Introduction to Point Set Topology, (Part I)

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Lecture - 01
An Introductory Course on Point-Set Topology, Part-I

Welcome to NPTEL-NOC Course - An Introductory Course on Point-Set Topology, Part-I. So, that should also tell you that there is a Part-II also planned and that is true. So, I am Anant R Shastri, Retired Emeritus fellow from Department of Mathematics, IIT Bombay. I am well supported by my team members - Professor Angom Tiken Singh from NEHU Shillong.

Hello everyone, I am A Tiken Singh, I am from NEHU Shillong.

And Professor Ardeline Buhphang again NEHU Shillong.

Hello everyone, I am Ardeline Mary Buhphang.

Omkar Javadekar from IIT Bombay.

Hello everyone, I am Omkar Javadekar from IIT Bombay and I welcome you to this course.

Souvik Mandal from ISI Bangalore.

Hello everyone, I am from ISI Bangalore.

And Vinay Sipani, IIT Madras.

Hello all this is Vinay Sipani. If you have any questions or queries or any doubts feel free to ask us. Thank you.

Thank you all. I should first of all thank NPTEL portal as well as the staffs with from whom I have got this very nice opportunity to speak to you people and also it was a very enjoyable experience to work with NPTEL staff here at IIT Bombay. So, thank you all for that. And now let me tell you about this course in general and how to cope up with this course when you want to study this course, ok.

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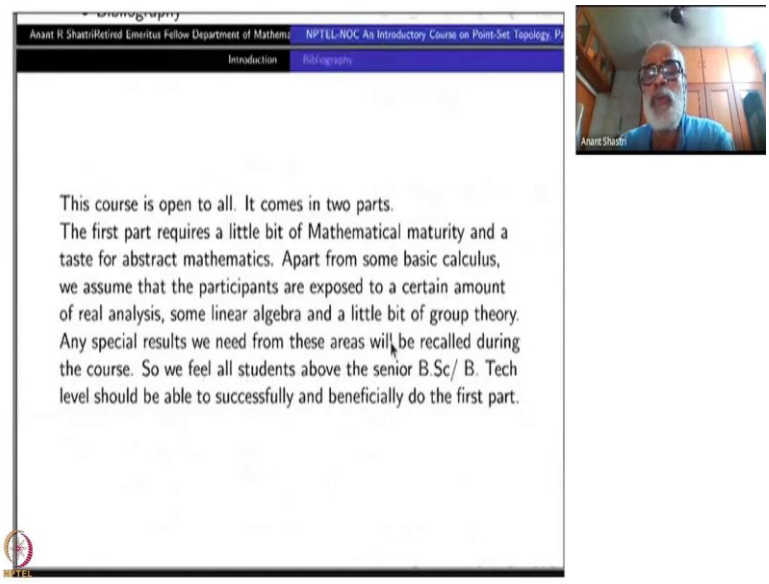

So, this course is open to everybody; everybody who wants to learn is welcome. It comes in two parts as I have told you. This part is Part-I that will be I have presented right now in this semester. The first part requires a little bit of Mathematical maturity and a taste for abstract mathematics. Apart from some basic calculus, we assume that the participants are exposed to a certain amount of real analysis, some linear algebra and little bit of group theory. If we need any special or specific results from these areas, we shall be very happy to recall them, ok, during the course.

We feel all students above the B.Sc or B. Tech level should be able to successfully and beneficially do this part of the course. So of course, they should knowing their calculus and basic things, that is all.

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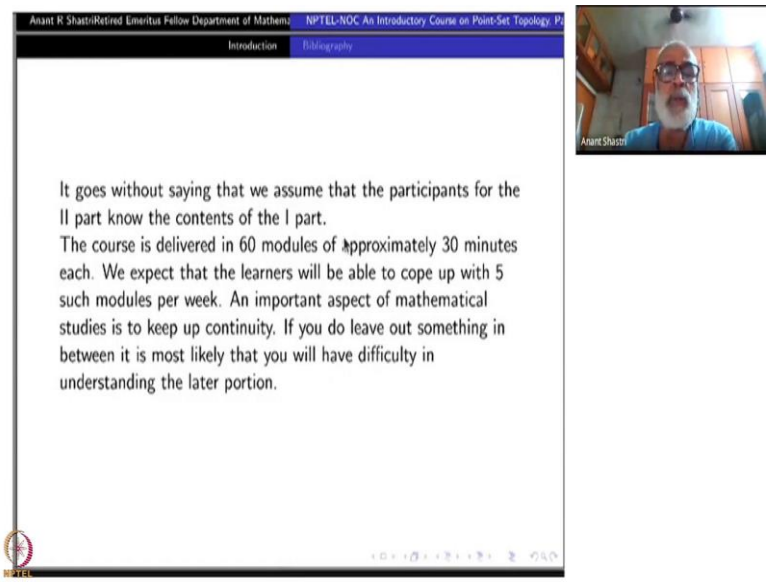

