**Introduction to Point Set Topology, (Part I)**
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**Lecture - 13**
**Definitions and examples**

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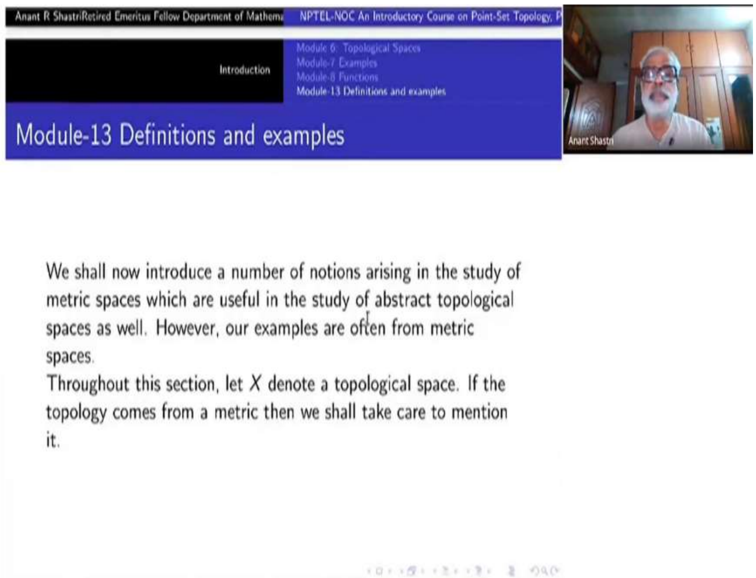


Welcome to module 13 of Point Set Topology course. We shall now introduce a number of notions. They all arise in the study of metric spaces as well, but they are useful also in the abstract topological setup. And most of the time we will have our examples from metric spaces.

Throughout this section, $X$ is a topological space. So, I may not mention it every time. For example, this symbol $X$ will not be used for any other thing in this section, in this lecture. If the topology comes from a metric then I will specially mentioned that $(X, d)$ is a metric space.

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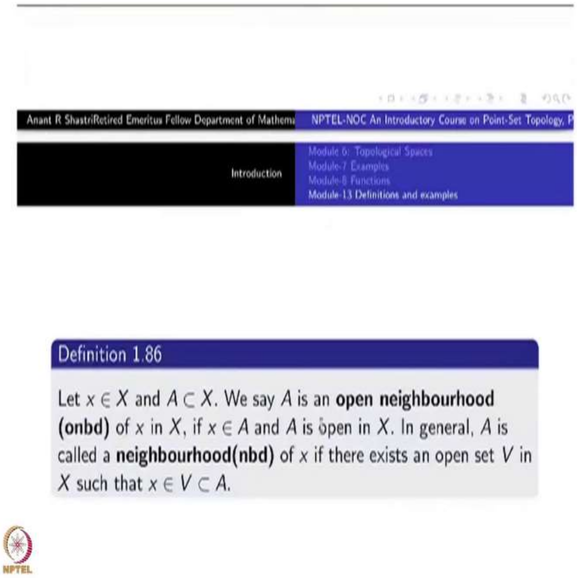


Take a point $x$ in $X$ and a subset $A$. We say $A$ is an open neighbourhood, quite often lazily we will write it as a onbd ok? So, $A$ is an open neighbourhood of $x$ in $X$, if first of all $x$ must be inside $A$ and $A$ secondly must be open. That is why the name and the notation onbd, o stands for open, neighbourhood means that $x$ is inside $A$ first of all.

In general $A$ is called neighbourhood without that o, if $x$ is inside $A$ for that is part is there ok, but you have an open subset $V$ such that $x$ is inside $V$ contained inside $A$. Of course, in this containment notation, equality is allowed. So, there is again another lazy notation, but it may happen that indeed quite often, that $V$ is smaller than $A$ and $A$ itself is not open. Even then we call it a neighbourhood. In that case, we do not call it an open neighbourhood. That is all the difference.

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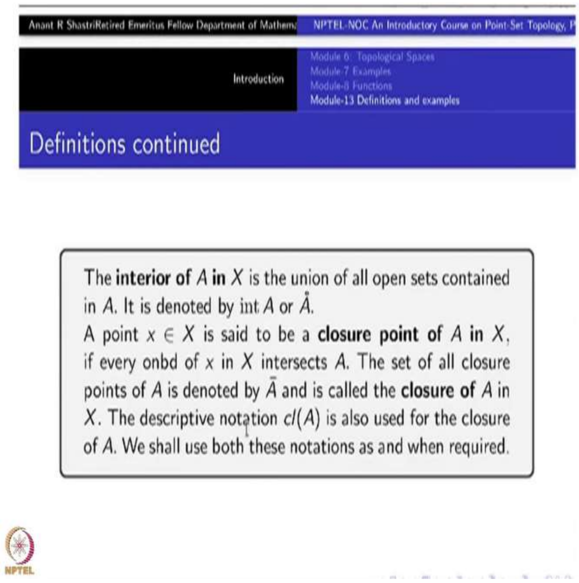


For a subset $A$, us now define interior of $A$ ok? So, interior of $A$ is the union of all open sets contained in $A$, ok? So, it is denoted by interior of $A$ or $A$ with circ on the top, but I will have to read it as interior of $A$ only.

So, this just means that if I take a point $x$ inside interior of $A$ ok that point is already a point of $A$ and there is a neighbourhood, open neighbourhood of the point contained in $A$. Therefore, interior of $A$ is the union of all open sets contained in $A$.

