

**Introduction to Point Set Topology, (Part I)**  
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**Module - 44**  
**Lecture - 44**  
**Productive Properties - Continued**

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Module-44 Productive Properties -Continued

Proposition 3.98

Let  $x \in X_J$ .  $X_J$  is  $l$ -countable at  $x$  iff

(a)  $X_j$  is first countable at  $x_j$  for all  $j \in J$  and

(b) The subset

$$s(x) = \{j \in J : x_j \text{ is not a Sierpinski point in } X_j\}$$

of  $J$  is countable.

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Welcome to module 44 on Point Set Topology course. So, today, we will continue our study of properties which are preserved under products, Productive Properties. So, let us begin with this proposition.

You start with an arbitrary product of topological spaces  $X_j$ , pick up a point in it. So,  $X_J$  will be first countable at that point if and only if each factor  $X_j$  is first countable at  $x_j$  for all  $j \in J$ . So, all coordinate points are having a countable base that is the first condition.

Second condition is that the subset  $s(x)$  of the indexing set all  $j$  belong to  $J$  such that this  $x_j$  is not a Sierpinski point in  $X_j$ . Look at all such indices. That set  $s(x)$  must be countable. Then the other part is if and only if. So, then the converse is also true. This is what the

proposition says. These are somewhat not very straightforward so, let us go through the proof carefully ok.

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$X_2$

$U$

$w$

$z$   $y$   $X_1$

This completes the proof of the theorem.

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**Proof:** Suppose  $X_J$  is  $I$ -countable at  $x \in X_J$ . Then (a) follows from the fact that the coordinate projections  $p_i : X_J \rightarrow X_i$  are open and surjective and  $I$ -countability is weakly co-hereditary (see remark 612).

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So, first, suppose  $X_J$  is  $I$ -countable, the product space, at the point  $x$  belonging to  $X_J$ . That means that we have a countable local base for  $X_J$  at the point  $x$ . That is the meaning of it is

first countable at  $x$ . Then, (a) follows from the fact that all the coordinate projections are open and surjective.

So, if you take  $p_i(B)$  where  $B$  ranges over the countable local base, that will give you a countable base for  $X_i$  at the point  $x_i$  ok? So, this we have seen that first countability is weakly hereditary in the sense that under open surjective maps, it is preserved. So, part (a) is proved, the second part is something peculiar ok?

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The screenshot shows a video lecture interface. At the top left is a table of contents with the following items:

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At the top right is a video of the speaker, Anant Shastri.

The main slide contains the following text:

For the converse, let  $I \subset J$  be such that  $I \subset J$  is countable and  $X_j$  is indiscrete for  $j \in I^c$ . Then for each  $x \in X_j$  we select a countable local base  $\mathcal{B}_j$  at  $x_j$  for each  $j \in I$  and consider the countable family

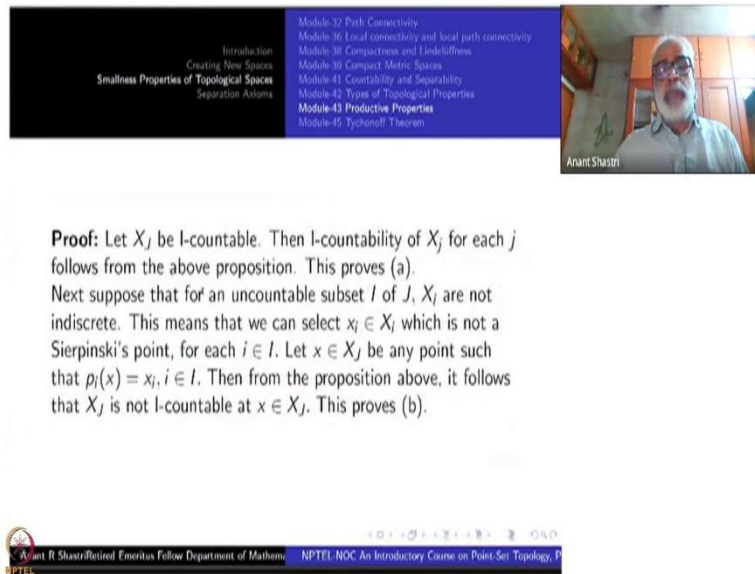
$$\{p_j^{-1}(U_j) : U_j \in \mathcal{B}_j, j \in I\}$$

and check that it forms a local subbase at  $x$ . From this it follows that there is a countable local base at each  $x \in X_j$ .

