is the amount of the person A has at the end of n th game. If you make out the random variable x n for the person A has the amount at the end of the nth game then at the way the game going on, the value of the x n will be keep changing. And if you make out the another random variable S n is the sum of x i where i is running from 1 to n. This gives what is the total amount the total amount of the person A. The diagram in which the S n gives what is the way the dynamics goes and over the n.

And if you see the diagram, you can make out the whole dynamics goes the how the game is going on in the first few games accordingly it changes the positive side or it goes to the negative side and if n is goes large, then the dynamics of the S n over the n. It will be keep changing over the time and you will get the realization realization of the S n over the time, and here I have given three different realization and this diagram is a taken out from the book by U. Bhat, the title of the book elements of applied stochastic processes.

So, this is one of the motivation behind the stochastic processes and from this our interest will be after the what is the distribution of S n at any n and also has n tends to infinity what could be the distribution of S n. That means, you need you need the distribution of the random variable and also you need what could be the distribution has n tends to infinity or the limiting distribution of S n. If you know the distribution then you can get all other moments has for different term as well as the asymptotic behavior of the random variable S n.

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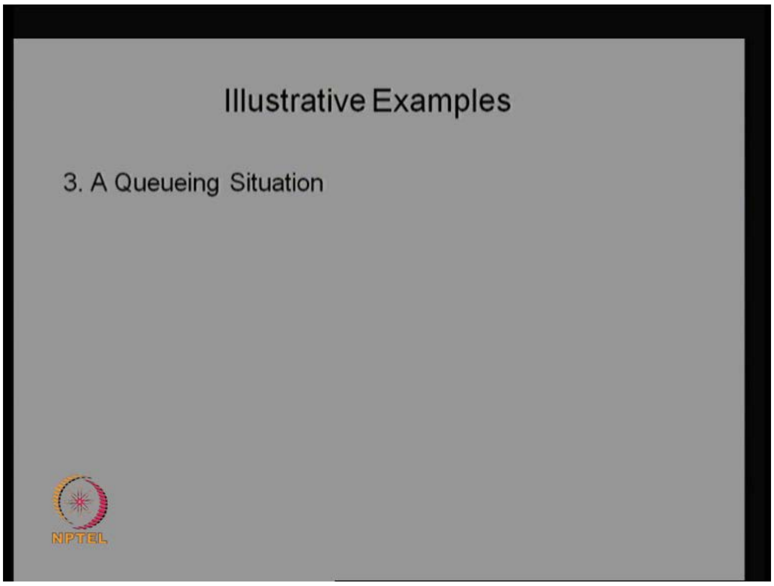

Next I will move into the another example in which it is a queuing situation.

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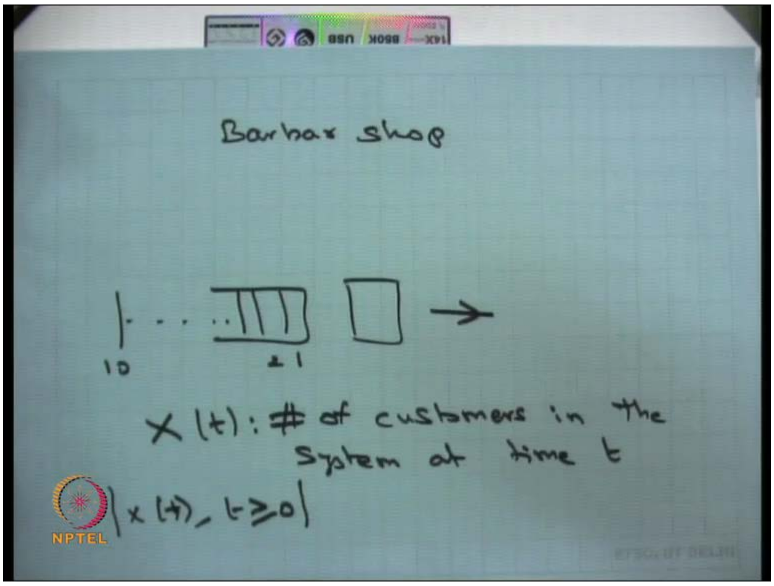

The queuing situation here I have taken it has a taken a simple example that is a Barbour shop example in which there is a only one Barbour shop a person, and who does the who does the service for the people whoever entering into the Barbour shop. And there are only a limiting capacity in which there is a maximum 10 people can stay in the Barbour shop and 1 person will be under service.

Once the service is over and the system will be the customer can leave the system. At any time maximum 10 people can be in the Barbour shop, and only one person is doing the service for the customer whoever enter into the Barbour shop. Suppose, you take the random variable has X t is the number of customers in the Barbour shop or in the system at time t, the way that dynamics goes. The possible values of X t will be starting from 0 to n. To study this system you need what is the way the people or the customers entering into the system and what is the way the service is going on for the customers and what is the discipline in which the customer are getting shout also.

Our interest will be suppose we have the capacity of 10 what could be the waiting time whenever the customers are entering into the system or if I increase the waiting time or if I increase sorry, if I increase the number of if I increase the way capacity of the system then how much I can reduce the waiting time.

So, that means, my interest will be one is how to reduce the waiting time on average. In the customers, this is the customer's point of view as the Barbour shop point of view how much I can get the more revenue. That means, how I can increase the capacity capacity of the system. So, that I can make a more profit over the time.

That means, if I know the dynamics of the X t over the t for t is varying from 0 to infinity. I can I can understand the system over the time as well as I can whatever the probabilistic measure or whatever other measures average number of the customers or average waiting time and so on. I can find out using this type of random variable. So, later we are going to say, this is going to be a one of the stochastic process for this example.

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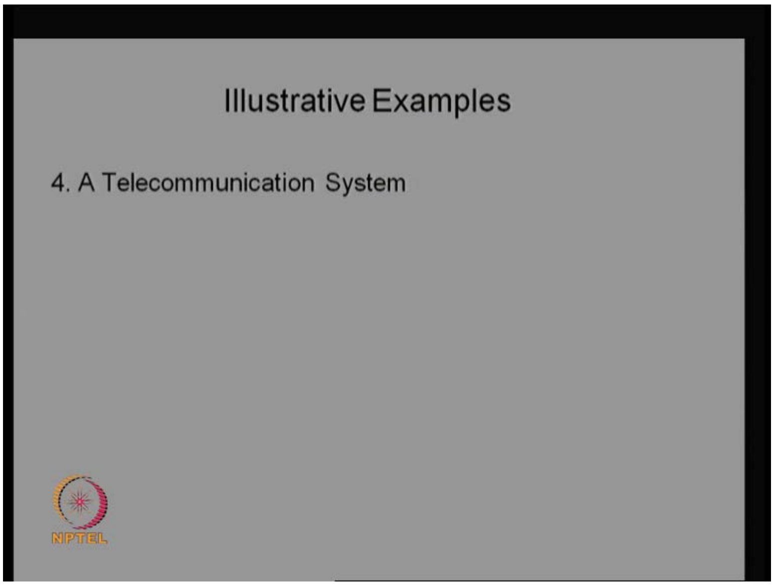

Next I am going to consider the 4 example has the telecommunication system. Suppose you think of system in which you have a n trunks are there. Trunks are nothing but it is a maximum numbers of a calls will be allowed at any time. whenever a call entering into the system and you have given 1 trunk to the call and at the end of the call is over the trunk will be a back.