So, interior of $A$ itself is open ok? Not only that; obviously, it became a union of all those sets, it will be the largest open set contained inside $A$ ok? That is another definition for interior of $A$, the largest open set contained inside $A$. So, it begins with $A$ which may not be open. So, that is why interior of $A$ is defined that is a subset of $A$ that is open and the largest one.

But if $A$ itself is open; obviously, then being the largest open subset contained inside $A$ interior of $A$ will be equal to $A$. Of course, converse is also true right? If interior of $A$ equal to $A$ then $A$ is open because just now I have proved that interior of $A$ is open ok? So, we will make use of these things again and again. So, it is better to byheart them. So, does not matter if I repeat it once or twice.

Let $x$ belong to $X$. It is said to be a closure point of $A$ ok? (so, everything is happening inside $X$) if every neighbourhood of $x$ ok intersects $A$. So, here I have put every open neighbourhood of $x$ intersects $A$ ok if every open neighbourhood of $x$ intersects then every neighbourhood intersects.

So, every neighbourhood intersects open neighbourhoods being neighbourhoods they will also intersect. So, these two are equivalent we can take whichever one you like. Every open neighbourhood of $x$ intersects $A$, is fine. The set of all closure points see when you take the closure points we are no longer inside $A$. Any point of $X$ you can take if it verifies this condition, then it will be simply call the closure point of $A$.

Look at all the closure points we will have a notation for that. It is denoted by A bar on the top. It is a very popular notation, but some people may not use that one. They may use $cl(A)$. And then some other people may use $A$ superscript $c$. But I will never use that notation ok? $cl(A)$ I may use for closure, $\bar{A}$ also I may use, but $A^c$ at the top that I will never use because many people use that for the complementation. So, that is a very confusing notation. So, I will not use that one. So, this descriptive notation $cl(A)$ is very easy to remember it is closure of $A$.

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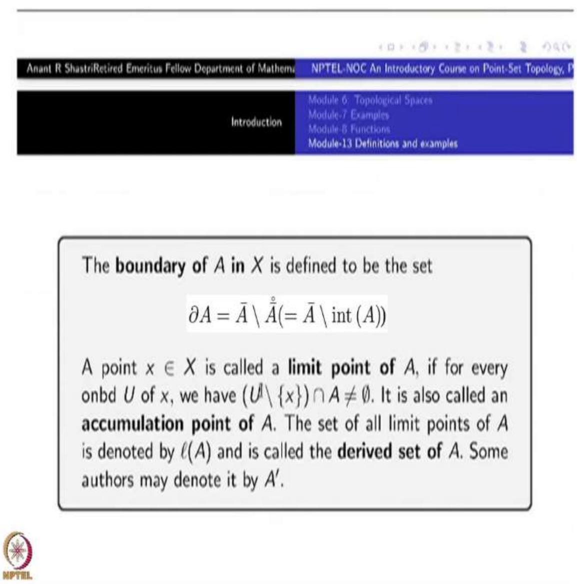

Now, another concept, the boundary of $A$ is nothing but all the closure points of $A, \bar{A}$ and throw away the interior. Interior is already subset of $A$. $\bar{A}$ is larger than $A$ perhaps or maybe equal to $A$, I do not know, but interior of $A$ definitely is contained in $\bar{A}$ ok? So, you throw that away what you get is boundary of $A$.

A point $x$ is called a limit point of $A$ if for every open neighbourhood $U$ of $x$, $(U \setminus \{x\}) \cap A$ is non empty. In the definition of closure point we did not have this $U \setminus \{x\}$. $U$ intersection $A$ is non empty was the condition. So, $(U \setminus \{x\}) \cap A$ is non empty is a stronger condition ok? If $(U \setminus \{x\}) \cap A$ is non empty $U \cap A$ is also non empty for every $A$ and for every $U$. Therefore, a limit point will be automatically a closure point, but not always vice versa ok? A limit point is a closure point.

There is another notion here that is called accumulation point of $A$ ok. Sorry, not another notion another name for limit point. The word accumulation point is used by some people. The set of all limit points of $A$, let us have a notation for that, viz., $l(A)$. So, take care, in geometry this l will be denoting length of set etc. So, do not confuse it for any length here. This $l(A)$ is the set of limit points of $A$.

But then there is another notation here. These notations are not very standard. Different people use different notations, but not too many. So, 50 percent of them may be using this and another 50 may the other. So, it has another name, derived set of $A$. Some people use the notation $A'$ for $l(A)$. What is $l(A)$? Set of limit points. What is a limit point? Every neighbourhood $U$ of $x$ has the property $(U \setminus \{x\}) \cap A$ is non empty.

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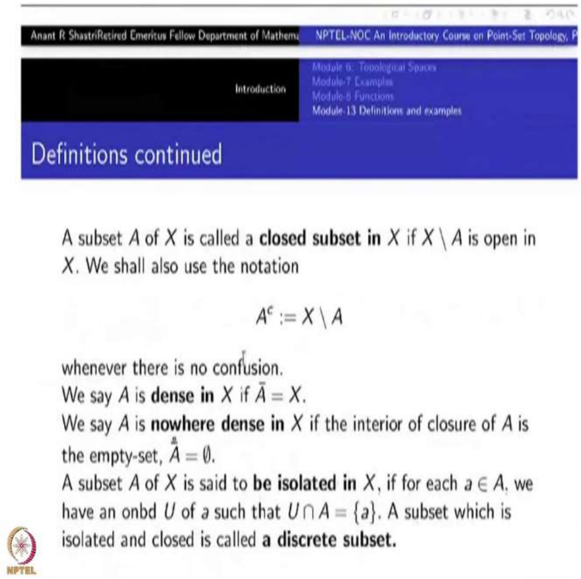


Let us have a couple of more definitions here. Because all of them are related to these open sets right? they are all interdependent. So, its better to study all of them together. So, let us have some more definitions.