So, it goes without saying that we assume that the participants for the 2nd part would have already attended this 1st part or would have acquired this much of knowledge from somewhere else.

So, Part-I is more or less compulsory for Part-II. We will not have any patience or anything to do with people who do not know the Part-I course in the Part-II course, ok. So, that is what the general plan is.

Now, the course is divided into 60 modules of approximately 30 minutes each. We expect the learners will be able to cope up with 5 such modules every week. Ok that is like two and half hours of lecture courses, ok.

An important aspect of all mathematical studies is to keep up the continuity. If you do leave something in between, it is most likely that you will have difficulty in understanding the later part of the course.

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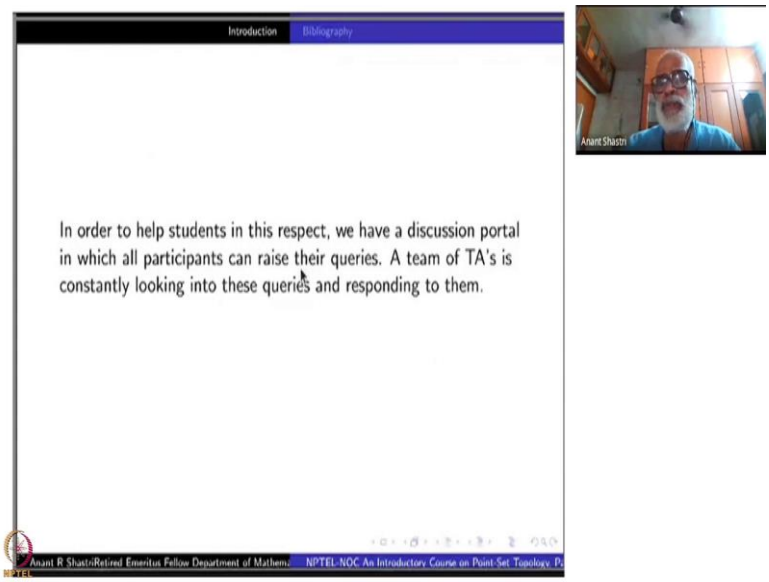

In order to help the students with this aspect, we will have a discussion portal. This is all available for all NPTEL courses in which all participants can raise any kind of queries regarding the course and whatever help they want and so on. A team of TAs will be constantly looking into these queries and responding to them.

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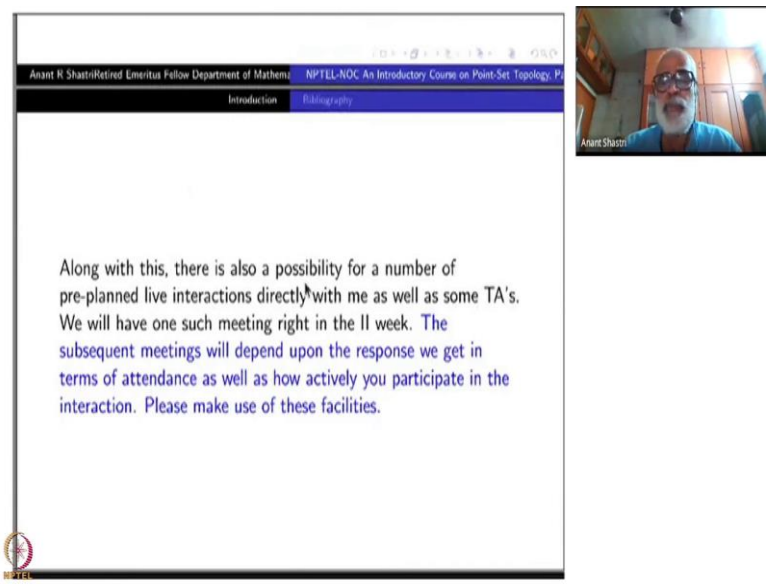

Along with this, especially for this course there is a possibility of number of pre-planned live interactions directly with me as well as some of the TAs. We will have one such meeting right in the 2nd week of whenever the course begins, ok.