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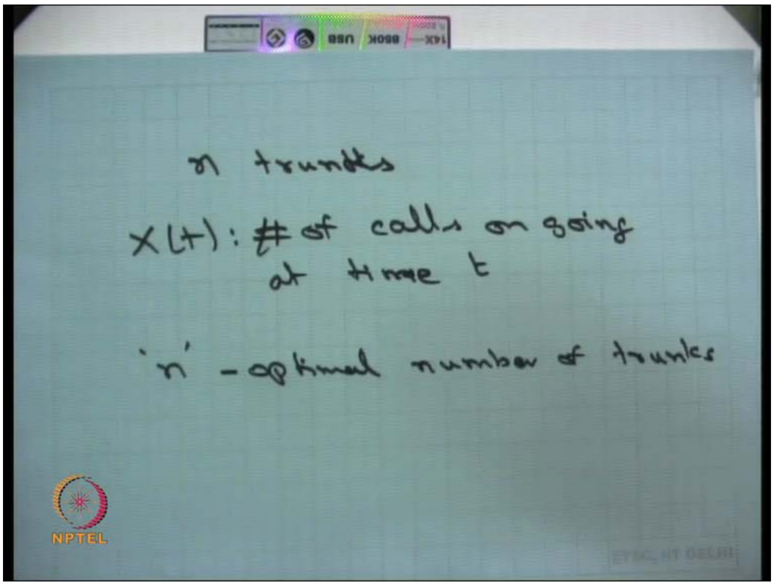

So, you have telecommunication system in which n trunks are available at any not at any time n trunks are available. Suppose I make a random variable X t has the number of calls ongoing at time t. See here also the dynamics of X t is going to be keep changing from 0 to small n over the time and my interest will be how I can how I can do the service such a way that the more calls will be entertained as well as how I can find out the optimal n. Such a way that what is the optimal optimal number of trunks such that I can minimize the waiting time or I can maximize the revenue. So, this is also one of the problems which we come across in the usual daily life and so on. So, my interest is to introduce the stochastic process. So, that I can study this type of system in a better way.

So, for that we need the probability theory in detailed. So, even though we cannot explain the whole probability theory in a complete. I am just going to give a in a I am just making a refresh a type of defining words of probability and what is the random variable and so on. And I will cover up whatever the probability theory knowledge is needed for the stochastic process that I will explain in another this lecture as well as the next lecture, and some of the in detailed probability theory concepts which will be used related that I am going to explain whenever the problem comes into the picture.

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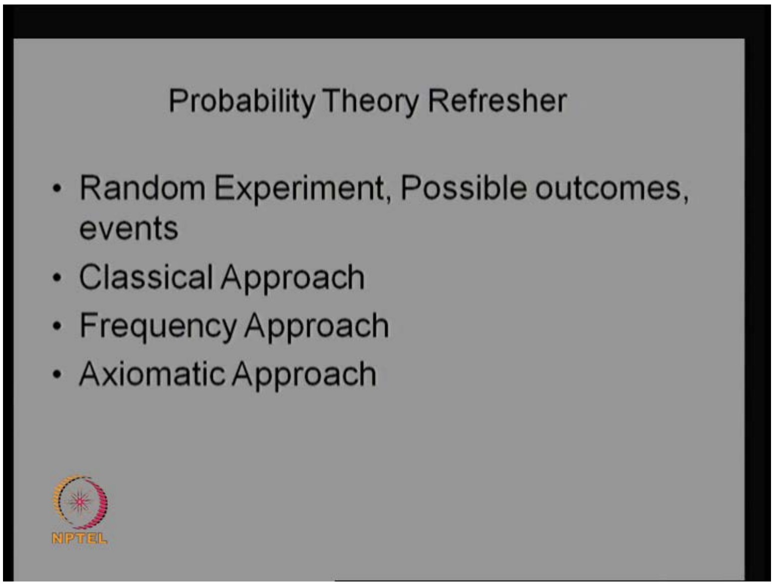

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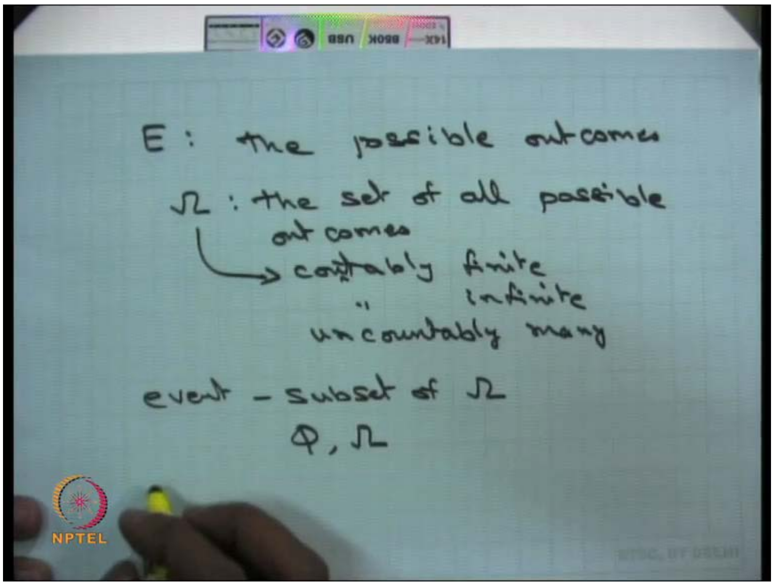

So, for that first we need, what is random experiment? The random experiment is a experiment in which you can able to list out what are all the possible outcomes can going to come if the experiment is going to actually takes place. That means, before the experiment is takes place, you can always able to list out the possible outcomes. So, the possible outcomes that I am going to make it has the collection with a word called omega. So, the omega is the set of all the set of all possible outcomes.

The outcomes could be a numerals or non numerals as well as the outcomes the set the omega could be a countably finite. It could be a countably finite or it could be countably infinite or it could be uncountably many also. So, the way you have chosen the random experiment when you start collecting the possible outcomes that is collection I am going to use the I am going to put it in the collection called omega.

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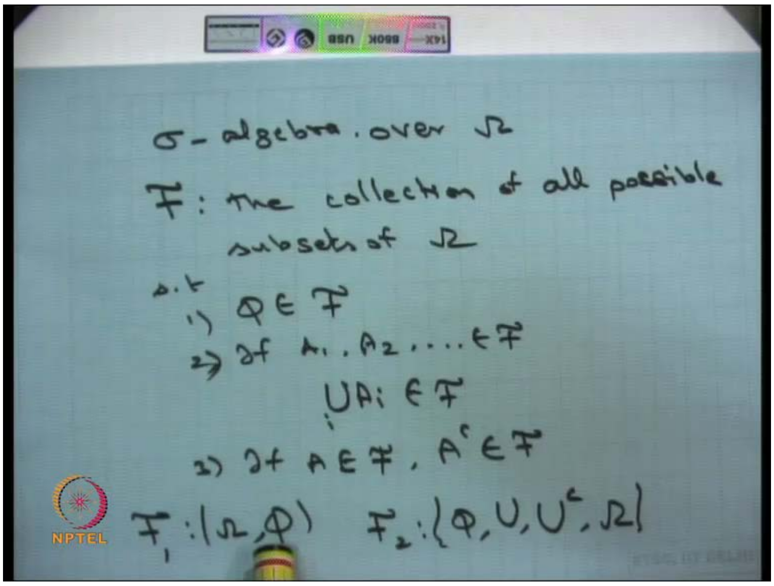

Once you have the omega then we can go for creating the event the event is nothing but the subset of omega. So, the event the possible events are starting from the empty set as well as these are all the these are all the just we can get it like that. So, the empty set as well as the omega and we can create all the possible subsets of omega that is also going to form a events. Next we are going to make a probability space. I will just take out. So, to define the probability space you need sigma algebra.

