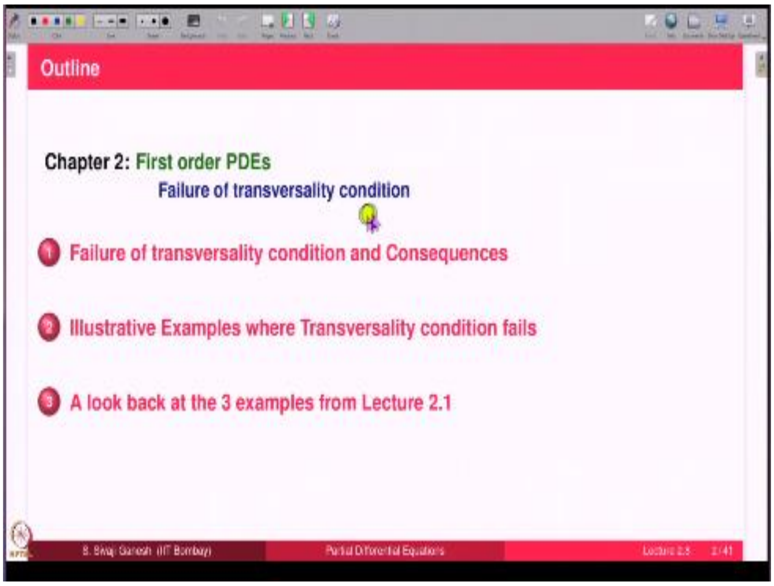
Partial Differential Equations
Prof. Sivaji Ganesh
Department of Mathematics
Indian Institute of Technology – Bombay

Lecture – 2.8
First Order Partial Differential Equations
Failure of Transversality Condition

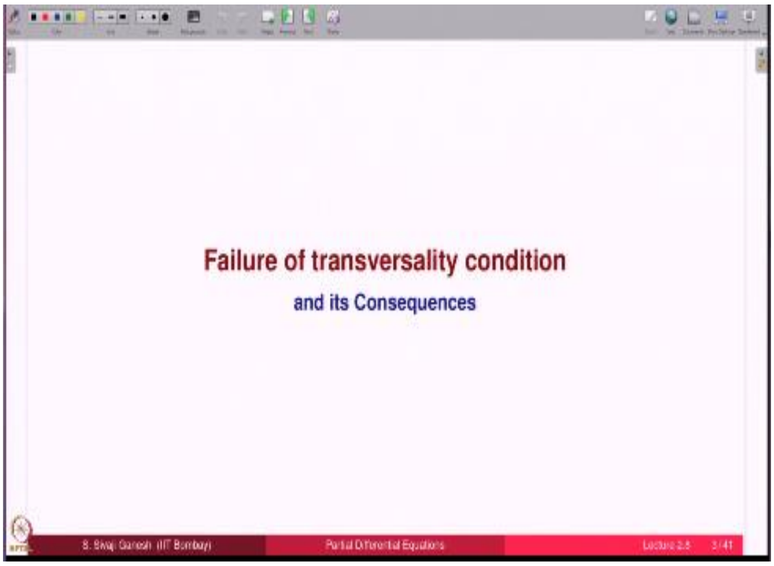
(Refer Slide Time: 00:25)



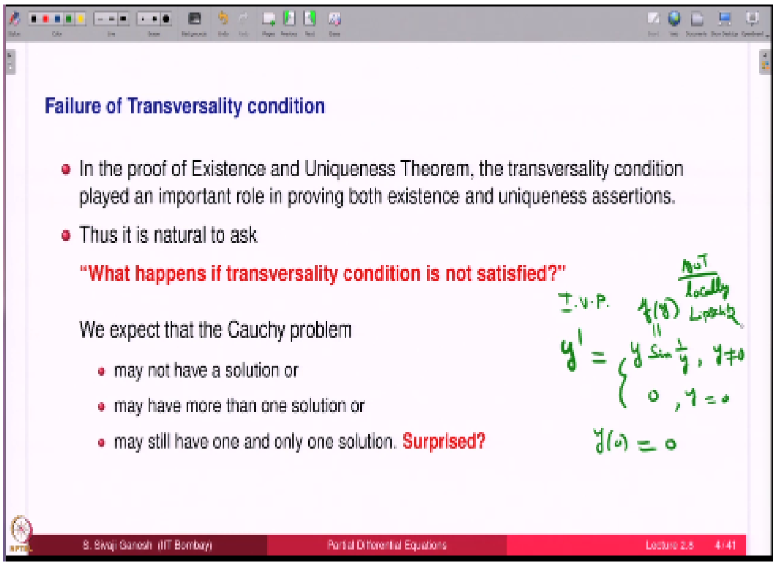
In this lecture, we are going to discuss the situation when the transversality condition fails. The outline for the lecture is as follows. First, we discussed failure of transversality condition and its consequences, what can be expected in when there is a failure of transversality condition. Then we look at the illustrative examples where transversality condition fails. There are 2 examples, they illustrate the 2 possibilities that we can conclude in the case when transversality condition fails.