The subsequent meetings will depend upon the response we get in terms of attendance as well as how actively you participate in the interaction. So please make use of these facilities, you know to the best of your abilities.

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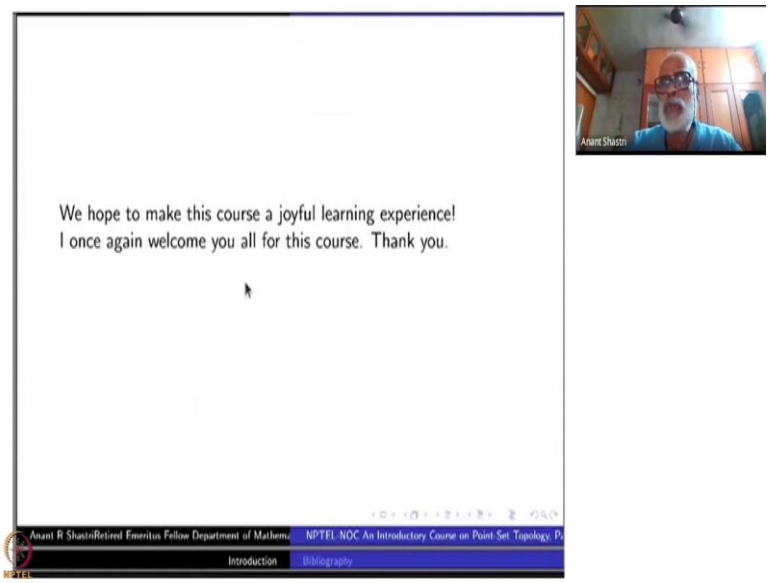

So, we hope to make this course a joyful learning experience. I once again welcome all of you to this course.

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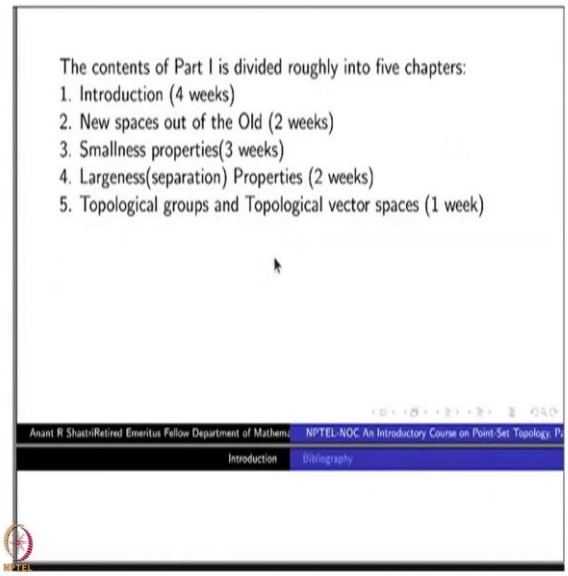

The contents of Part I is divided roughly into five chapters. This is only for convenience, there is nothing more than that. The first 4 weeks I have just called as Introduction. So

what it is, I will tell you a little more ok. The second part the 2 weeks we have I have called it as New topological spaces out of the Old.

So, it is a technique how to create new topological spaces. Smallness properties takes the third chapter for another 3 weeks. Then there are some so called Largeness properties that we will discuss for 2 more weeks. In the last week, we will give you some glimpse of Topological groups and Topological vector spaces. So, this is roughly the division of this entire course.