So, What is sigma algebra? what we are going to create a sigma algebra over omega. So, that I am going to use the word F. F is the sigma algebra over the omega that is the collection of all possible subsets of omega such that the empty set is belonging to F, and if I take few elements in the omega then the union of A i also belonging to F. The third condition, if I take one element from the F, then that complement is also belonging to F.

So, that means the sigma algebra over the F over the omega that F contains collection of all possible subsets of omega such that these three conditions are satisfied. That means, we can go for making the trivial F that is going to be contains only the empty set as well as whole set this is also going to be one of the sigma algebra's over the omega, that is the default one.

Like that I can go for creating many sigma algebra that by making a few elements of few elements of possible outcomes that I make it has a set A. Then I can make it the another sigma algebra that has empty set, and I can make a one set called U and U consists of few elements of omega and U complement then I can have a omega also.

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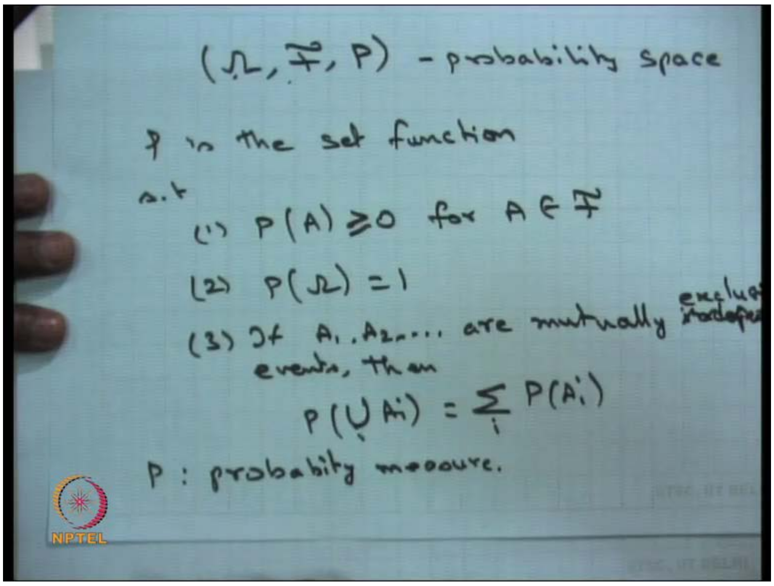

So, like that I can keep creating the different sigma algebra over omega and the trivial one is the empty set with the omega set that is going to be the trivial one and now I am going to define the probability space. What is probability space? The probability space is a triplet in which the omega is the collection of possible outcomes and F is the sigma algebra over omega and P is the set function.

Such that P is the set function such that the P of A is always going to be greater than or equal to 0 for any A belonging to F. The second condition the P of omega is going to be one always. The third condition if I take few A i are mutually independent events, then the P of union of A i is same as summation of P of A i. Let me just explain the probability space in a better way. This triplet is going to be call it has a probability space as long as you have the collection of possible outcomes and you have a sigma algebra.

So, this sigma algebra can be anything and you can go for the default one is the largest sigma algebra which you have created and P is the set function, such that whatever the element you are going to take it from F. Any elements of F is going to be event. So, the P of any event that is going to be always greater than or equal to 0 and if you take the event is going to be omega.

Therefore, the omega is also one of the element in the F and the P of omega is equal to 1 and the third condition if you take A i are mutually exclusive. Sorry, if you take A i are mutually exclusive events then the probability of union is going to be the summation of a probabilities summation of P of A i, then this P is going to be the set function and the P is going to be the probability measure. This P is going to be call it has this P is the probability measure and this P pro is a non the measure also because of the condition P of omega is equal to 1. There are many definitions over the probability theory.

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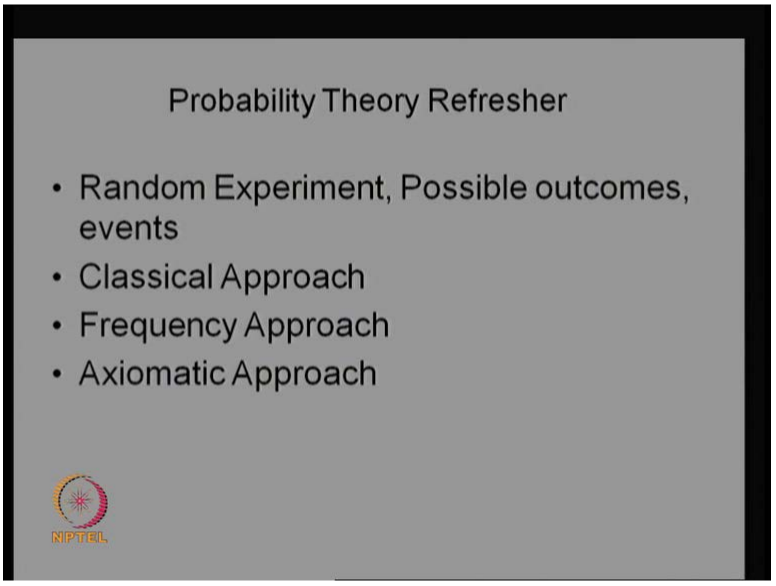

The classical approach or the frequency approach and what we have we have given is a axiomatic approach.

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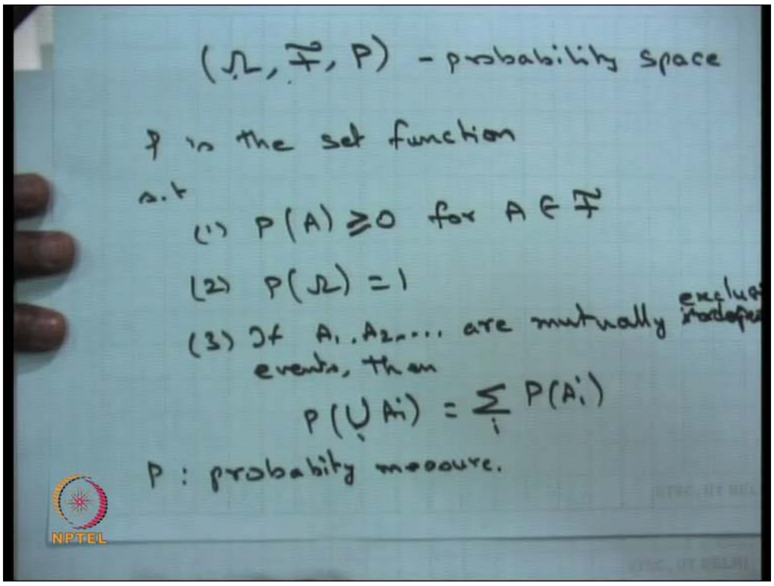

So, that of the way I have given the definition that is the probability space with the omega F and P, and this is called the axiomatic approach and we are going to use axiomatic approach not the frequency approach or the classical approach. And you should note that the classical approach is going to be the special case of the axiomatic approach in which you make the collection of possible outcomes are going to be equally likely, then the classical approach is going to be the special case of the axiomatic approach. Therefore, throughout our course we are going to use the axiomatic approach not the classical approach.