A subset $A$ of $X$ is called a closed subset, this I am repeating I have already told you what is closed subset, if $X \setminus A$ is open in $X$, just the complement ok? We shall use the notation this $A^c$ for the complement if there is no confusion, ok? Usually, this is the best notation, you can take complement of $A$ in different subsets $B$ containing it right? Instead of just $X$. So, then for all of them if you use $A^c$ that will be confusing whereas, this will be $B \setminus A$. Suppose, $A$ is contained in $B$, $B \setminus A$ makes sense. In fact, $B \setminus A$ makes sense even if $A$ is not contained in $B$ all that you have to do throw away from $B$ all the points of $A$ which are inside $B$.

If they are not there in $B$ then you do not have to throw them away. If they are there means what you take the intersection $A$ with $B$ then throw away $A \cap B$ from $B$ that is $B \setminus A$. That notation is a set theoretic notation I am not introducing that I am just recalling that.

So, a set a subset of $X$ is dense in $X$ if the closure is the whole space $X$. So, every point of $X$ is a closure point ok. In that case $A$ is called $A$ dense set in $X$.

Exactly opposite, sorry, actually something stronger than the negation of this one of course, these are not the negation of each other, it is not just not-dense that another notion I am going to define now. Nowhere dense is very strong notion.

We say $A$ is nowhere dense in $X$ if interior of the closure of $A$ is empty. That is, first you take the closure $A, \bar{A}$, and then put a circle circ over it right? That is the interior, interior of the closure is empty ok?

So, closure of $A$ you have to take and then look at the interior of it, that is empty means what? Take any set $B$, its interior is empty means what? No non empty open set is contained in $B$, because interior of $B$ is a union of all of them ok? So, that is happening for $\bar{A}$, where $A$ is nowhere dense subset $A$ of $X$. A subset $A$ is said to be isolated set if for each $a \in A$, we have an open neighbourhood of $a$ such that if you take $U \cap A$, it is just a singleton $\{a\}$. See $U$ is a neighbourhood of $a$, and $a$ is already inside $A$.

So, $U \cap A$ contains $a$. It should not have any other element, it is singleton $\{a\}$. If you can find for each $a$, an open set $U$ like this, then the set $A$ is called isolated in $X$ ok? Clearly, each singleton set is isolated. A subset which is isolated and closed also that will be called a discrete subset. This is just a terminology some people may not put this closedness and say isolated is same thing as discrete. Or some people may interchange them also ok? Wherever I have called isolated they may call discrete and wherever I have called discrete they may call isolated. This can with some authors, but this is my definition. I am going to stick with this ok?

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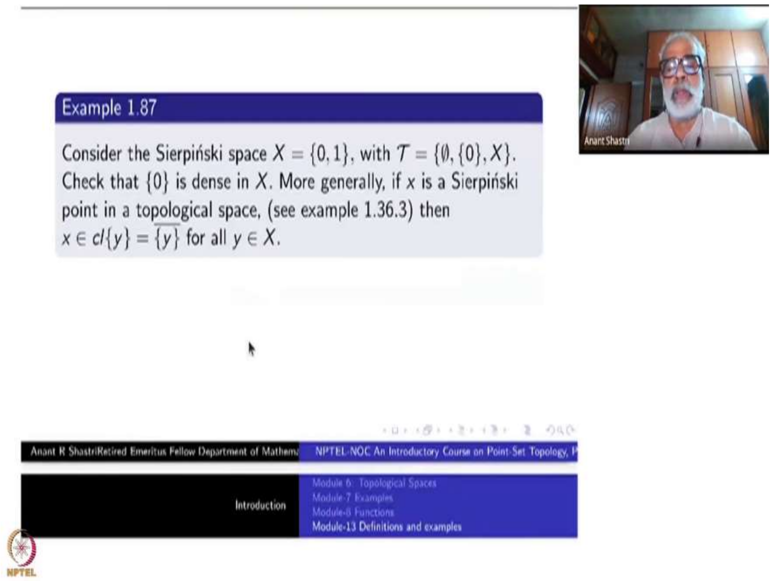


One general remark is that all these terminologies occurred even before the concept of topology. As such the definition of topology that we have introduce was adopted ok, from the works of Weierstrass. But the ideas, the terminologies took some time of course, and then there were different terminologies by different authors. So, that is why even today there are slightly different terminologies by different authors, but many things have come to in a central stream many things have stayed and other things have fallen. Now only a few things have survived ok? So, many of these notions were there long since. Some people were using different terms. You know Weierstrass was not using any of these terms, but every time he simply will explain the whole condition.

So, it is a very torturous thing you know reading those original papers is very difficult. So, let me first give you the simplest examples.  Discrete sapce and indiscrete space are two simple ones. The next simplest example is the Sierpinski space that we have introduced right? Sierpinski space consists of just 2 elements 0 and 1, and three open sets. What are the open sets? Empty set and $X$ must be there. So, you put one more, namely either singleton 0 or singleton 1.