At the bottom of the slide, there is a footer with the following text:

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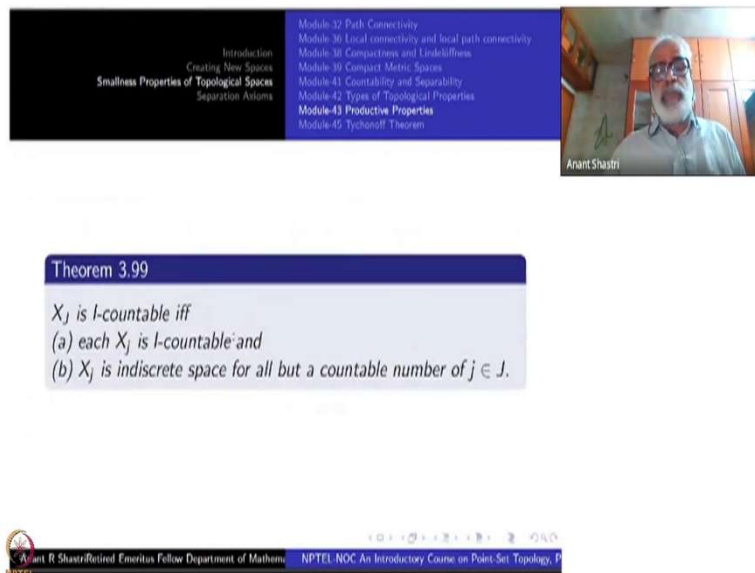
The slide displays a navigation menu on the left with the following items: Introduction, Creating New Spaces, Smallness Properties of Topological Spaces, Separation Axioms, Module 32 Path Connectivity, Module 36 Local connectivity and local path connectivity, Module 38 Compactness and Lindelöfness, Module 39 Compact Metric Spaces, Module 41 Countability and Separability, Module 42 Types of Topological Properties, Module 43 Productive Properties, and Module 45 Tychonoff Theorem. A video feed of Anant Shastri is visible in the top right corner. The main text of the slide is a proof:

**Proof:** Let  $X_j$  be l-countable. Then l-countability of  $X_j$  for each  $j$  follows from the above proposition. This proves (a).  
Next suppose that for an uncountable subset  $I$  of  $J$ ,  $X_i$  are not indiscrete. This means that we can select  $x_i \in X_i$  which is not a Sierpinski's point, for each  $i \in I$ . Let  $x \in X_j$  be any point such that  $p_i(x) = x_i, i \in I$ . Then from the proposition above, it follows that  $X_j$  is not l-countable at  $x \in X_j$ . This proves (b).

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Suppose that for some uncountable subset  $I$  of  $J$ ,  $X_i$  is not a Sierpinski's point ok? ....

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The slide displays the same navigation menu as the previous slide. A video feed of Anant Shastri is visible in the top right corner. The main text of the slide is Theorem 3.99:

**Theorem 3.99**  
 $X_J$  is l-countable iff  
(a) each  $X_j$  is l-countable and  
(b)  $X_j$  is indiscrete space for all but a countable number of  $j \in J$ .

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To prove (b): let  $\mathcal{B}$  be a countable neighbourhood system for  $X_j$  at the point  $x$ . Suppose  $s(x)$  is not countable. Choose a proper open set  $x_j \in U_j \subset X_j, \forall j \in s(x)$ . Then for each  $p_j^{-1}(U_j)$ , there must be  $V \in \mathcal{B}$  such that  $V \subset p_j^{-1}(U_j), j \in s(x)$ . By pigeon hole principle, it follows that there is one  $V$  in  $\mathcal{B}$  for which this happens for an uncountable subset  $I \subset s(x)$ . But that means  $p_j(V) \subset U_j \neq X_j$ , for  $j \in I$  which is a contradiction to (29).