And then we look back at the 3 examples, which were motivating examples, which were dealt in lecture 2.1. It was Cauchy problem, 3 Cauchy problem for the same differential equation, same PDE wherein, we saw one has exactly one solution, other Cauchy problem has infinitely many solutions and the third one had no solutions. So, we are going to analyse those examples in the light of this failure of transversality condition.

(Refer Slide Time: 01:22)



(Refer Slide Time: 01:25)



In the proof of existence and uniqueness theorem, the transversality condition played an important role in both existence and uniqueness assertions in proving both of them. So, therefore, it is natural to ask what happens if transversality condition is not satisfied. We expect that the Cauchy problem may not have a solution that is existence is violated no existence, second aspect is uniqueness is violated.

That means it has more than one solution or it may still have one and only one solution. Are you surprised? No need to surprise because the theorem, if you remember the assumption of transversality condition was only a sufficient condition under that condition we have proved existence and uniqueness. So, if the sufficient condition is not satisfied, still we may have the existence of a unique solution, I would like to recall your attention to one of the very important ordinary differential equation problem.

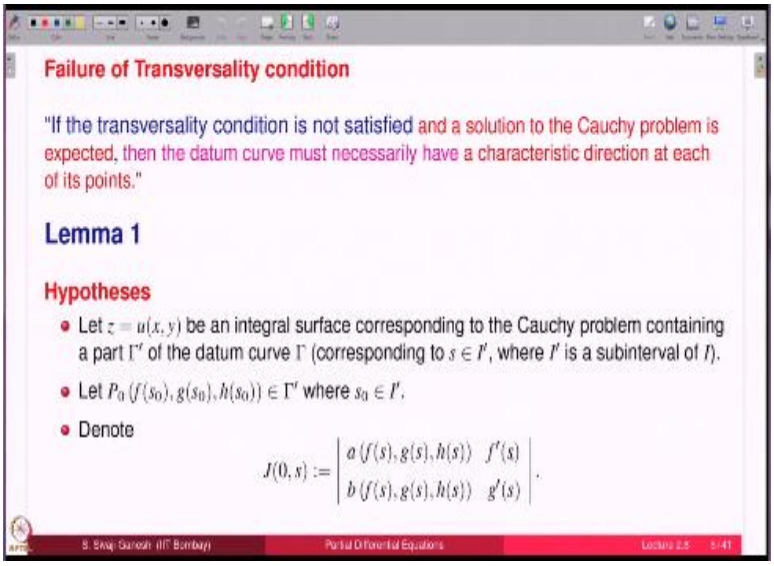
Let me write that equation y dash = y sin 1 by y obviously, when y is not 0 and 0 when y = 0, that is to be expected and then y of 0 = 0. So, this initial value problem, the theorem that we have is a Peano's theorem and Cauchy Lipschitz Picard's theorem. When we apply Peano's theorem, yes, the right hand side is a continuous function. Therefore, there is no problem. It has a solution.

Now, the right hand side is not a Lipschitz function that you can check it because not ellipses, Cauchy Lipschitz Picard's theorem cannot be applied and uniqueness, we concluded. We cannot do that. So therefore, we can expect that maybe there are more than one solution. In ODEs, it is so happens that when you have 2 different solutions for the same initial problem. There is infinitely many solutions, actually. So, that is the only possibility.

So, you have no solution or one solution or infinitely many solutions, these only possibilities for ODEs. Now here, it so happened this problem, the right hand side is not ellipses function, this is not local ellipses. If you call this f of y, there is no x dependent, so I do not try it. So, this is a locally not locally ellipses. It is not local ellipses. Therefore, we cannot apply the theorem but still it so happens. This has a unique solution.

So, that I leave it for you if you are not come across this question already, please look at it and show that solution is unique. So, violation of sufficient condition means nothing, no conditions can be done that is why you may still have one and only one solution. It can happen.

(Refer Slide Time: 04:25)

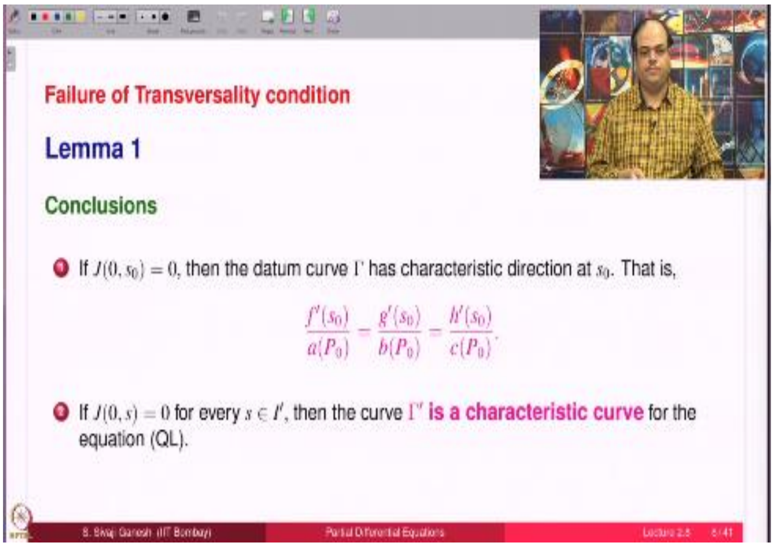


So, if the transversality condition is not satisfied and a solution to the Cauchy problem is expected that means the Cauchy problem has a solution, then something should happen. The datum curve must necessarily have a characteristic direction at each of its points, which means that it is a characteristic curve. Now, we have a result lemma 1. It is a very simple result.