So, put singleton 0? Check that singleton 0 is dense in X. You see what I have said, but not sure of myself. So, I have to tell again. So, what is the meaning of this tell me. There is only one other point right? Namely 1. I am not sure that 1 is in the closure of this singleton 0. So, what does that mean? I must take an open set containing 1, but by this definition X is the only set which contains 1 which contains 0 also. Therefore that open set intersects this one that is the condition for 1 to be in the closure. Therefore, 0 closure will contain 1 also. Of course, it contains 0 as well. Therefore, it is the whole of X. Was that easy to verify? That singleton 0 closure is this whole space that is the meaning of a denseness ok?

Now, you can do a little more generally. Namely take any Sierpinski point in a topological space, you remember what is a Sierpinski point? Sierpinski point is a point where the only open set containing that point is the whole space. Therefore it will be in the closure of every other point. Take y to be any other point x will be inside the closure of that point ok? This does not mean that y bar is the whole of X of course. The Sierpinski point will be there in the closure of singleton y. So, that is similar to this, but when I say more generally I am not concluding this one namely the closure of y is the whole space. closure of y contains x ok.

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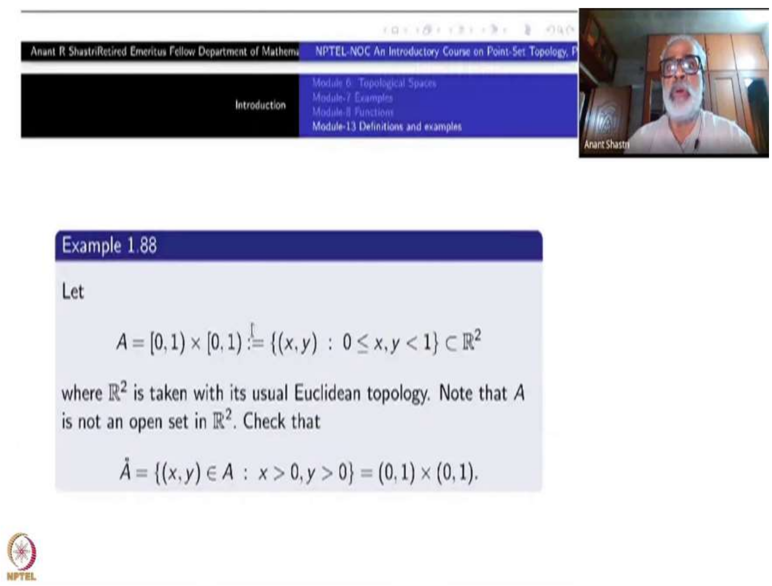


Now, let us come to, as I told you to, examples inside $\mathbb{R}$, $\mathbb{R}^2$,$\mathbb{R}^3$ and so on. So, I am taking an open interval sorry, a half closed interval $[0, 1)$ or you may call it half closed interval, cross with the same thing, $[0, 1)$. So, namely take $A$ to be all points $(x, y)$ such that $0 \leq x, y < 1$. Both $x$ and y are less than 1 and greater than or equal to 0. Here equality allowed, here strictly inequality ok?

But what is the topology? I am taking it as the subsapce of the usual Euclidean topology on $\mathbb{R}^2$. So, that is a metric topology also right? Try to work out whatever we want to do without using the concept of the metric directly. We have defined the topology what is an open set etc. You just use them, try to use them that is all. Note that this $A$ is not an open set in $\mathbb{R}^2$, why? To see that some set is not open you have to produce one element in the set such that no neighbourhood of that element is contained inside the given set right?

For example, I can take $(0,0)$ that is a point of $A$. Every neighbourhood of $(0,0)$ first of all will contain an open ball of some positive radius. No open ball of positive radius will be contained in this $A.A$ is just a part of first quadrant, inside inside $\mathbb{R}^2$ right?

So, this set $A$ is not open in $\mathbb{R}^2$, ok? You can do the same thing with all the points on the $x-$ axis part or $y-$axis part ok? If $y$ is $0$ or $x$ is $0$, those points are not in the interior. Therefore, the interior does not contain those two sides. You have to throw away those sides. Now , look at points with $x$ positive and $y$ positive, you take all those points, they are in the interior. Why? Because as soon as $x$ is positive $y$ is positive ok. you can take the minimum of them as radius and take a ball around that point $(x,y)$ of that radius that will not intersect the $x-$axis and $y-$axis part, and so that would be contained in $A$ ok? So, you want see that the ball does not go out of $A$. So, you should also take care of that alright. So, this is actually an open set because its $A$ interior ok its a largest open set contained inside $A$ ok?