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What I am doing here yeah to prove (b), suppose  $\mathcal{B}$  is a countable neighbourhood system of  $X_j$  at each point and  $s(x)$  is not countable. You want to prove that  $s(x)$  is not countable; that means, what? There are uncountably many  $j$  such that  $x_j$  is a not Sierpinski's point means that they have proper open subsets as neighbourhoods, Sieripinski point is defined to be such a point wherein the only open set containing that point is the whole space right?

So, not a Sierpinski's point means  $x_j$  belongs to  $U_j$ ;  $U_j$  open and  $U_j$  is a proper subset of  $X_j$ , for every  $j$  inside  $s(x)$  with  $s(x)$  is uncountable. So, now I am assuming that this is not countable. Now, what happens? This is countable, that is not countable, so something happens.

Then, for each  $p_j^{-1}(U_j), p_j^{-1}(U_j)$  is a subbasic open set right. There must be a  $V$  belonging to  $\mathcal{B}$  that is  $\mathcal{B}$  is the countable base at the point  $x$  for  $X$  right? So, this  $V$  must be inside  $p_j^{-1}(U_j)$  for  $j$  in  $s(x)$ , ok?

By pigeon hole principle, while for each  $V$  there is such a thing I mean for each, but number of members in  $\mathcal{B}$  this is only countable, but these  $j$ 's are coming from an uncountable set  $s(x)$ . So, it follows that one of the  $V$  inside  $\mathcal{B}$  for which this happens for an uncountable subset  $I$  of  $s(x)$ , right?

If all of them are countable, countable union of countable sets will be countable so  $s(x)$  will be countable. So, that means, that  $p_j(V)$  when you project  $j^{th}$  coordinate that is contained inside  $U_i$  because  $V$  is contained in the  $p_j^{-1}(U_i)$  and this is happening and  $U_i$ 's are not the whole space. So,  $p_j(V)$  are proper open subsets, containing proper open subset for every  $j$  that is a contradiction to this basic fact that we have observed earlier ok.

This must happen at most for finitely many right? at all other coordinates it must be equal to whole of  $X_j$ , but now we have got uncountably many. That is a contradiction. So, that proves what? One way. So, I have to prove the converse. Suppose (a) and (b) are true. Then I have to show that  $X_J$  has a countable base at the point  $x$ . So, that is easier actually.

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Conversely, choose a countable local base  $\mathcal{B}_i$  at  $x_i$  for  $X_i$ , where  $i \in s(x)$ . Then it follows that

$$\mathcal{S}_x = \{p_i^{-1}(B) : B \in \mathcal{B}_i, i \in s(x)\}$$

is countable. Therefore the collection  $\mathcal{B}_x$  of all intersections of finitely many members of  $\mathcal{S}_x$  is also countable. Check that it is a local base at  $x$  for  $X_J$ .

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Choose a countable local base  $\mathcal{B}_i$  at  $x_i$  for every  $X_i$ , where  $i$  is inside  $s(x)$ . Remember what  $s(x)$  is?  $s(x)$  is set of points wherein these  $x_j$ 's are not Sierpinski's point. For those indices I am choosing countable base. For others what should I choose? It does not matter because the only open set containing  $x_j$  will be the whole of  $X_j$ , it does not matter. you can choose singleton  $X_j$ , that itself is the countable base. So, you can ignore them, that is the meaning ok.

But now, I have chosen for each  $j$ , I have chosen a countable base at  $X_j$  and  $s(x)$  is countable. So, this  $\mathcal{S}_x$  is the set of all  $p_i^{-1}(B)$ , where  $B$  is inside this countable family and  $i$  runs over all of  $s(x)$ , this is countable. So, these are countably many open subsets So, this is a countable family ok.