Let z = u x y be an integral surface corresponding to the Cauchy problem for Quasilinear equation, QL containing a part of the datum curve which is called gamma dash that corresponds to, see, gamma corresponds to x belongs to y; gamma dash corresponds to x belongs to I dash, where I is a subinterval. Now, take a point on gamma dash and denote J 0 s is the precisely that determinant which comes in transversality condition; a b here, f dash g dash.

a b c tangential direction for the base characteristic curve. Base characteristic passing through the point f s, g s but because it is Quasilinear, it is tied up with the value h s also. f prime g prime is a tangential direction to the base characteristic curve, not sorry, not base characteristic curve, to projection of gamma which is gamma 2. It is a tangent for that.

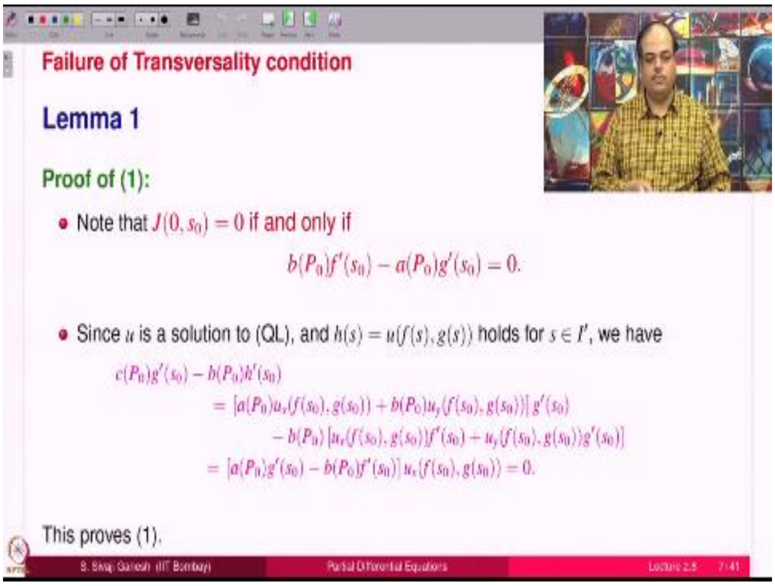
(Refer Slide Time: 06:04)



Conclusion: If J of 0, s 0 that means that determines 0 at one point, then the datum curve has a characteristic direction at that point. What does that mean? Datum curve has characteristic direction at s 0. What is characteristic direction at s 0? a P 0, b P 0, c P 0 because P 0 is a point f s 0, g s 0. And what is the saying the datum curve has a characteristic direction?

It is tangent which is a f prime s 0, g prime s 0, h prime s 0 that tuple is proportional to a b c. And if it is 0, J 0 s is 0 for every s in a subinterval I dash, then gamma dash is a characteristic curve for the Quasilinear equation.

(Refer Slide Time: 06:53)

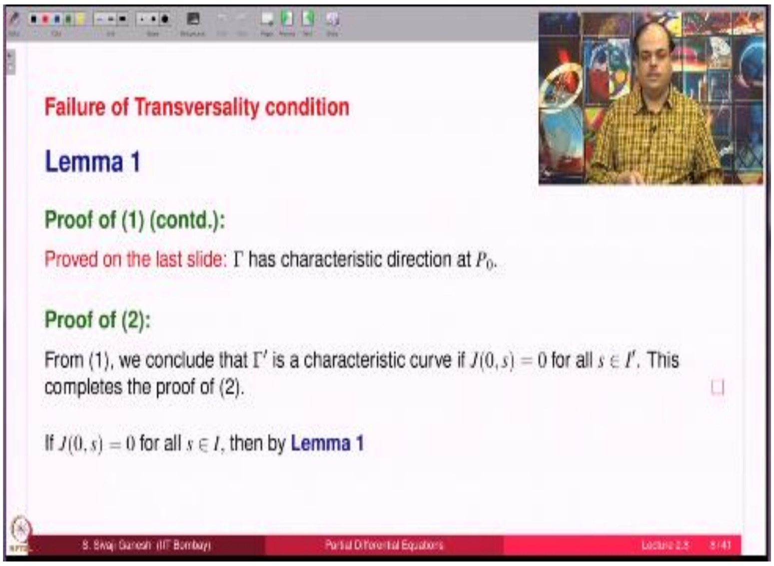


Proof of 1: So, J 0 s 0 is given to be 0. And we want to show that datum curve has a characteristic direction at that point P 0. So, J 0 s 0 is 0 if and only if this is just expansion of that determinant. a b one column, f prime g prime on other column that determinant is precisely the left hand side that is equal to 0. Since, u is the solution to Quasilinear equation and h s = u of f s, g s wholes, we know this, we have.