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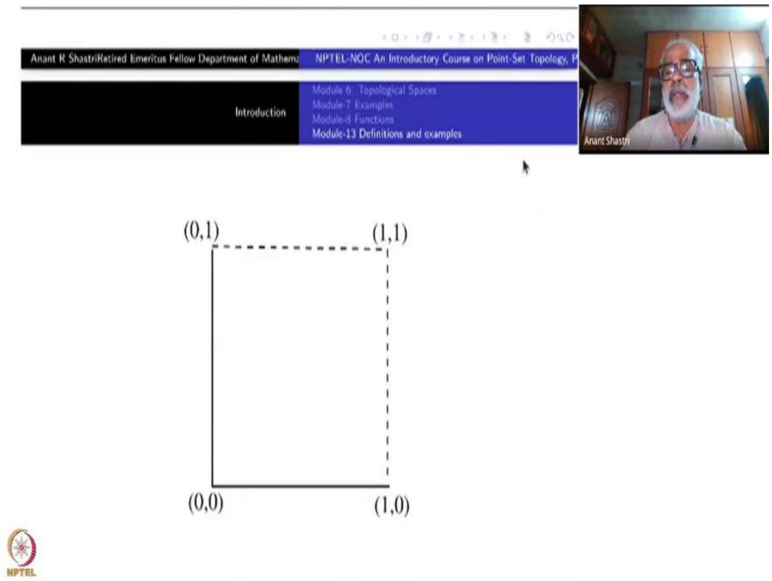


Let me now show you the picture here. So, in this picture I have taken $[0,1) \times [0,1)$, that is why I have put these two dot dot dot here ok. What does it mean? $1 \times I$ is not contained in the set. Similarly, this one $I \times 1$ is not contained inside that set ok? Everything else is here, but these bold lines here full lines they are contained inside that is a set. Just now I explained that to take a $(0,0)$ here then any neighbourhood of that will contain a ball around that right? So, it would not be contained inside the set $A$. Once I am inside here I can just look at distances of that point from various points of the four sides in the boundary, take the minimum of all of them. Then the ball of that radius will be contained inside $A$. Therefore, every point other than these solid lines here they are inside, what? the interior of $A$. Therefore, interior is precisely equal to that and that is all ok?

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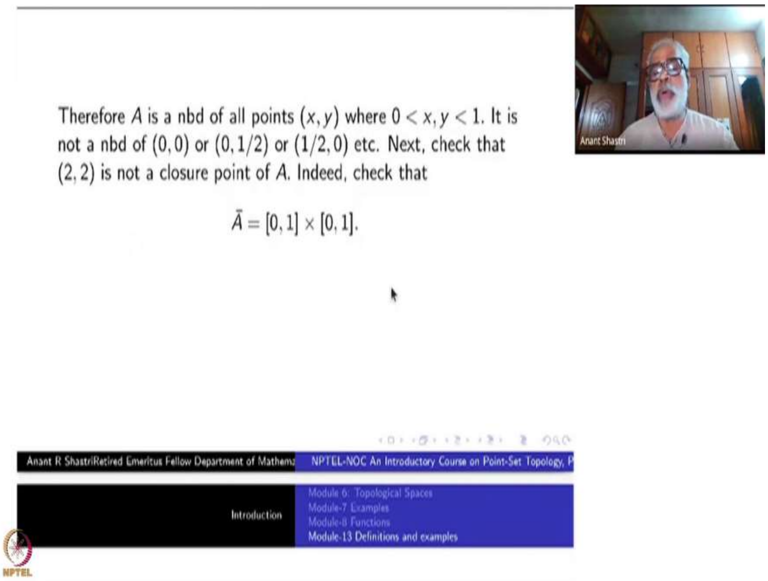


Next thing is observe that $(2, 2)$ is not a closure point of A. $(2, 2)$ is somewhere far away here, ok? that will not be in the closure of $A$. Why I am saying that? Of course, points of this interior here. They are in the closure even this boundary dot dot dot line will be also in the closure, why? Because if I take a neighbourhood of one of these points, some ball right? Half of the ball will lie inside here, like that here also.

So, this entire dot dot dot dot dot dot here that will be in the closure. But if I take some far away point ok? far away means what? Anything which is not in this not in this $I \times I, [0, 1] \times [0, 1]$; both closed. So, once you out here, you can choose its distance from there to here, the open ball of that radius will not intersect this $A$ at all. One open set does not intersect this one means that the point is not in a closure ok? So, closure of $A$ is precisely equal to $[0, 1] \times [0, 1]$?

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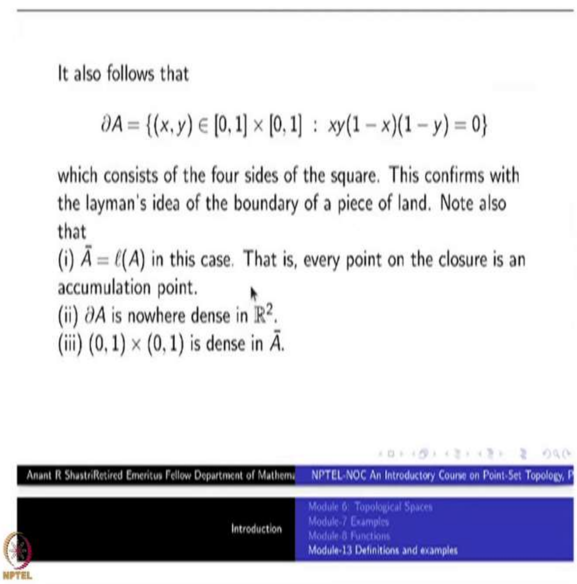


Next let us look at the boundary of $A$, see boundary of $A$ was defined as $\bar{A} \setminus A^{\circ}$. We have computed both $\bar{A}$ and $A^{\circ}$ therefore, what is boundary of $A$? All those points lying on these four sides here. So, this can be defined as: this is $y$ equal to $0$, this is $x$ equal to $0$, this is $x$ equal to $1$, that is $y$ equal to $1$.