Therefore, if you take finite intersections of members of this family, that will be also countable ok, but once you take all finite intersections that becomes a local base at  $x_J$  for the product space  $X_J$ .

The Sierpinski points where you see all those indices, they do not trouble you at all ok because, as soon as you take some open subset around corresponding  $X_j$  there around  $x_j$ , it will be the entire space  $X_j$ . ok? So, for those things you do not have to take finite intersections so only on this family you have to take. So, that will become a countable base at  $x$ .

Now, why I have proved this one so carefully is that this is a pointwise statement. Now, the same proof we will go through if you want to do globally ok, proof will be the same, but statement will be slightly different. What is that? Because now, you are taking for all points this is happening ok, you want to do that; that is for first countability at all the points of  $X_J$ . This was first countability at a single point right? First countability at all the points now. So, that is our next theorem.

$X_J$  is I-countable if and only if each  $X_j$  is I-countable that is the first part, this is pointwise; we saw this first part. But the second part says that  $X_j$  is indiscrete space for all, but a countable number of  $j \in J$ .

If all the points are Sierpinski's points in a space right that is an indiscrete space, that is the difference between condition (b) here and condition (b) in the previous proposition. For a countable subset of  $J$ , anything can happen. Other than that, all of them must be indiscrete spaces ok? Yeah, product with indiscrete space, does not disturb the rest of the things that is what the theme is here ok.

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The screenshot shows a presentation slide. At the top left is a table of contents with the following items: Introduction, Creating New Spaces, Smallness Properties of Topological Spaces, Separation Axioms, Module 32 Path Connectivity, Module 33 Local connectivity and local path connectivity, Module 34 Compactness and Lindelöfness, Module 35 Compact Metric Spaces, Module 41 Countability and Separability, Module 42 Types of Topological Properties, Module 43 Productive Properties, and Module 45 Tychonoff Theorem. At the top right is a video feed of Anant Shastri. The main body of the slide contains the following text:

**Proof:** Let  $X_J$  be I-countable. Then I-countability of  $X_j$  for each  $j$  follows from the above proposition. This proves (a).  
Next suppose that for an uncountable subset  $I$  of  $J$ ,  $X_i$  are not indiscrete. This means that we can select  $x_i \in X_i$  which is not a Sierpinski's point, for each  $i \in I$ . Let  $x \in X_J$  be any point such that  $p_i(x) = x_i, i \in I$ . Then from the proposition above, it follows that  $X_J$  is not I-countable at  $x \in X_J$ . This proves (b).

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So, let us go through this one, but this is more or less like the previous proposition. Let  $X_J$  be I-countable. Then I-countability of each  $X_j$  for  $j$  follows from the above proposition ok? Same argument like open surjective maps preserve the I-countability. This proves (a).

Next suppose for an uncountable set  $I$  of  $J$ ,  $X_i$ 's are not indiscrete ok. This means that we can select  $x_i$  belonging to  $X_i$  which is not a Sierpinski point for each  $i \in I$  and this is uncountable set ok. So, having chosen  $x_i$ 's, you take a point  $x$  which has these as the coordinates  $i^{th}$  coordinate is equal to  $x_i$ , you choose an  $x$  belong to  $X_J$  such that  $p_i(x)$  is equal to  $x_i$  on this uncountable set, other things can be anything.

Then from the proposition above it follows that  $X_J$  is not I-countable at that point  $x$  because condition (b) also proposition is not satisfied. So, by contradiction, you have proved property (b), ok?



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For the converse, let  $I \subset J$  be such that  $I \subset J$  is countable and  $X_j$  is indiscrete for  $j \in I^c$ . Then for each  $x \in X_J$  we select a countable local base  $\mathcal{B}_j$  at  $x_j$  for each  $j \in I$  and consider the countable family

$$\{p_j^{-1}(U_j) : U_j \in \mathcal{B}_j, j \in I\}$$

and check that it forms a local subbase at  $x$ . From this it follows that there is a countable local base at each  $x \in X_J$ .