Let us compute this c P 0 g prime minus b P 0 h prime. What is c P 0 from the equation? Here, a u x + b u y = c therefore, that c P 0 is equal to this quantity in the brackets. a u x + b u y at the respective points a and b or a b and u x, u y are evaluated into g prime s 0 minus b P 0 into h prime s 0. How do I get a h prime s 0? From here, differentiate this with respect to s, take s = s 0 that will give you this by chain rule, u x f prime + u y g prime.

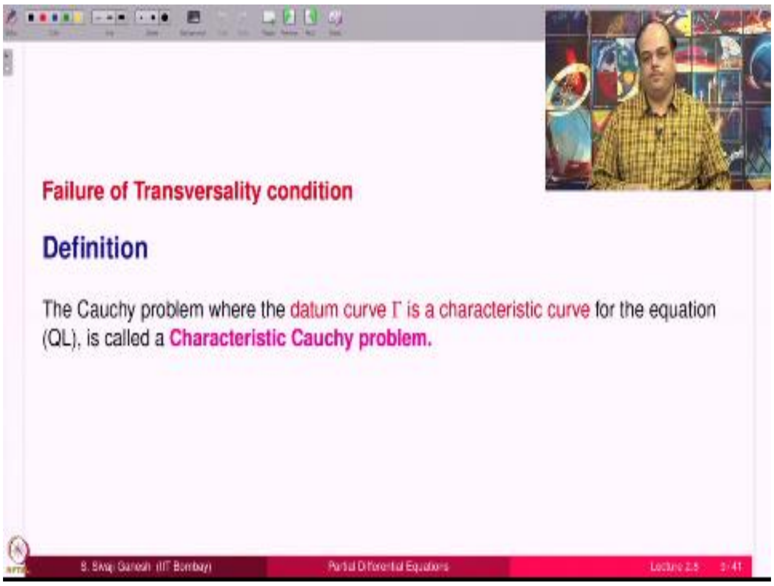
Now, that is equal to after simplification is this, but here, you observe this thing in the bracket is precisely the left hand side with a minus sign. So, that is 0. Therefore, this is 0. So, this proves 1.

(Refer Slide Time: 08:27)



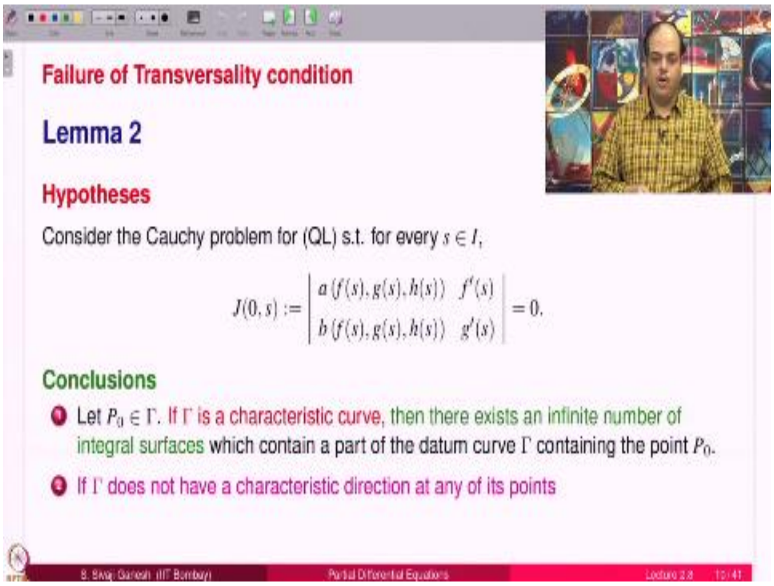
So, we have proved on the last slide that gamma has a characteristic direction at the point P 0. Proof of 2 from 1, we conclude that gamma dash is a characteristic direction if J 0 s = 0 for all s in I prime. What is the part 1 says? Whenever J 0 s = 0 and we have a solution, then that has to be a characteristic curve that the datum curve has characteristic direction at that point, but in this case, it happens for s in I dash. Therefore, gamma dash is a characteristic curve that is the definition of a characteristic. If J 0 s = 0 for all s, then by lemma gamma itself is a characteristic curve.

(Refer Slide Time: 09:15)



So, when such a thing happens the Cauchy problem is called characteristic Cauchy problem. A Cauchy problem where the datum curve is a characteristic curve that is called a characteristic Cauchy problem.