So, all these four conditions can be put in one single condition $xy(1-x)(1-y)$ equal to $0$. If this product is $0$ one of them must be $0$. That $x$ is $0$ or $y$ is $0$ or $1-x$ is $0$ (which is same as $x$ is 1) or finally, $1-y$ is $0$ which is same as $y$ is $1$. This consists of four sides of square ok? So, why this example? I wanted to tell you that this concerns with the Layman's idea of a boundary of a piece of land.

So, here a piece of land, a square shaped one, and the points on the four sides of the square are the boundary points. So, that is the kind of boundary you must have already used in your fourth standard geometry. ok? Boundary of a triangle, the length of it is the perimeter right? The boundary is a geometric concept there. It refers to the empty triangle, when you say triangle, you probably take all the points in the interior as well right? At least when you talk about the area of a triangle. So, that is what it is. This is an area of a triangle, here is the perimeter, perimeter is what? Total length of the boundary ok? So, that is the same thing for us now here for nice objects, but now we have defined it for arbitrary subspaces. So, you should have some properties which are inherent in this pictures and so on, but some other properties which may be strange which are not depicted by the pictures.

So, you have to be careful because finally, it is just the logic here of sets nothing more. Topology is always that. Strange that it produces so many ideas, so many you know visual things are reflected there and so many strange ideas that is the power of the language of set theory ok?

Let us make a few more points here one by one. In this case I have computed $\bar{A}$ ok. We have computed interior of $A$, we computed the boundary of $A$ also. One more thing I want to say what are the limit points of $A$? That was the set $l(A)$. Here, $l(A)$ will be equal to the entire of $\bar{A}$. It is not always true of course. If that is the case, then there is no need to define $l(A)$. So, let us verify that. So, why that is true? What is the meaning of $l(A)$? Take any point you must produce $A$ neighbourhood such that from the neighbourhood even if you throw away that point, it should intersect the set. And that is very clear: points inside here it is clear. Even at this point this point if you throw away that point, but you keep the half the disk will be inside this one. So, it will intersect right? So, it is very easy to see that every point in the closure is a limit point here for this set ok? That is every point the closure is a limit point or accumulation point in other words.

Second thing is if you look at the boundary of $A$ that is nowhere dense in $\mathbb{R}^2$. See I start with a set $A$, namely this time boundary of $A$. That is my set, a closed set. When is it nowhere dense? Its interior must be empty ok? In general, note that you must take the closure first and then take the interior. But here I do not have to take the closure because boundary of $A$ is a closed set and so the closure is the same thing as boundary of $A$, ok?

So, if you take this set and look at its interior I want to say that the interior is empty there is no point which is in the interior. What does that mean? That is very easy to verify ok? Take a point on the boundary this boundary of $A$. Ok? Take a point here or here. Now, can I produce an open set at least one open set, at least open ball around this point contained in that line? No open ball will be contained in the boundary of the square. It will overflow both inside and outside the square. Both sides. That is the beauty. See take any point here take a ball here it will intersect both $A$ as well as the complement of $A$ in $\mathbb{R}^2$. That is the characteristic property of a boundary point of $A$. I will come to that one again later on. Right now, concentrate on the fact that boundary of $A$ has no interior. No open ball in $\mathbb{R}^2$ is contained in here.

So, no non empty open set is contained in here. How to prove that? No open ball is contained in here, that is enough. Because if there is one open set non empty then there will be a ball also. So, since no open ball is contained is very clear right. Therefore, this boundary of the square is nowhere dense. Similarly if you take a triangle, the boundary of a triangle will be nowhere dense alright. Next, if you take the open part $(0, 1) \times (0, 1)$ that is a subset of $A$ that is dense in $\bar{A}$ ok.

Now, I am not talking about denseness inside $XX$ equal to $\mathbb{R}^2$ ok? I am just saying dense inside $X$ equal to $\bar{A}$. Therefore, I am thinking of this $\bar{A}$ itself as a space. All that I have to do is, what I have to do? Take any point of $\bar{A}$ sure that it is in the closure of this one $(0, 1) \times (0, 1)$; that means, take any point you take a neighbourhood it should intersect it and that is precisely what we have been showing all the time.

These two lines are not necessary ok? When you take the closure they will automatically come take a point here, ok, in $\bar{A}$ anywhere here here here to take a neighbourhood if you take an open ball it will intersect the interior right. So, these two lines these two these and along with this line also, finally, all the four lines are in the closure of this $(0, 1) \times (0, 1)$.

So, much easier to see that if you do this inside $\mathbb{R}$, viz., if you take the open interval $(0, 1)$ both 0 and 1 are in the closure. And when you take them you get a closed interval ok? So, that is easy to see. I have taken middle course by taking subsets of $\mathbb{R}^2$, so that both cases $\mathbb{R}$ as well as $\mathbb{R}^3$ and $\mathbb{R}^4$ etc will get explained.