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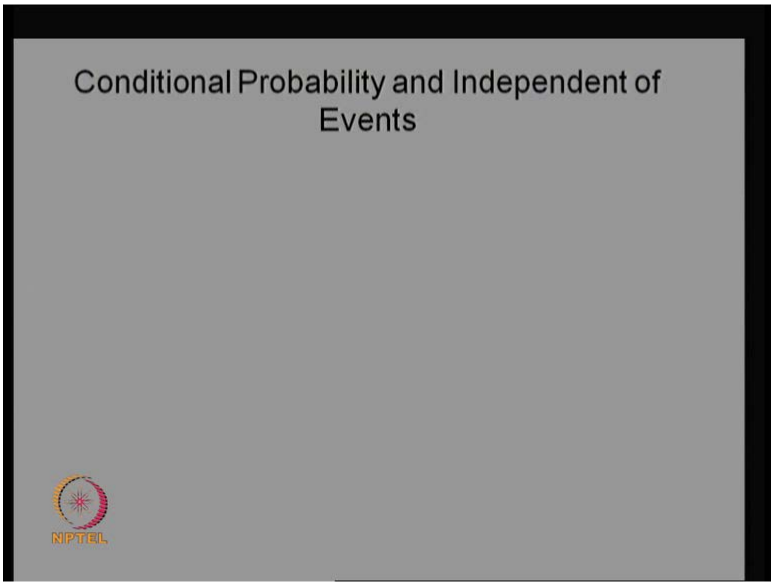

And next I am moving into the concept called conditional probability.

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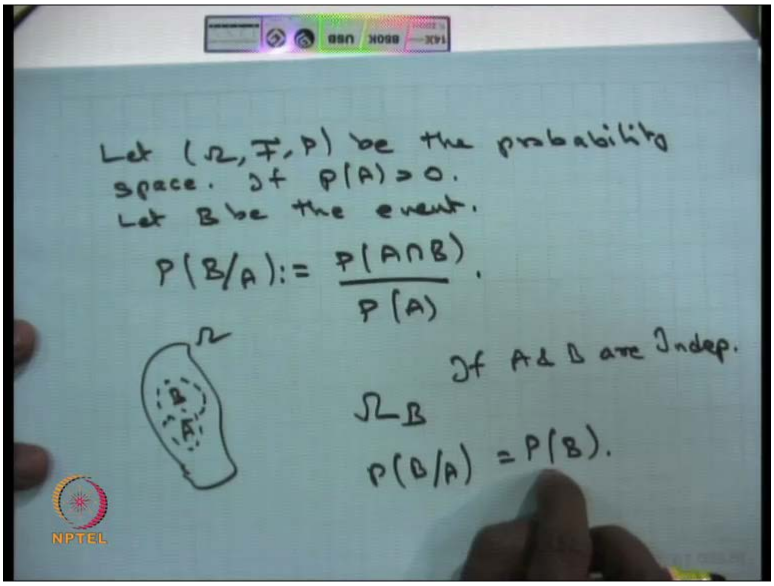

So, suppose you have a probability space be the probability space if you take if P of A greater than or equal to 0 then let B be the event. You can define the probability of B given A is same as the probability of probability of A intersection B divided by probability of A. That means, if already the event A occurs with the positive probability then you can find out what is the probability of the event B given that already the event

A occurs that is same as what is the probability that A intersection B divided by probability of A.

So, these can be this is by the definition and this can be visualized from the reduced sample space also; that means, you have a sample space omega and from the omega you take a event A. Suppose this is going to be the event A and what you are saying is the event A is already occurred; that means, with this given condition and suppose you make another event, that is event B and you are asking what is the what is the proportion in which already the event A occur. And you are asking what is the probability of event B, that means; you find out what is the reduced sample space omega B, and you find out what is the proportion in which or what is the probability of event B occurs in the reduced sample space is same has by using the definition of probability B given A. That means, you find out what is the intersection A intersection B; that means, you find out what is the event which corresponds to A intersection B, and what is the ratio in which probability of A intersection B with the probability of that gives the conditional probability.

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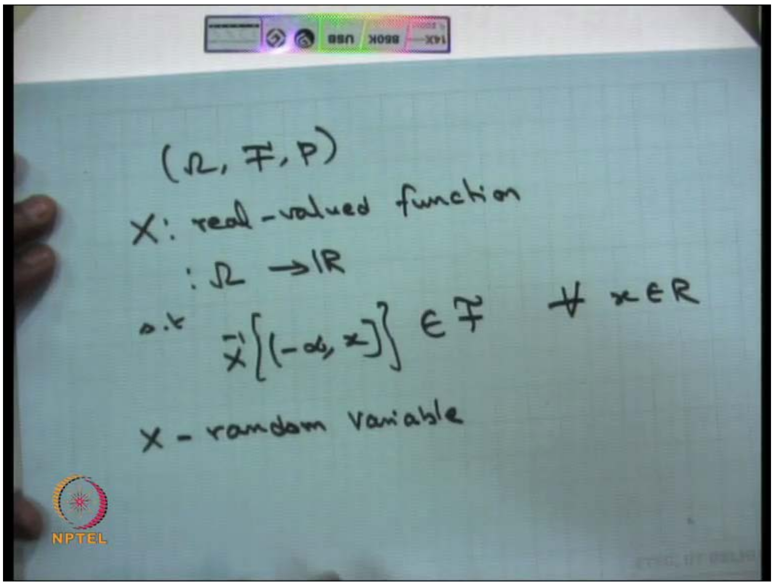

If the event A and B are independent, if A and B are independent event, then there is no way of relating the probability of B given A then the probability of B given A is same as probability of A probability of B, sorry. That means, there is no dependency over, there is no dependency over the event B and A, therefore it is not going to cause anything with the event B by occurring the event A. Therefore, the probability of B is same as the probability of A intersection, sorry probability of B intersection A.

Let me go to the next one. Sorry, now you are moving into the next concept that is called a random variable; that means, you have a probability space started with the probability space. And if you are defining a real valued function which maps omega to R, such that if you find out the inverse image of any x in the real line that inverse image between minus infinity to x belonging to F. If this condition is satisfied by any real valued function which maps from omega to R then that is going to be called it has a random variable.

That means after you have collection of possible outcomes you are finding one sigma algebra you can make a more than one sigma algebra over the omega. So, you have one fixed sigma algebra it could be trivial one or the non trivial one and so on. So, you have a fixed F. After fixing the F, you have a probability measure and the probability measure is nothing to do with the random variable at all. Still you have a probability space and the from the probability space, you are defining the real valued function.

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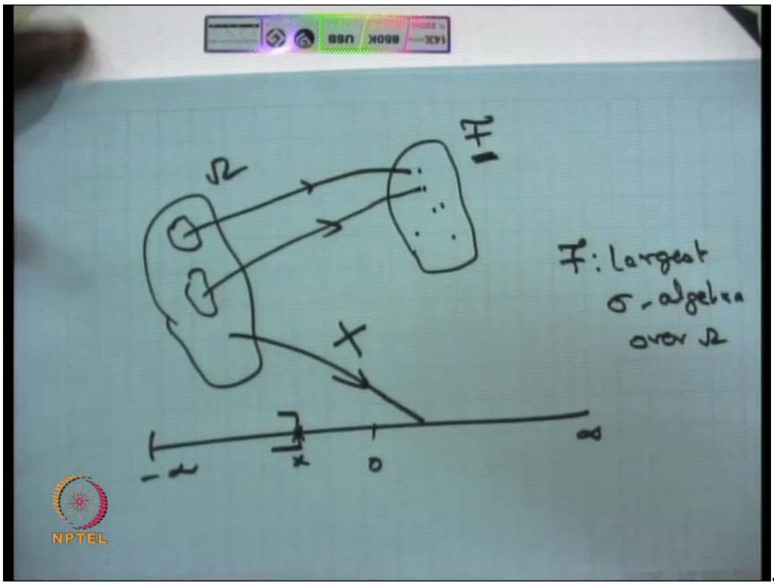

Such that the inverse images belonging to F that I can make out the simple diagram. This is the omega and from the omega you have created the F. F means it has the events and the events are nothing but the few possible outcomes; that means, this possible outcomes you will land up with the one element and this possible outcomes you will land up with the another event and so on. So, like that the different few possible outcomes that is going to be one of the elements in the F. You have created a another real valued function that is X from omega to R is a real valued function and you take any point some x in the real line, and if you find out what is the inverse image from minus infinity to till x. You collect what is the inverse image you got it under the mapping x from minus infinity to the closed interval x. You collect the all the possible outcomes that is going to give the value between minus infinity to closed interval x. You collect such a possible outcomes.