(Refer Slide Time: 09:28)

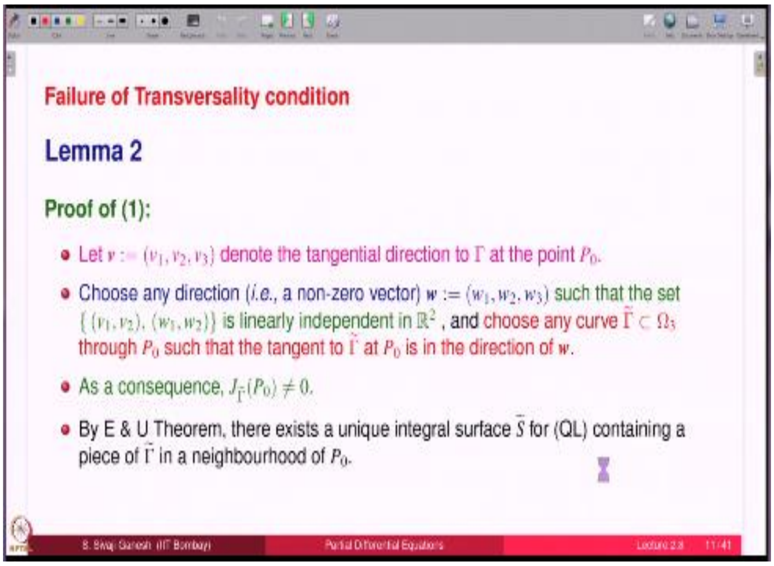


Now, we have lemma 2. This is what is the result which is what happens when transversality conditions, what is to be expected of solutions on how many in number? So, consider the Cauchy problem for QL such that for every s, J is 0. Conclusions: take a point in gamma. If gamma is a characteristic curve, then there exists infinite number of integral surfaces, infinite number which contains a part of the datum curve gamma containing the point P 0 that is always there, local solution we have been talking about.

So, these will always be there, but here, the interesting the infinite number of integral surfaces if gamma is a characteristic curve. Earlier, lemma 1 said if you have a solution, then curve gamma must be characteristic curve if the J is identically equal to 0. Now, this is opposite kind of converse if J is 0 throughout and gamma is a characteristic curve, then not only there is a solution, but actually infinite number of solutions exists.

And if gamma does not have a characteristic direction at any of its points, then no solution. Cauchy problem does not admit a solution. Note: in this theorem, we are assuming that J is 0 throughout for all s in I.

(Refer Slide Time: 11:00)



So, how do we show infinitely many solutions exists? What we do is this. Let v denote v 1, v 2, v 3, denote the tangential direction to gamma at the point P 0. So, imagine, this is your gamma, this point is P 0, this is a tangential direction which is this one. Now, choose any direction w such that the set v 1, v 2 and w 1, w 2, this is in R 2, is linearly independent in R 2. We will see in a moment why this we are assuming this. Is it possible to choose?

Yes, it is possible to choose because once you know v 1, v 2 that is after all one vector in R 2, you can find infinitely many such vectors w 1, w 2 so, that this set is linearly independent and w 3, you can simply add to that. There is no condition on w 3. The only condition is on w 1, w 2 and v 1, v 2. So, it can be done. In fact, infinitely many ways you can do this that is the reason why we are going to get infinitely many solutions. We will see that.

So, choose any curve gamma tilde in omega 3 that is we had a gamma, we had a point P 0 where we had some tangential direction. Now, what we are asking is choose anybody else, tangent has to be different and something like that. This is another curve. In fact, it can be a curve. It can be a straight line as well we are going to see that, suppose your curve details, no problem. So, this is gamma delta.

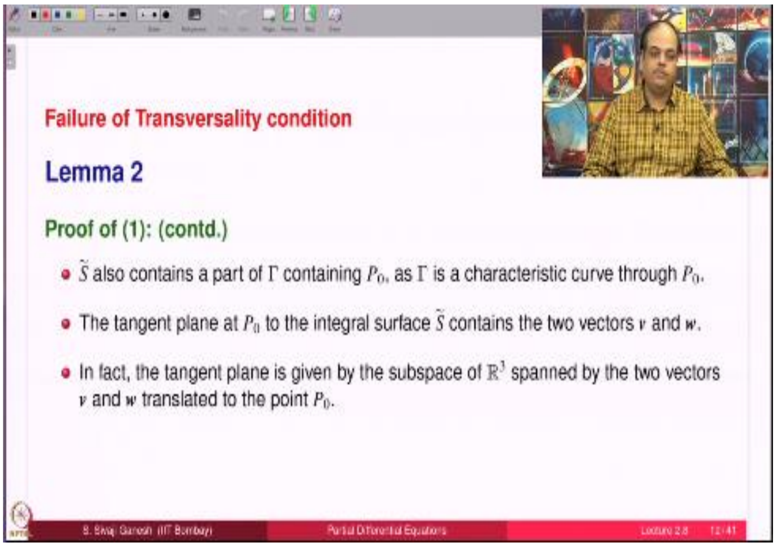
Take any curve gamma delta in omega 3 through the same point P 0 such that the tangent to gamma dash at P 0 is in the direction of w that one easy way of ensuring that is to take straight line. So, you have gamma here, this is the point P 0, you take some curve, I know, I do not want to take curve. So, you had a gamma, you had a point P 0 and here through P 0, you fixed a direction w, you take a line that is it.

A line passing through P 0 with the direction w that is simply given by P 0 plus some parameter alpha times w. These are vector form of the line. So, of course, the tangle will be the direction w. So, the consequence J gamma tilde P 0 will be nonzero because on one hand, you had f prime g prime right earlier and here you had that a b because of that, you got things 0. Now, you are taken somebody who is lead.