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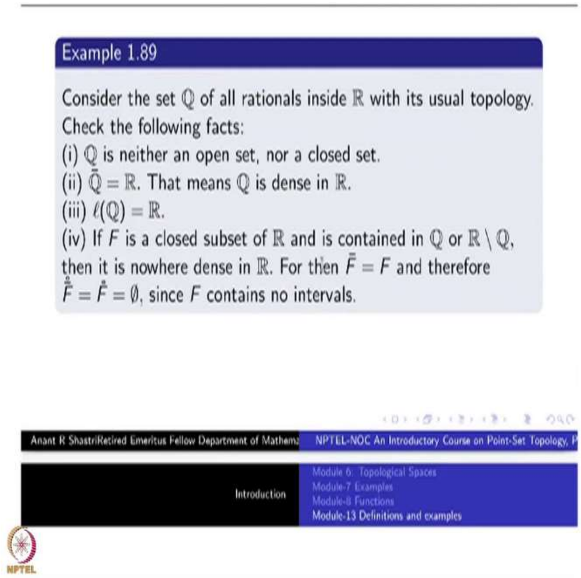


Now, we will come to a little more serious example. Look at the set $\mathbb{Q}$ of all rational numbers inside $\mathbb{R}$. Let us check that these points one by one. This set of rational numbers is neither open nor closed, in $\mathbb{R}$. Why? Why it is not open? If it were open, for each point you must get an interval ok? Because an open ball in $\mathbb{R}$ is nothing but an open interval, an open interval must be contained inside $\mathbb{Q}$. But you know that rational numbers do not contain any interval any open interval ok? Of course, if you want to take empty interval, it is a there but we are interested in non empty.

Nor it is closed. Same thing as saying that complement of $\mathbb{Q}$ inside $\mathbb{R}$ is not open. Same reason. All the rational number, throw away them, what you get? All irrational numbers. They also do not contain any non empty open interval ok? So, neither $\mathbb{Q}$ nor its complement is open. However, if you look at the closure of $\mathbb{Q}$ that is the whole of $\mathbb{R}$ ok? So, every point in $\mathbb{R}$ is a limit point of rational numbers. Take any point, take any interval around it there will be always some rational number. That is all I am using here ok? Every open interval non empty open interval will intersect $\mathbb{Q}$. Therefore, $\bar{\mathbb{Q}}$ is the whole of $\mathbb{R}$. But in terms of our terminology this just means at $\mathbb{Q}$ is dense in $\mathbb{R}$, ok?

Let us look at the limit points of $\mathbb{Q}$. That is also the whole of $\mathbb{R}$. Just now, I said that if you take any point and take a neighbourhood around it, an open interval, throw away that point, even then there will be many rational numbers there ok. Therefore, every point is a limit point. Not only just $\mathbb{Q}$ all real numbers, You take any point rational or irrational, ok? Take an interval around that point throw away that point still you will have two non empty open intervals there right? So, they will intersect $\mathbb{Q}$.

The next one is: if $F$ is a closed subset of $\mathbb{R}$, we take a closed subset $F$ of $\mathbb{R}$ ok? Note that $\mathbb{Q}$ is not closed neither complement of $\mathbb{Q}$ is closed. Now we will take a closed subset of $\mathbb{R}$ and suppose it is contained in $\mathbb{Q}$. Like you know you can just take a single point which is a rational or $1, 2, 3, 4, 5, 6$ all the integers. there are lots of closed subsets contained in $\mathbb{Q}$. You take any closed subset inside $\mathbb{Q}$. Or the same thing you can do in irrational numbers also. $\pi, 2\pi, 3\pi$ anything like that ok? So, take a closed subset $F$ of $\mathbb{Q}$ or $\mathbb{R} \setminus \mathbb{Q}$. Then that set is nowhere dense. So, I am giving examples of every concept here at least one example to begin with. That is nowhere dense, nowhere dense means what? If $F$ is already closed, so I do not have to take the closure, first of all, and then what I have to take? Then you have to take the interior.

So, interior means what? $F$ should contain some non empty open set right? But $F$ is contained in $\mathbb{Q}$ and $\mathbb{Q}$ does not contain any open set open interval therefore, $F$ also does not contain. Same thing if F is contained inside $\mathbb{R} \setminus \mathbb{Q}$ also. It does not contain any no empty open set. So, it is a consequence of this observation, the last one is that every closed subset of $\mathbb{Q}$ or $\mathbb{R} \setminus \mathbb{Q}$ is nowhere dense ok?

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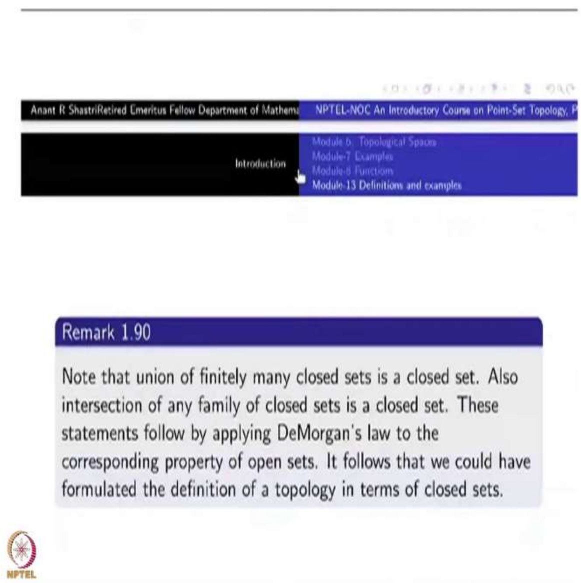

Note that union of finitely many closed sets is a closed set. Also intersection of any family of closed sets is a closed set. These are two axioms dual to (AU) and (FI) right? (FI) says what? Finite intersection of open sets is open. So, you take DeMorgan law, finite union of closed sets is closed that is what you get. So, that is the DeMorgan law. So, what it amounts to saying is that the (T) axiom, (AU) axiom and (FI) axiom could have been replaced by three other axioms which you may call (T'). (AU') and (FI'), which are in terms of closed sets ok? So, our foundational definition of topology could have been in terms of closed sets. And then we would have defined open sets as complements of closed sets. That would be exactly parallel to whatever we have done so far. So, there is no surprise no change at all in the theory ok?