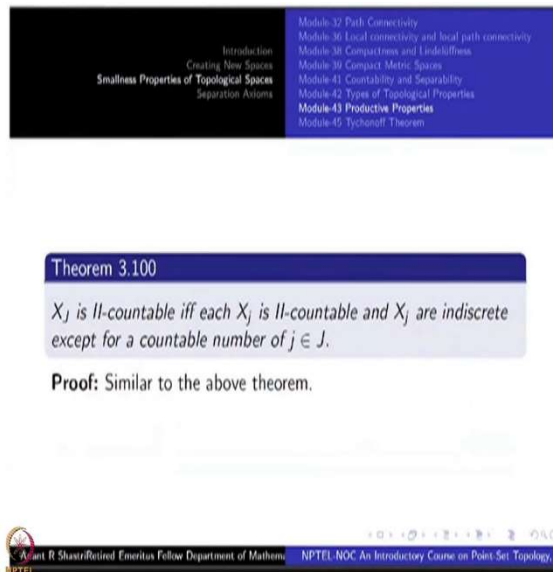
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The converse is also easy here. Suppose  $I$  contained inside  $J$  is countable and  $X_j$  is indiscrete for  $j$  in the complement. This is the condition we want, away from a countable set it all of them  $X_j$ 's should be indiscrete. Then for each  $x \in X_J$ , we select a countable local base  $\mathcal{B}_j$  at  $x_j$  for each  $j \in I$ .

See again you have to do it by pointwise construction ok? The indiscreetness gives you for all point the Sieripinski condition (b) is satisfied. Therefore, the same conclusion as in the proposition will work for case also, ok?  $p_j^{-1}(U_j)$ , where  $U_j$ 's are inside  $\mathcal{B}_j$  and  $j \in I$ , this  $I$  is countable, you check that this forms a local base ok.

This is repetition of the previous part, the only thing is having an indiscrete space away from a countable set will give you the Sieripinski points ok just naturally the condition  $s(x)$  is countable will be satisfied.

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The slide features a navigation menu at the top with two columns. The left column lists: Introduction, Creating New Spaces, Smallness Properties of Topological Spaces, and Separation Axioms. The right column lists: Module 32 Path Connectivity, Module 36 Local connectivity and local path connectivity, Module 38 Compactness and Lindelöfness, Module 39 Compact Metric Spaces, Module 41 Countability and Separability, Module 42 Types of Topological Properties, Module 43 Productive Properties, and Module 45 Tychonoff Theorem. The main content area contains the text of Theorem 3.100 and its proof. The footer includes the NPTEL logo and the text: 'Amit R. Shastri/Retired Emeritus Fellow Department of Mathematics, NPTEL, NOC An Introductory Course on Point Set Topology, P'.

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**Theorem 3.100**

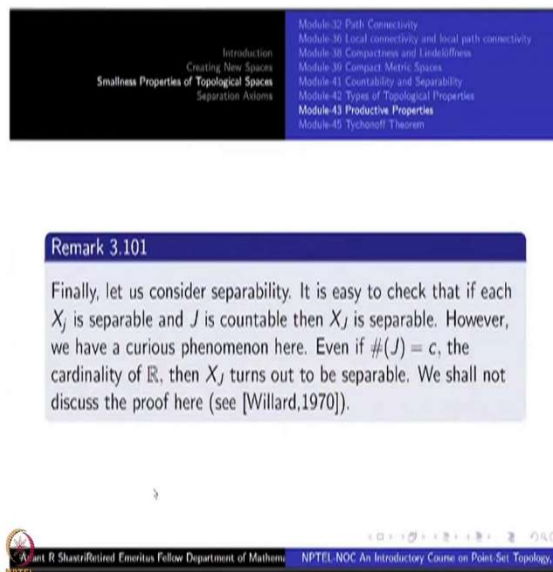
$X_J$  is II-countable iff each  $X_j$  is II-countable and  $X_j$  are indiscrete except for a countable number of  $j \in J$ .