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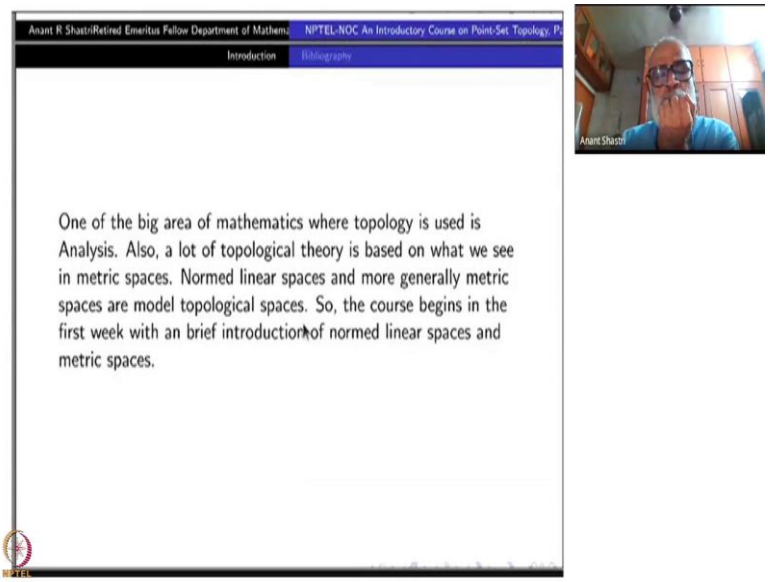

Out of this, what are the kind of things that you get. So, I will give you a little more elaboration on these topics. One of the big area of mathematics where topology is frequently used is Analysis. Also, a lot of topological theory is based on what we see, what we experience in metric spaces. So, the natural thing for us to start with is the normed linear spaces just like our prototype namely the Euclidean space so called $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^n$ and so on, ok.

And more generally drop out the linear part and just go to the metric part, those space are called metric spaces and then and those are the ones which give you a large class of topological spaces. So, the course begins in the first week with a brief introduction to normed linear spaces and the metric spaces, ok.

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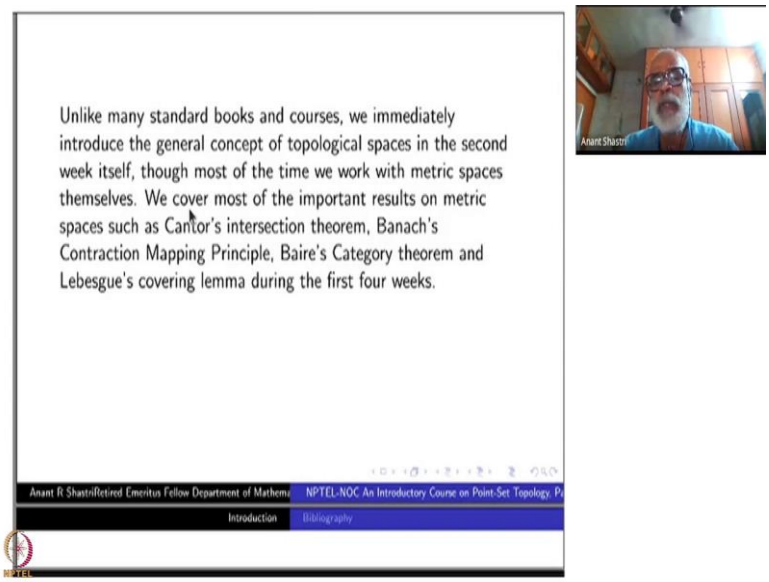

Unlike many standard books and courses, ok unlike I am telling you, we immediately introduce the general concept of topological spaces right in the second week, though most of the time we work with metric spaces themselves.

We cover most of the important results on metric spaces such as Cantor's intersection theorem, Banach Contraction Mapping Principle, Baire's Category theorem and Lebesgue's covering lemma during the first four weeks.