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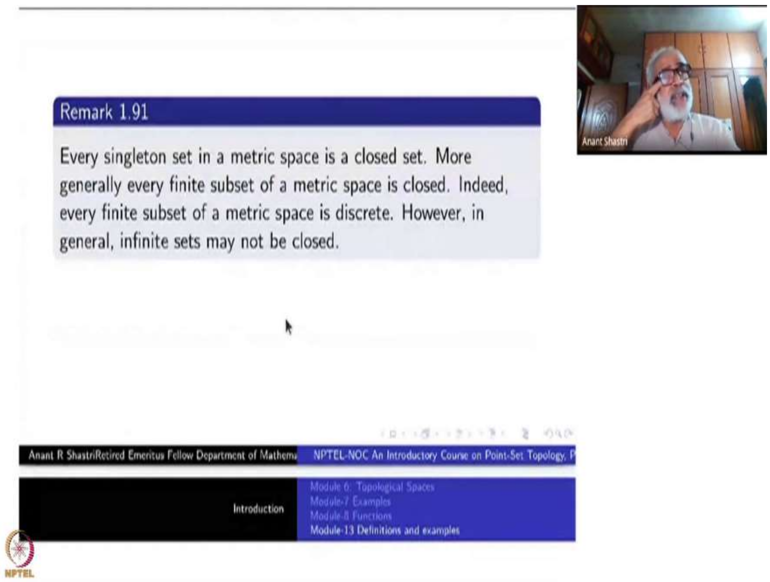


Inside a metric space every singleton is a closed set why? Because, the complement is open. Take any point other than the given point. So, let us say given point is $x_0$, take any other point $y$. Then the distance between that $y$ and $x_0$ is positive, say it is $\epsilon$ or say $\delta$ whatever. Now take $\delta/2$. And take the ball around $y$ which is of radius $\delta/2$ that will not contain the point $x_0$, right? That shows that the complement of a single point is open, therefore the single point is closed, ok?

Every finite subset $A$ of a metric space is actually discrete. Whatever I have just proved above actually proves this one. Because I have taken a finite set what you can do is look at all the various distances between $x_i$ and $y_i$, points of $A$. You know these finitely many numbers are all positive. So, you take the minimum ok? Now you take a ball around each of these points with that radius equal to this minimum. That will not contain any other point. Therefore $A$ is an isolated set. But being a finite set, it is also closed. So, it is a discrete set. However, if you take an infinite set, that may not be closed, may not be discrete ok? So, let us have some examples. Earlier we have taken very large sets like to $[0, 1] \times [0, 1]$. Definitely that is not an isolated set neither it is discrete ok?

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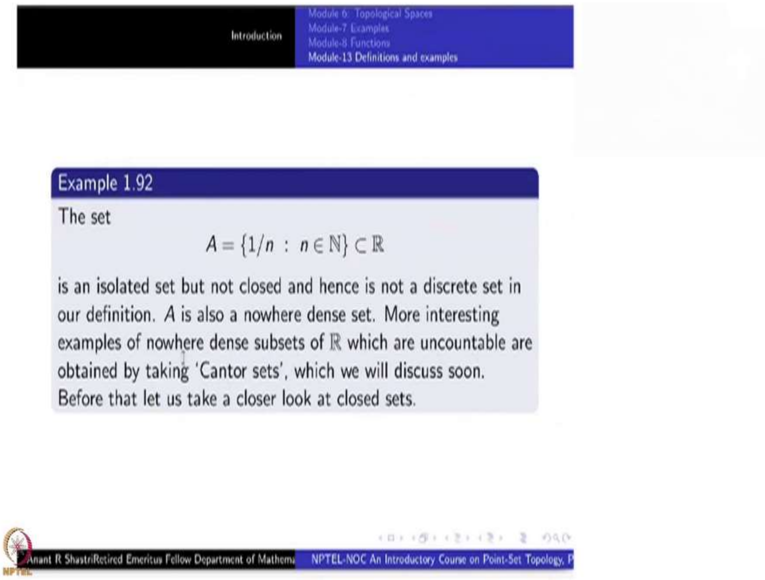


So, that is very easy to see, but even more complicated sets can be taken may violate this condition. So, here is a very simple example which is isolated yet not discrete. Namely, look at all $1, 1/2, 1/3, 1/4$ and so on ok? inverse of the positive integers. That is an infinite set ok? It is an isolated set, because you take any say $1/m$. Then I have to take something between $1/(m+1)$ and $1/(m-1)$, take the open interval between these two that will contain $1/m$ and will not contain any other $1/n$, right?

Therefore, it is an isolated set. However, it is not a closed set. That is very easy to see because 0 is in the closure of this set. Look at 0 take any interval around 0 it will have these some $1/n$ after some large $n$. So, in any case every open interval around 0 will intersect this set. So, 0 is a limit point, 0 is in the closure. If you have an extra point which is not in $A$ and is in the closure, $A$ cannot be closed ok? Alright.

More interesting examples will be seen, namely, like Cantor sets. Cantor set is an uncountable set. It is nowhere dense. It is not like $[0,1] \times [0,1]$ ok? Yet you know it is uncountable yet it is nowhere dense. There are many interesting properties of the cantor set. We will discuss that when it comes to, but right now we want to study the closed sets interiors, etc more appropriately.

So, let us stop here today.