**Proof:** Similar to the above theorem.

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Now, we just make a statement, and its proof is straightforward. Let  $X_J$  is countable product of an arbitrary family of topological spaces. It will be II-countable if and only if each factor  $X_j$  is II-countable and  $X_j$ 's are indiscrete except for a countable number of  $j \in J$ . So exactly same thing as I-countability. So, this time you do not have to worry about pointwise, you take a base here, go back come back and so on, the same proof will work.

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The slide features a navigation menu at the top with two columns. The left column lists: Introduction, Creating New Spaces, Smallness Properties of Topological Spaces, and Separation Axioms. The right column lists: Module 32 Path Connectivity, Module 36 Local connectivity and local path connectivity, Module 38 Compactness and Lindelöfness, Module 39 Compact Metric Spaces, Module 41 Countability and Separability, Module 42 Types of Topological Properties, Module 43 Productive Properties, and Module 45 Tychonoff Theorem. The main content area contains the text of Remark 3.101. The footer includes the NPTEL logo and the text: 'Amit R. Shastri/Retired Emeritus Fellow Department of Mathematics, NPTEL, NOC An Introductory Course on Point Set Topology, P'.

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**Remark 3.101**

Finally, let us consider separability. It is easy to check that if each  $X_j$  is separable and  $J$  is countable then  $X_J$  is separable. However, we have a curious phenomenon here. Even if  $\#(J) = c$ , the cardinality of  $\mathbb{R}$ , then  $X_J$  turns out to be separable. We shall not discuss the proof here (see [Willard,1970]).

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Let us come to now separability in our list. Separability is a tricky business ok? One has to be a bit careful here, but whenever things happen, it happens easily also. It is easy to check that if each  $X_j$  is separable and  $J$  is countable, then  $X_J$  is separable ok? All that you have to observe is that product of  $A_i$  closure is equal to the closure of the product of  $A_i$ 's ok? First take the product and then, take the closure, that is the same thing as first take all the closures and then, take the product.

So, if  $A_i$ 's are dense in  $X_i$ ,  $\bar{A}_i$  will be the whole of  $X_i$  ok? Then, you take the countable product, then take the closure that will be the whole space. The only thing is (this is always true of course), that a product of countable set is countable. That has to be used. If you take uncountable product, it need not be countable alright?

So, countability has to be preserved so, you have to take  $J$  to be countable that is all. However, there is a curious phenomenon here, namely, even if the cardinality of  $J$  is the first uncountable, which is denoted by the letter  $c$  (this is the cardinality of the reals, For example, ok?) I-continuum ok, then  $X_J$  turns out to be separable though the argument I have given does not work and it is not an easy argument here.

So, we have no time to do that one and you know it is not used by us anyway anywhere. So, I have given you a reference you can look into that namely Willard's book. However, if cardinality of  $J$  is bigger than  $c$ , then even this will fail. Each of  $X_J$  may be separable, but the product may not be separable ok?

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The screenshot shows a presentation slide with a table of contents on the left and a video feed of the presenter on the right. The table of contents lists modules 32 through 46. The main content area contains 'Exercise 3.102' which asks to show that every separable metric space is II-countable. The presenter, Anant Shastri, is visible in the video feed.

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**Exercise 3.102**

Show that every separable metric space is II-countable. Recall that a topological space is said to be metrizable, if there exists a metric on the underlying set such that the metric topology coincides with the given topology. Let  $J$  be an uncountable set. For each  $j \in J$ , let  $X_j$  be a metric space with at least two points. Show that the product topology  $\prod_{j \in J} X_j$  is not metrizable.

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So, let us stop here for today, just summing up what we have done so far. So, after hereditary property and co-hereditary properties checking for various things namely connectivity, path-connectivity, compactness, lindelofness, I-countability, II-countability and separability. Then, yesterday and today, we checked about I-countability and II-countability and separability ok.

So, this is the list of the various properties of topological properties that we have studied so far alright. So, we have still more thing to do with product properties namely compactness and lindelofness, we have to worry about that. So, that is another topic. So, that will be taken next time ok.

So, thank you.