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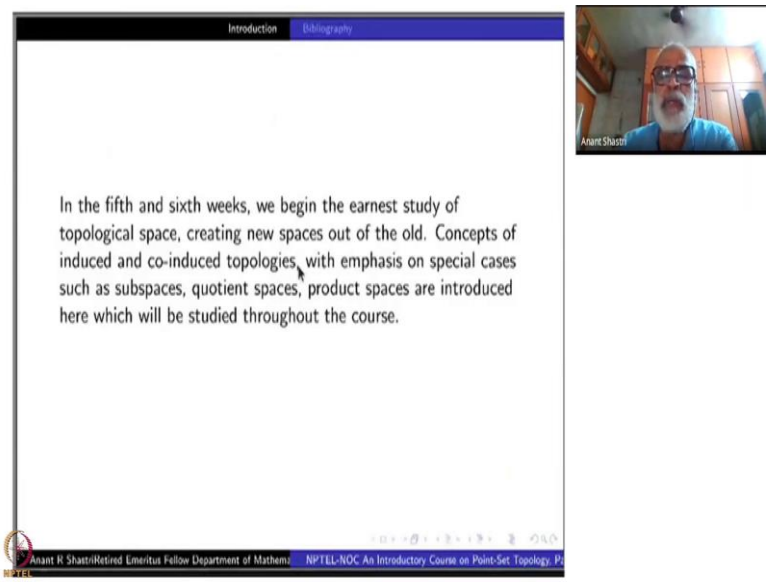

The fifth and sixth week, we begin the earnest study of topological spaces. Now, the metric spaces and normed new spaces are at the background now, creating new spaces out of the old. Concepts such as induced and co-induced topologies, be with emphasis on special cases such as subspaces, quotient spaces, product spaces. These are introduced here and studied and they will be again and again occurring within the course all over, ok.

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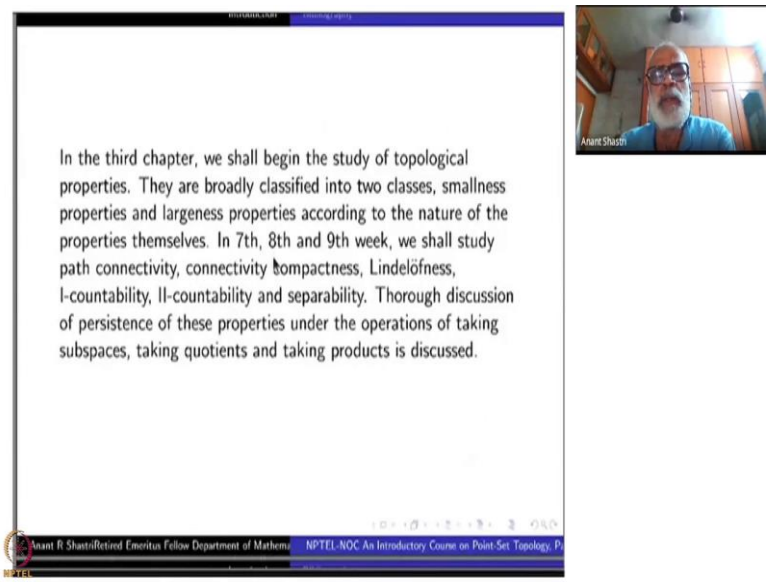

In the third chapter, that is in 7th and 8th and 9th week, we shall begin the study of topological properties. They are broadly classified into two classes, smallness properties and largeness properties, according to the nature of the properties that I will not go into detail now. So, that is the portion for 7th, 8th, 9th week. We shall study what are called as path connected spaces, connected spaces, then compact spaces, Lindelof and I-countability, II-countability, separability, ok.

So, then some inter-relation between these concepts throughout with a thorough discussion of the persistence of these properties. What is the meaning of persistence? Suppose you take a subspace, suppose you take a quotient space will the same property will hold there also? Suppose you take product of two spaces if each of them has that property, will the product will have that? These are called persistence properties. So, this will be discussed very thoroughly in this course, ok.

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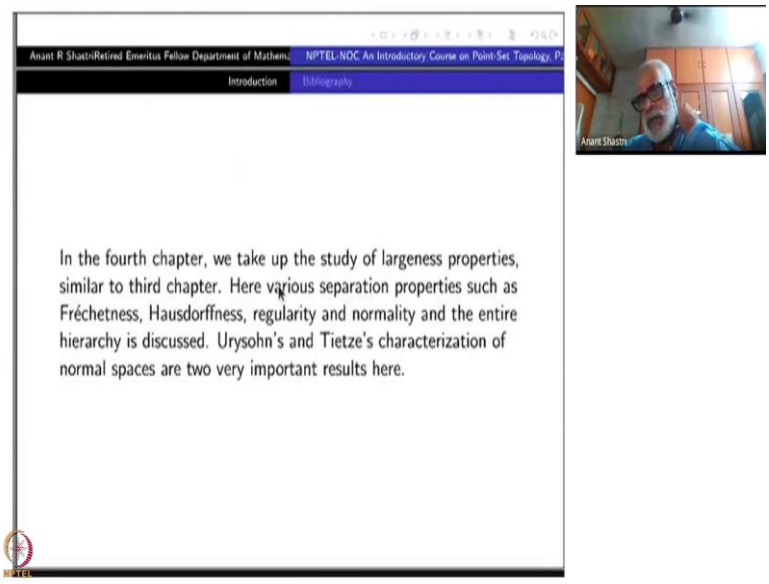