So, this was 0 earlier that means f prime g prime means proportional to a b. Now, I replaced this with a new thing w 1, w 2. This will be nonzero because this is not linearly independent, this sorry, w 1, w 2, v 1, v 2 are linearly independent. Therefore, as a consequence w 1, w 2 and f prime s 0, g prime s 0, both these columns are linearly independent. So, determinant is going to be nonzero.

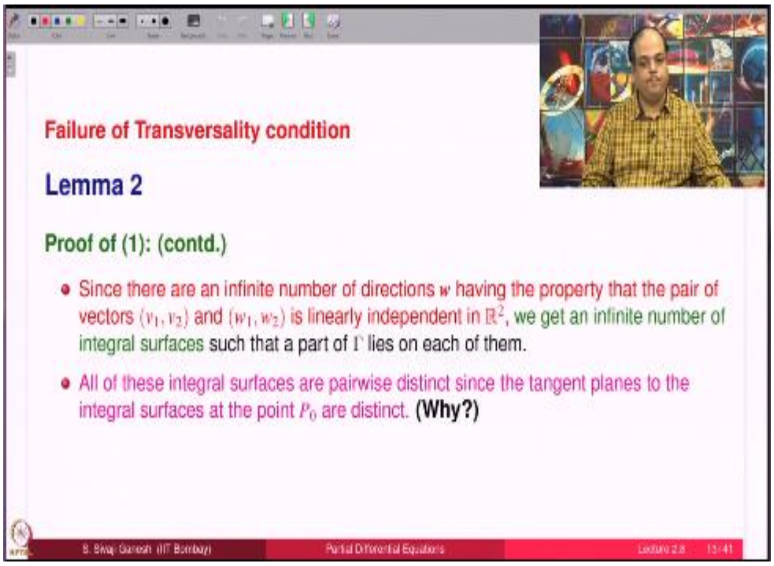
Now, I can apply existence uniqueness theorem. So, what happened is that initially we had a curve gamma, there was some problem because J was 0, we could not apply the existence uniqueness theorem. Then we chose the another curve gamma tilde for which existence theorem can be applied. As a consequence, we get a kind of surface, which contains part of this gamma tilde and as a consequence, a part of gamma will also be there, because that is a characteristic curve.

(Refer Slide Time: 15:19)



So, the surface is called S tilde that contains a part of gamma alpha, because gamma is a characteristic curve. So, the tangent plane at P 0 to a S tilde contains 2 vectors v and w; v is there because the curve gamma is there on the surface and v is a tangential direction basically it is a characteristic division at v 0 and w is also there in the tangent. So, in fact, the tangent plane is given by the subspace of R 3 spanned by the 2 vectors v and w that is the tangent space and tangent plane will be at the point P 0, origin will be at p 0.

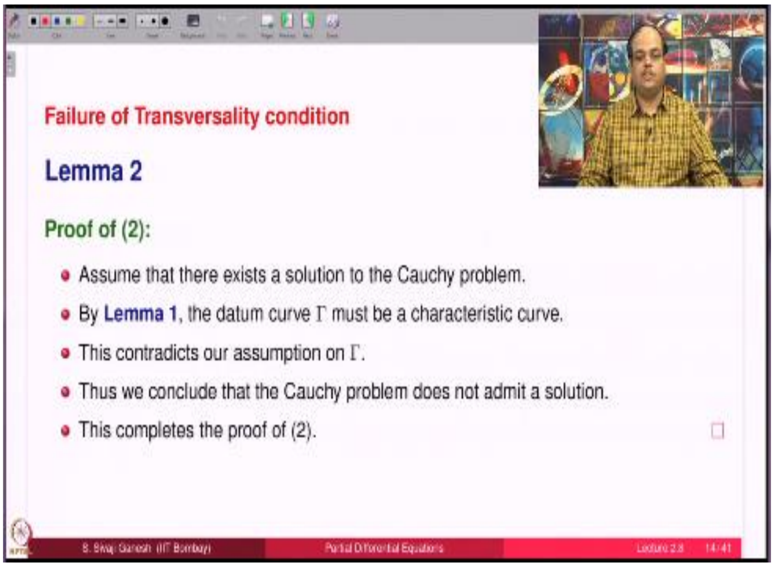
(Refer Slide Time: 16:06)



So, we are gone that. So, there are infinite number of directions w having the property that v 1, v 2, w 1 and w 2 is linear independent. We already saw this. Therefore, we get an infinite number of integral surfaces such that a part of gamma lies on each of them. The part might vary from surface to surface. All of these integral surface are pairwise distinct because the tangent planes are having different directions.

The orientation of the tangent of course, tangent plane is always a 2 dimensional quantity, but it will be changing. One guy who is always be there in that direction in the tangent plane is the direction of the characteristic direction, but the other direction is the w. As you change w, 2 planes are not same.