If you collect such a possible outcomes and that is going to be one of the possible elements in the F for the different values of x, then the real valued function is going to be call it has a random variable; that means, once you know the F, if you created the real valued function. After checking that condition, you can conclude that the real valued function is going to be a random variable; that means, if you have some other F there is possibility some real valued function may not be the random variable. That means, how will you choose F that is going to play a role of a come to the conclusion the real valued function is going to be a random variable or not.

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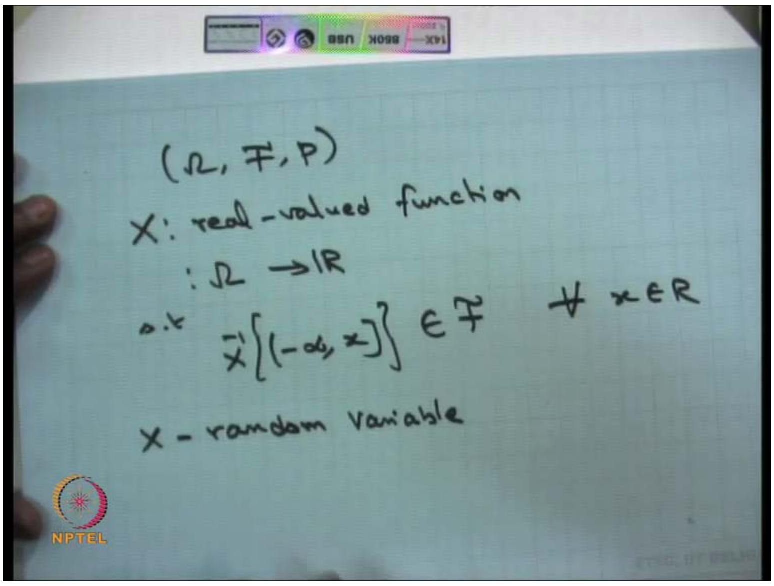

If you take the F is going to be the largest one. The largest sigma algebra over F over omega then any real valued function is going to satisfies this property. Suppose you take the sigma algebra over omega which is in between the trivial one and the largest one then the few real valued function may be a random variable, and few other real valued function may not be the random variable. So, in the usual scenario whenever you see the random variable definition in many books. They use real valued function is going to be a random variable just like that.

That means they did not they have taken the F is going to be the largest sigma algebra. So, whenever F is going to be the largest sigma algebra then any real valued function is going to be a random variable and going back to the the previous slide. This condition is going to be the, if and only if condition is also. Suppose you have a real valued function is going to be a random variable then this condition will be satisfied, and if this condition is satisfied then that real valued function is going to be a random variable also.

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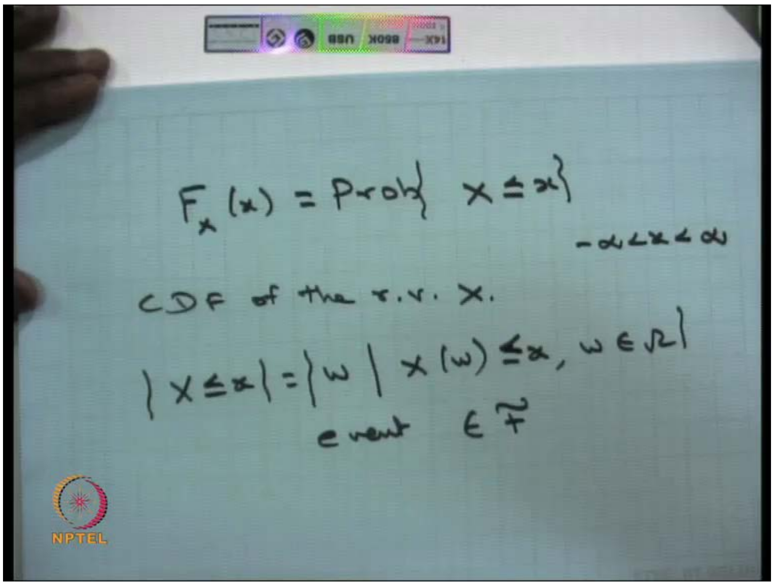

Now, we are moving into next concept called cumulative distribution function. So, the cumulative distribution function for the random variable x can be defined has a capital F suffix x is for the random variable x, and the small x is the variable x; that is going to be probability of X is less than or equal to small x, and here the x lies between minus infinity to infinity. So, this is going to be call it has a CDF of the random variable x. So, the way I relate with the probability of X is less than or equal to x this X. X is less than or less than or equal to x is nothing but you collect few possible outcomes such that under the operation X of w that gives the value less than or equal to x for all w belonging to omega. That means, you collect the few possible outcomes w such that under the mapping x X of w should give the value maximum x. So, that is less than or equal to therefore, this is nothing but a event.

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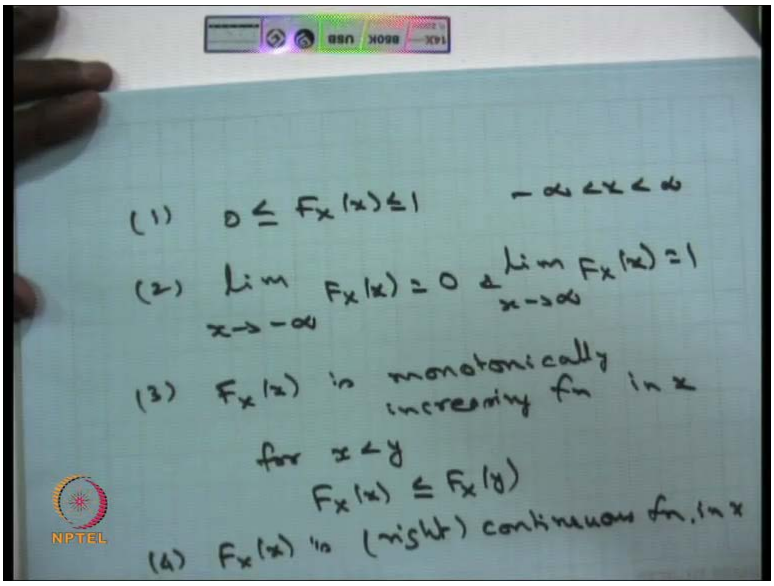

And this event is belonging to the capital F. Therefore, the way you taken the probability of X is lesser than or equal to the x; therefore, has the x moves from minus infinity to infinity. You are keep on including some more possible outcomes over the x, therefore the probability of X is less than or equal to x varies over the x you are going to get more probability values therefore, this F of x is going to satisfied few properties. So, if you see the properties of the F of x this values will always lies between 0 to 1 for all x.