In the fourth chapter, the same kind of study will be done for what are called as largeness properties in each. Some of these largeness properties are precisely Frechet space, Hausdorff space, regular space, normal space etc. And then again interactions in, interrelations between them. Here two important results are Urysohn's lemma, it is called Urysohn's lemma, it is actually characterization. Similarly, Tietze's characterization of normal spaces.

So, the normal spaces are the sort of heros here with Urysohn's and Tietze's characterizations taking care of them as important results in this part.

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Finally in the last week, we briefly introduce the concept of topological groups and topological vector spaces. The first topic lays foundation for studies of Lie groups and the second one is a useful introduction to Functional Analysis wherein a lot of topology is used anyway, ok.

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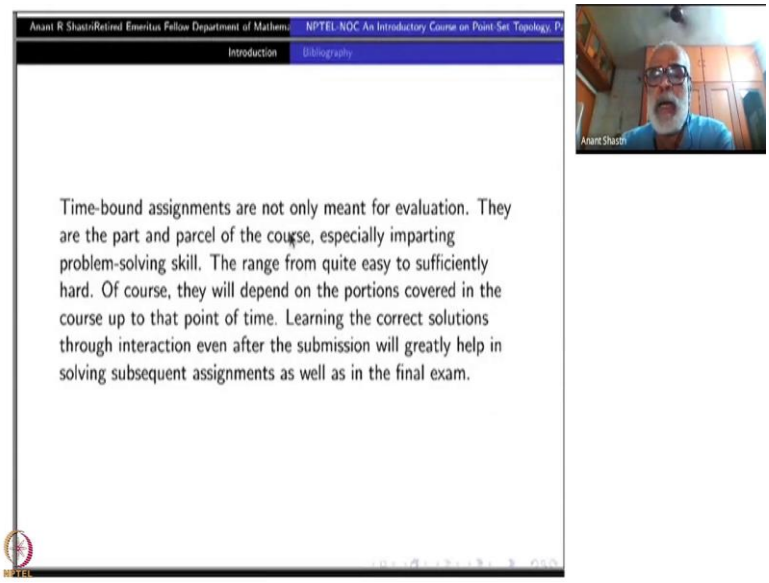

Time bound assignments are not only meant for evaluation. They are part and parcel of the course, especially imparting problem solving skills. So, we want all of you to take seriously this part of the course, ok. So, this is part and parcel of the course. The level at

which these things are there, it will be quite easy also, but a moderate level is also there, somewhat difficult things are also there. Especially as the coruse proceeds, you may find those things more and more difficult especially if you have ignored the earlier things. If you have learnt earlier thing properly, so throughout you may find a whole thing easy also.

So, learning the correct solutions through interaction or interactive sessions, asking questions, getting the answers and so on, that will help you not only in solving the problems later on the following assignment, they will be very very useful at the final exam also. That is the only way to consolidate things that you have learnt properly, ok.

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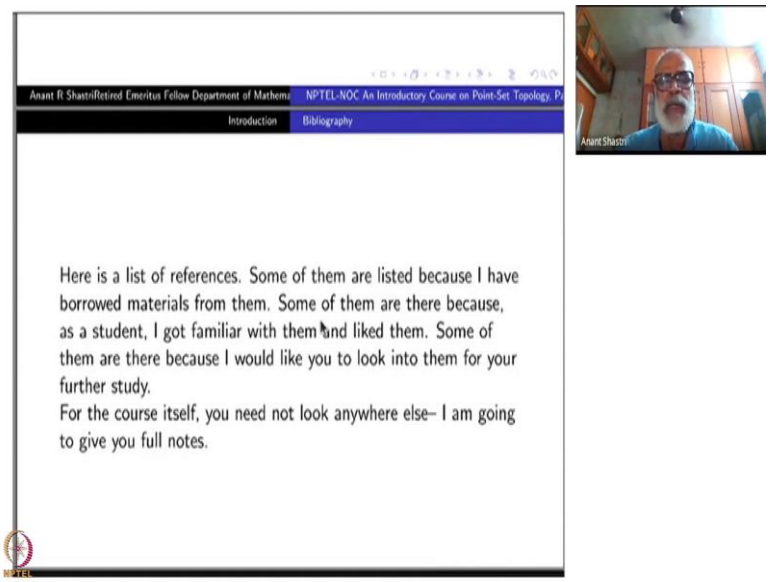