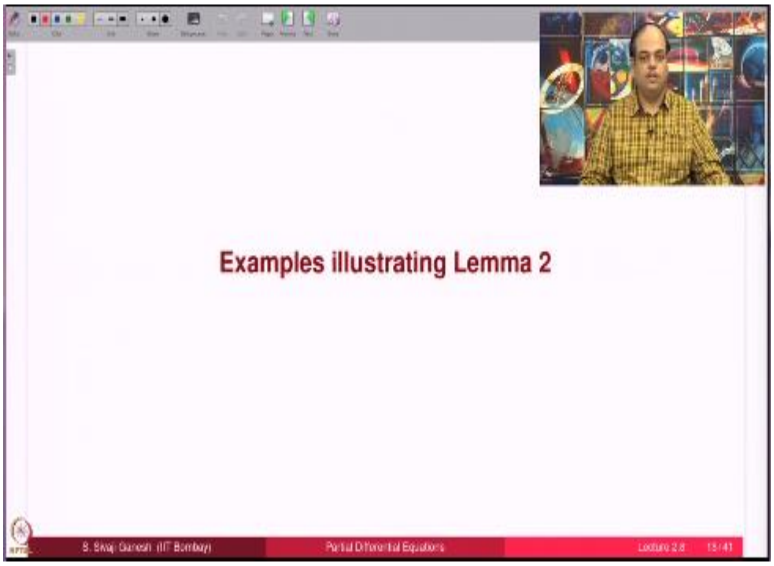
(Refer Slide Time: 16:55)



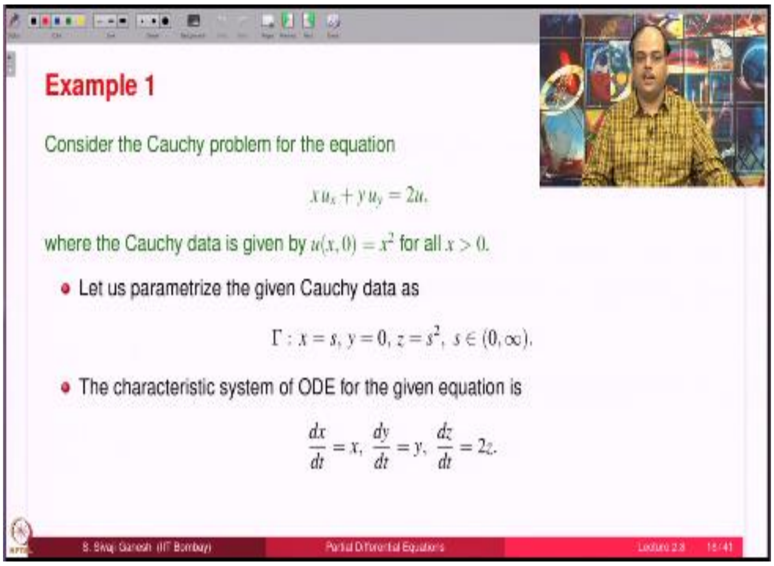
You have infinitely many solutions, because you have infinitely many choices for w. So, second proof of second one assume that there is a solution. Then by lemma 1, datum curve must be a characteristic curve. And we have assumed, it does not, not only it is not a characteristic, it has, it does not have a characteristic direction at any of its points that is a hypothesis of part 2 of lemma 2.

So, it contradicts this. Therefore, Cauchy problem does not admit a solution. So, this completes the proof of 2.

(Refer Slide Time: 17:31)

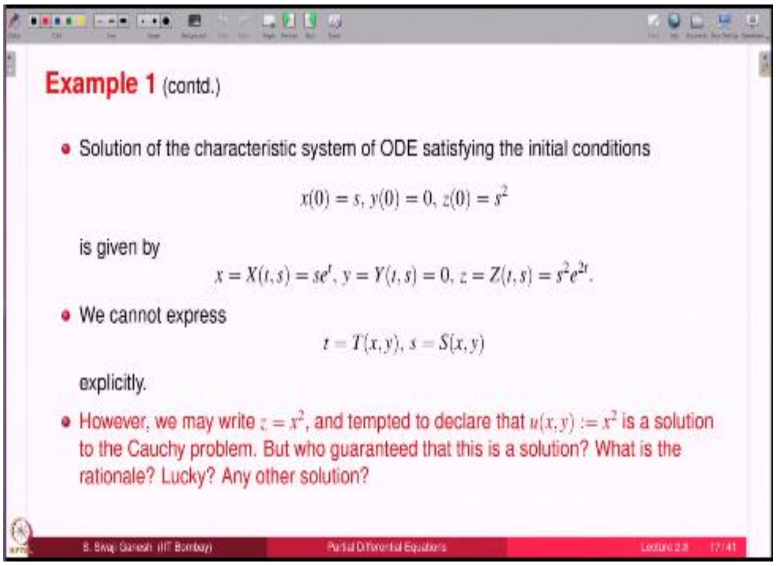


(Refer Slide Time: 17:36)



Now, we are going to illustrate these 2 conclusions of lemma 2 with 2 examples. The first example, we are going to consider is this equation x u x + y u y = 2 u, u x 0 = x square is the Cauchy data given for x positive. Let us start solving. So, first thing is you need to parameterize Cauchy data which is this. x = s, y = 0, z = s square, then we have to write down the characteristic differential equations, which is this. Now, we have to solve this with the initial conditions, so, that at time t = 0, this, it passes through a points of gamma.