Suppose you take x is almost minus infinity then that is a going to be 0 and if it is a towards in the infinity, then it is going to be 1; that means, I make out a limit x tends to minus infinity this is going to be 0 and the limit x tends to infinity the F of x is going to be 1. The third property the way we are keep on accumulating the possible outcomes and trying to find out the probability and that we make it has a F of x, therefore the F of x is a monotonically increasing function in x. That means over the x, if you take two values x is less than or equal to y, then the F of x value will be less than or equal to F of f x y; that means, as a x is less than y either it takes as same value or greater than value. Therefore, it is going to be a in the way it is called a monotonically increasing function in x. The fourth one it is going to be right continuous function in x; that means, either which is going to be continuous function. If it is not a continuous function, it is a right continuous.

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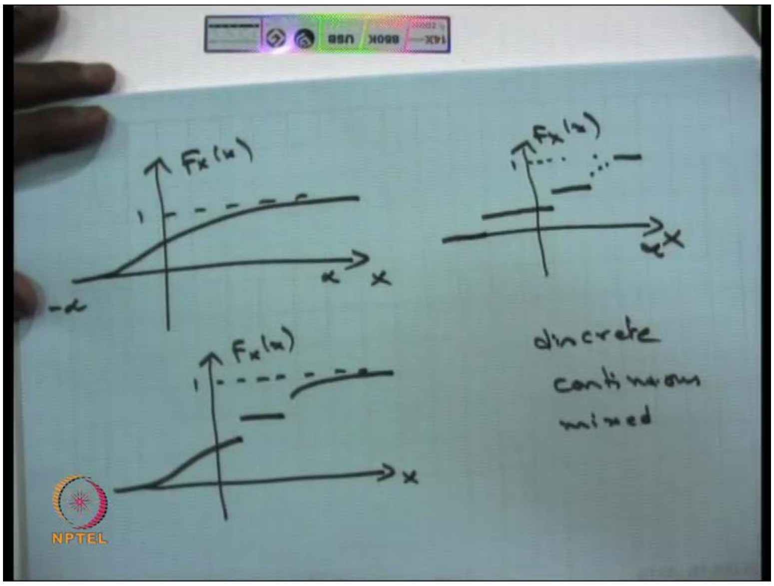

That means; the left limit exists for any x as well as the right limit exists and either it is going to be a left limit same as a right limit and value defined at that point or the left limit is different from the value defined that point which is equal to the right value. Therefore, the function is going to be call it as a right continuous. So, the CDF is going to be a continuous function or it is going to be a right continuous function. I can show few diagrams of the CDF has the x goes the F of x will start from 0 and land up 1.

So, this is going to be a F(x) continuous as well as it satisfies the condition of minus infinity to infinity. It is going to be 0 minus infinity to 0 infinity is 1 and it is a monotonically increasing and continuous function and I can give the another example of CDF. It is start from 0 and it has a discontinuity, so that means it has the it is a it is a right continuous function and monotonically increasing function, and it has the countably infinite jumps or countably infinite discontinuity and it reaches at infinity 1.

So, based on the way CDF goes I can give one more example, in which this is going to be continuous in some then it has jumps that is also possible. So, the way the CDF is going to be a continuous function from minus infinity to infinity or the CDF is going to have a countably finite jumps or countably infinite jumps or it has a both type, then you can classify the random variable has a discrete random variable, continuous random variable or mixed type random variable. So, the random variable is going to be call it has a discrete type random variable.

If the CDF is going to be have a countably finite or countably infinite jumps in the CDF then it is called the discrete random variable, If any random variable has a CDF has the continuous function from minus infinity to infinity then that random variable is call it has a continuous random variable. If any random variable is CDF has both continuous between some interval, and countably finite or countably infinite jumps in some interval then that random variable is going to be call it as a mixed type random variable.

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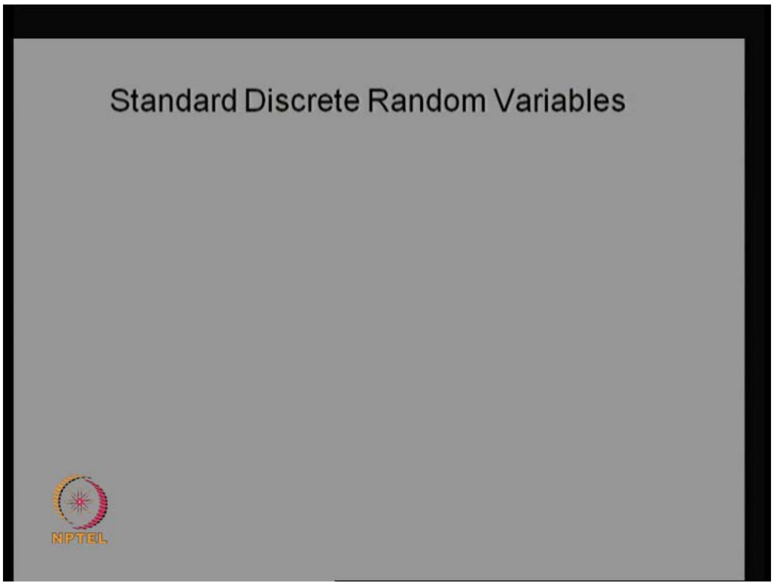

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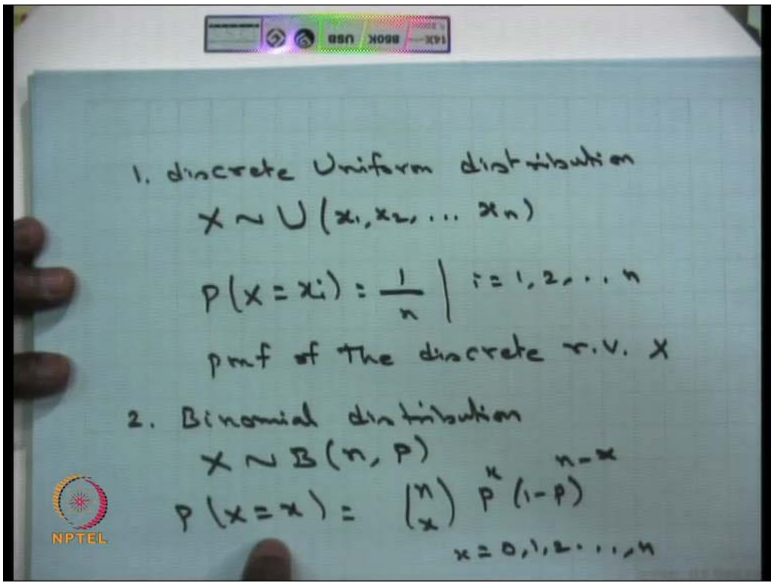

So, here I am going to list out few standard discrete and continuous random variable. So, these are all the standard one; we are going to use it in a our course. So, the first 1 is a discrete uniform distribution or the random variable is a discrete uniform distributed random variable. Suppose I make the random variable X is uniformly discrete uniformly distributed with the with the discrete points x n. That means, the random variable takes the possible values x 1 to x n, and it has the masses at the x i of equal mass for i is vary from 1 to n and all otherwise it is going to be 0.

Then in that case we say the random variable is going to be call it has discrete uniform distribution; that means, it is going to be satisfies the property. The summation of all the x i are going to be 1 and the probability of X is equal to x i is going to be greater than or equal to 0; that means, for these x i it is going to be greater than 0 and all other points it is going to be 0.

Therefore it satisfying the probability mass function of the discrete random variable therefore, this is the probability mass function of the random variable of the discrete random variable x. So, the P of probability of X equal to x i is going to be the probability mass function of the discrete random variable x n, the second one the discrete case that is binomial distribution. When we say the random variable x is going to be call it has a binomial distributed with the parameters n and p then the probability mass function for the random variable is going to be n c x P power x 1 minus p power n minus x where x takes the value from 0 1 2 and so on.