Here is a list of references. Some of them are listed because I have borrowed materials from them. Some of them are there because as a student, I got familiar with them and like them even today. Some of them are there because I would like you to look into them for your further study, actually I might not have studied them ok.

Of course, you need not look anywhere just for this course itself ok. I am going to give you the full notes right from the first day itself. So, that will be in PDF format not in this beamer file ok.

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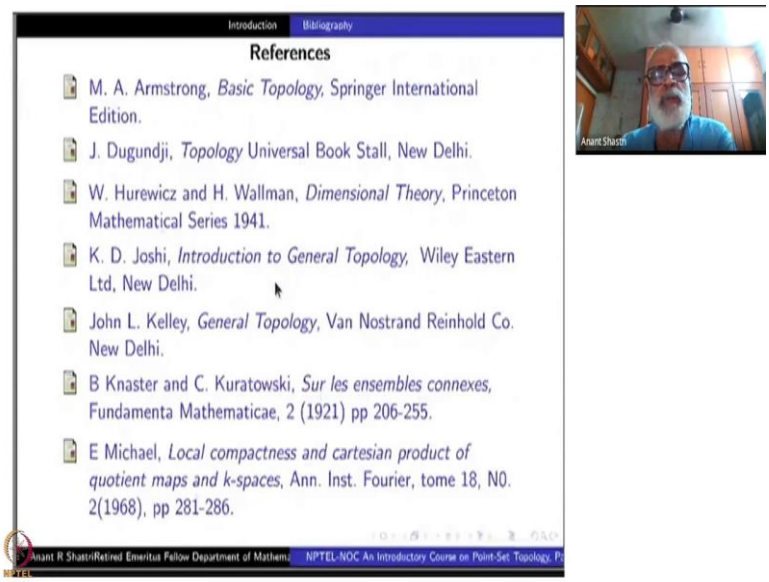

So, here is some of these things I will quote. I would like you to go through this one anyway, this list will be also there for you later on.

So, Armstrong's Basic Topology, Dugundji is a classical one Topology from Universal Book Stalls. Hurewicz and Wallman, this is a ancient book which is very very serious book on dimension theory. My own colleague, ex-colleague now they are all retired, K D Joshi wonderful book Introduction to General Topology. I warn you know, this Kelly's book is a hard book. This was the book, famous book at when we were students and I am quite thorough with this book.

Then this is some paper I have quoted because I have presented a presented some part of this one. Then this, E Michael is a wonderful topologist, he has made lot of research work. So, I have referred to one of his theorems here.

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Then there are these books Gregory Moore's book, this is this not a book, this is an article which will give you some historical background for open sets and closed sets. Recently, I have been familiar with this Wayne Patty's Foundations of Topology, it is a nice book. I have not studied this but I like this book.

Then our Functional Analysis by Rudin. My own book on Complex Analysis and Element of Differential Topology and Algebraic Topology. Then, there is this little book by Shirali and Vasudeva on Metric Spaces. So, I have borrowed some material from here also, it is a wonderful book ok.

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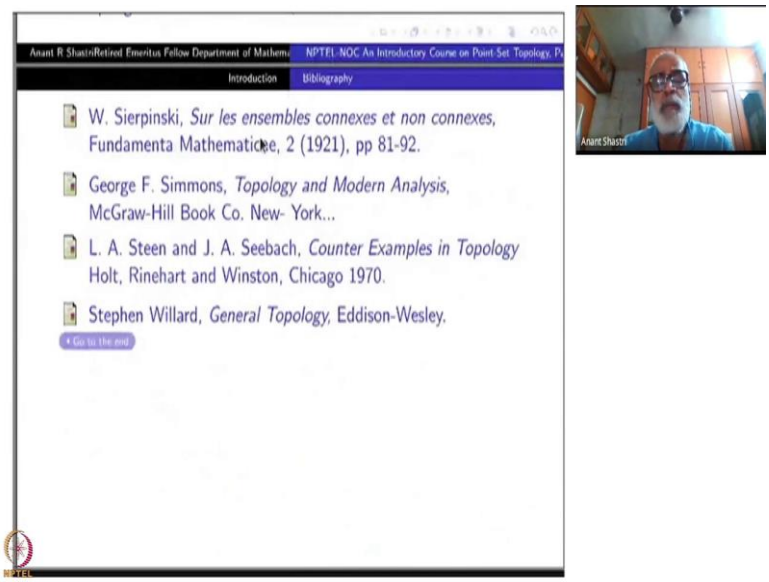

Lastly, but not leastly this George Simmon's book is the book I studied for my M.Sc and this is the very wonderful book, ok. Finally, there is this book by Steen and Seebach - Counter examples in Topology. This is a like a dictionary of all counter example, this is a wonderful book. Nowadays, it is available online and the present edition is much much larger than the edition which I had seen as a student, you know 30 years back.

And this is this Stephen Willard General Topology, which is a standard book in many university, but I have myself not studied this one. So, that is roughly the thing.

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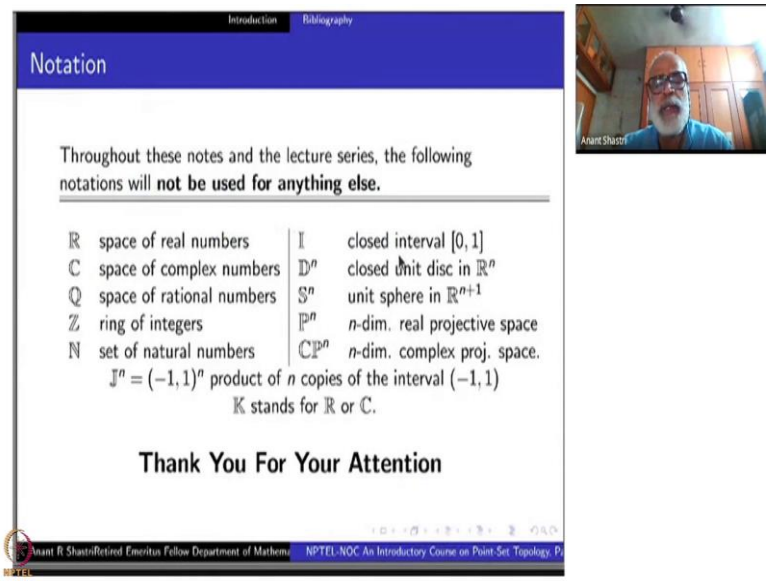

Then finally, this I will repeat again when the course starts, again and again I will tell you.

So these are the some standard Euler fonts I am using for nothing else during the course, ok. So, $\mathbb{R}$ this Euler font $\mathbb{R}$ is always used for real numbers, $\mathbb{C}$ is space of complex numbers, $\mathbb{Q}$ is space of rational number, $\mathbb{Z}$ is ring of integer, $\mathbb{N}$ is a set of natural number, this $\mathbb{J}$ is [-1,1] the interval; $\mathbb{J}^n$ is n Cartesian coordinate space now taken n times, ok. And then the common notation $\mathbb{K}$ standing for either $\mathbb{R}$ or $\mathbb{C}$, quite often we will work with real numbers or complex numbers both of them it will be true so, I will be use the notation $\mathbb{K}$.

This $\mathbb{I}$ always introduce the closed interval [0,1], $\mathbb{D}^n$ the closed unit disc, $\mathbb{S}^n$ the unit sphere in $\mathbb{R}^{n+1}$, $\mathbb{P}^n$ and $\mathbb{CP}^n$ are real and complex projective spaces, this you may not know right now but later on I will introduce them, ok. So, these are the standard notations I am going to use, ok. So, thank you for your attention. I hope you will all enjoy this course, ok.

Thank you.