That means; this is the probability mass function of the binomial distribution. It takes the value 0 to n; that means, it has a jump points n plus 1 points. It has a jump points n plus 1 jump points and this we call it has a binomial distribution. If you put n is equal to 1 then that is going to be the Bernoulli distribution random variable, and here the p is nothing but the probability of success in each trial, and you can create the binomials trial by having a n independent Bernoulli trials and each trial the probability of success is going to be p.

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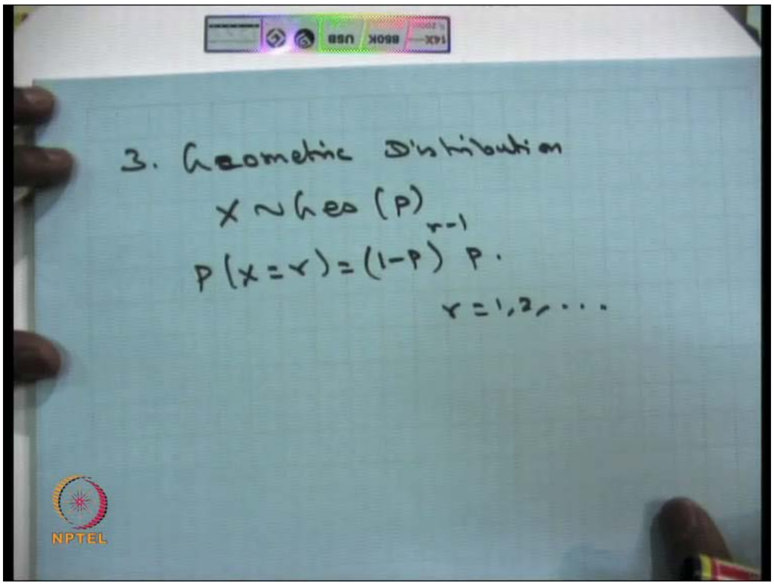

The third discrete random variable which we are going to use that is a geometric distribution, when we say random variable x is geometrically distributed with the parameter p. Then probability mass function of this random variable is going to be 1 minus p power r minus 1 into p, where r can take the value from 1 2 and so on; that means, if you have a any discrete random variable and that random variable probability mass function is going to be of this form. Then we say that random variable is geometrically distributed with the parameter p and here the p can be treated as the probability of success in each trial, and you can say what is the probability that the r th trial getting the first success that is same has all the trials are independent.

Therefore, you have a r minus 1 trials you have the success subsequently the failures subsequently and you get the success first time in the r th trial. Therefore, you land up 1 minus p power r minus 1 for all such failures all such non success r minus 1 trials and the first success if the r th trial.

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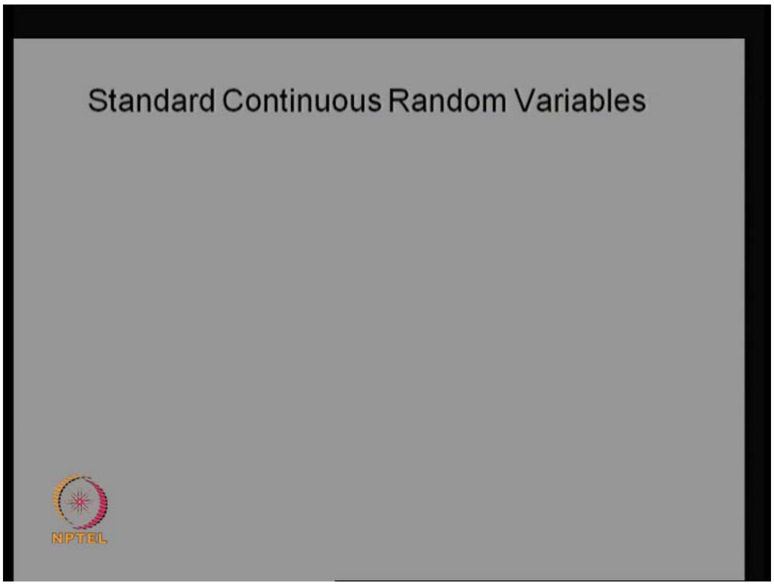

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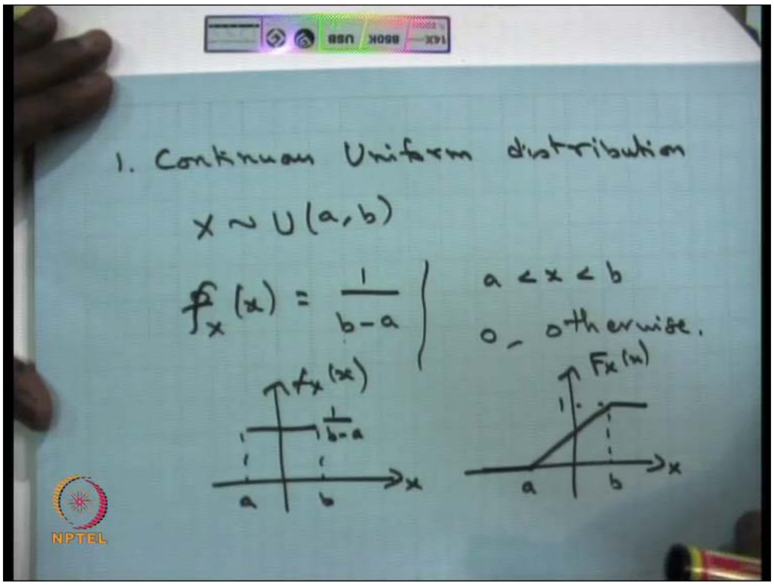

Next we are moving into the discrete continuous random variables. The first one is continuous uniform distribution, when we say the random variable x is continuous uniform distribution between the interval a to b, then the probability density function for the random variable x is going to be of the form 1 divided b minus a between the interval a to b, and all other is going to be 0. That means the probability density function for this random variable is have the height a, and if you treated has the b, and this height is 1 divided by b minus a. That means, if you find out the integration between the range a to b of height 1 divided b minus a then that is going to be 1 and this is going to be greater than or equal to 0 always.

Therefore, this is going to be the probability density function of the continuous random variable, and for any continuous random variable the probability density function is going to be 1 divided by length of the interval in which takes the value 1 divided this much, and all other it is 0, then that random variable is going to be call it has a continuous uniform distribution between the interval a to b. And if you see the CDF of this random variable till a is going to be 0, and after a it is going to be increasing and at the point b it reaches 1.

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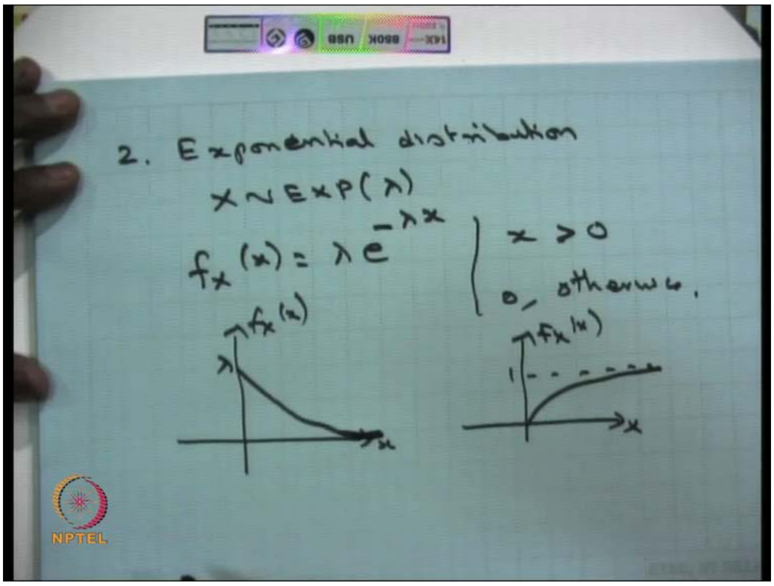

That means, you come to the conclusion if any random variable CDF is going to be between 0 to 1, and in the interval a to b with the standing line then we come out. What is the point in which in a and b, and you can come to the find out what is the random variable in which it is going to be continuous, it is going to be a uniform distribution between the interval a to b. The second one is a exponential distribution when we say the continuous random variable x is going to be exponentially distributed with the parameter lambda.

If the probability density function for that the random variables is going to be lambda times e power minus lambda x and between the x is going to be greater than 0 or if it is going it is going to be 0, otherwise where lambda is greater than… That means, within the range of 0 to infinity in the f of x is going to be lambda times e power minus lambda x otherwise it is going to be 0.

So, if you see the probability density function of that continuous random variable it is going to start from lambda and asymptotically it touches 0. So, this the probability density function of the exponential distribution and if you see the CDF of this it reaches 1 at infinity. So, this exponential distribution is going to be used in many of our problems later, therefore all the properties of the exponential distribution that I will discuss when we come up when we discuss the stochastic processes in detailed.

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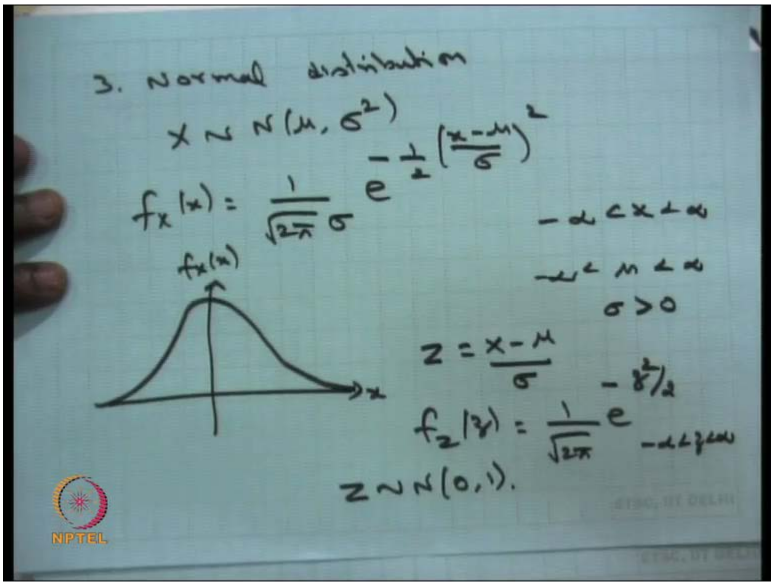

The third distribution is a normal distribution or Gaussian distribution. So, when we say the random variable is normally distributed with the parameters mu, and sigma squared the probability density function is going to be 1 divided by square root of 2 pi sigma e power minus half times x minus mu by sigma whole squared. Here the x can lie between minus infinity to infinity and the mu also can lie between minus infinity to infinity and the sigma is a strictly positive quantity, and the mu is nothing but the mean of the normal distribution. And the sigma squared is the variance of the normal distribution, and the sigma is the standard deviation and the standard deviation is always strictly greater than 0. And if you see the probability density function of f of x asymptotically it start. So, I made it with mu is equal to 0, and this is the probability density. So, it looks like a bell shape.

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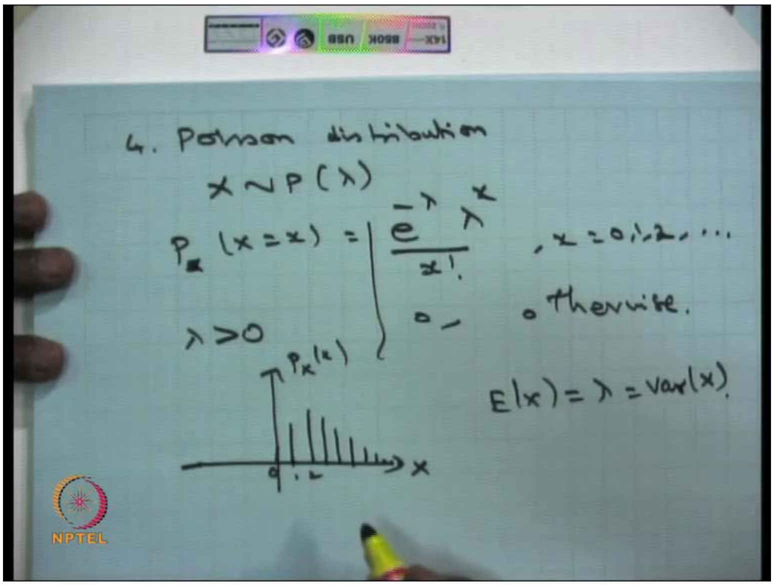

So, this is going to be a normal distribution, and you can always convert the normal distribution into the standard normal by using this substitution z is equal to x minus mu by sigma. So, you land up with the standard normal 1 that is 1 divided by square root of 2 pi e power minus z squared by 2, where z lies between minus infinity to infinity. So, this is going to be a standard normal distribution in which the mean is 0 and the variance is 1. So, other than discrete standard distributions we have discuss only the discrete uniform distribution then second we discuss the binomial distribution then we discuss the geometric distribution.

The fourth one that is a very important that is a poisson distribution. When we say the discrete random variable x is going to be poisson distribution with parameter lambda, if the probability mass function for the random variable x is going to be of the form e power minus lambda lambda power x divided by x factorial, where x can take the value from 0 1 2 and so on. So, that means this is the discrete type random variable in which it has the countably infinite masses and these are all the jump points, and the masses are going to be e power minus lambda lambda power x divided by x factorial here the lambda is strictly greater than 0.

That means if any discrete random variable has the probability mass function of this form, then we can say that that random variable is poisson distributed with the parameter lambda, and if you see the probability mass function for the different values of x. So, whatever the lambda you have chosen. So, accordingly it is going to be at 0 it has some value and 1 it has a some other value and 2 and so on.

So, that means for fixed lambda you can just draw the probability mass function and this is going to be have a countably infinite mass, and if you add over the 0 to infinity that is going to be 1 and the masses are going to be always greater than 0, and all other points it is going to be 0 and this is going to be the very important distribution. Because using this we are going to create a one stochastic processes that is going to be call it has a poisson process; that means, in the poisson process the each random variable is going to be poisson distributed.

So, for that you should know what is the probability mass function of the poison distribution, and the properties and here the lambda is same as if you find out the mean for this poisson distribution. The mean is going to be lambda and the variance is also going to be lambda. So, this is a one particular distribution in which the mean variance is going to be same has parameter lambda.

So, in today's lecture what we have covered introduction of stochastic process by giving the motivation motivation of the by giving 4 different examples to motivate the stochastic process, then what we have covered is what is the probability theory knowledge is needed in that I have covered only the probability space, and the random variable and the discrete standard random variables as well as discrete standard continuous random variable.

There are some more standard discrete random variables as well as there are some more discrete, there are some more standard continuous random variable that I have not covered here, because it is a probability theory refresher. And some of the distribution if it is needed, then we will be covered at the time of the when we explain the stochastic processes itself.

Therefore giving with few discrete random variables and few standard continuous random variable I complete the today's lecture, and the next lecture I will cover some of the other probability theory concepts needed for the stochastic process, that I will cover it in the next lecture. Then third lecture onwards I will start the stochastic processes.

Thank you.