(Refer Slide Time: 18:11)



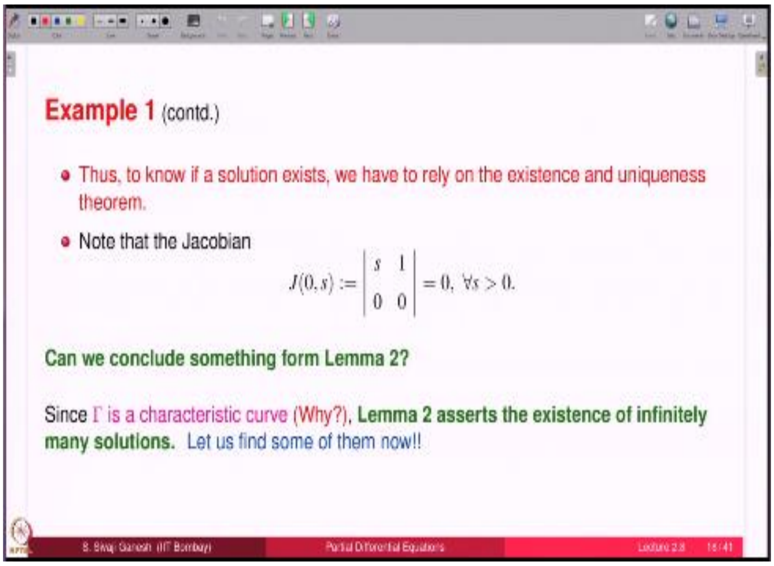
So, this is the initial conditions. Solutions are very simple. One can write this. Equations are linear equations, dx by dt = x solution is constant times the e power t. Here also constant times the e power t. Here constant times e power 2 t. What that constant turns out to be coming from this initial data. So, these are the solutions. Now, what is the next step?

That the existence uniqueness theorem told us using the first 2 equations, namely for x = X t s and y = Y t s, express t as a function of x y and s as a function of x y, go on substitute the third one and propose your solution. If all is well, that is going to be a solution. And here, we cannot express explicitly, you can see x = s e power t, y = 0, there is no way you can solve both s and t from this.

But certainly, we see that a we look at the third one that is s square e power 2 t that is s e power t whole square, therefore, Z t = s square is a solution. We have an I for a solution, maybe and tempted to declare that my solution is going to be u x y = s square. It is also Cauchy problem. Of course, we can substitute and see it actually solves, but who guaranteed that this will be a solution.

Because we have not done the application of the theorem, because if you want to apply the theorem, you have to check that the transversality condition is met. You have not done that. So, somehow you guessed lucky it is working that is it. Can you say this is the only solution? We do not know. So, this question is still to be settled.

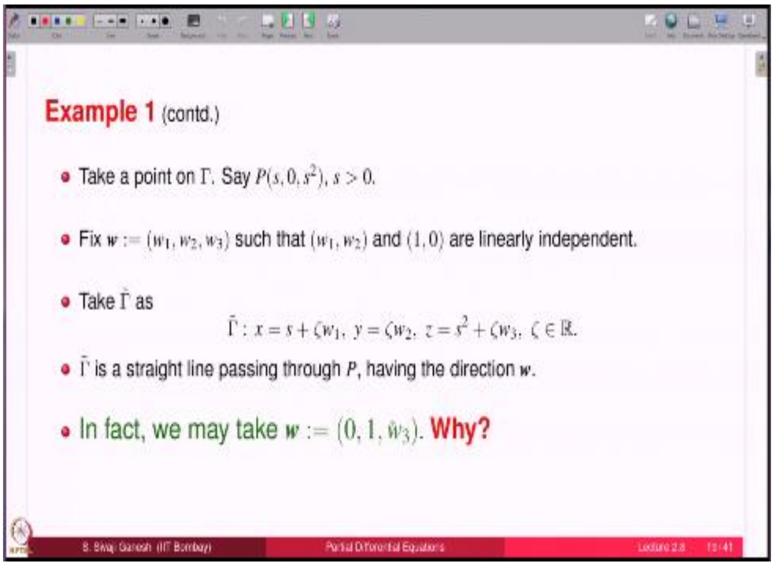
(Refer Slide Time: 19:57)



So, let us go through formally, go through the theorem to know that if a solution exists or not, we have to rely on the theorem. Look at the Jacobian that is 0 everywhere. So, I cannot apply the theorem. So, anything can happen. Now, can we conclude something from lemma 2? Gamma is a characteristic curve, one can check. It just means, one slightly one more thing to check.

What is the tangential direction to gamma? And what is the characteristic direction at this point? Both will be proportional. Please check that the lemma 2 asserts the existence of infinitely many solutions. Done. Let us find some of them, theorem says infinitely many; theorem also gives an algorithm which can be implemented. We will do that.

(Refer Slide Time: 20:48)

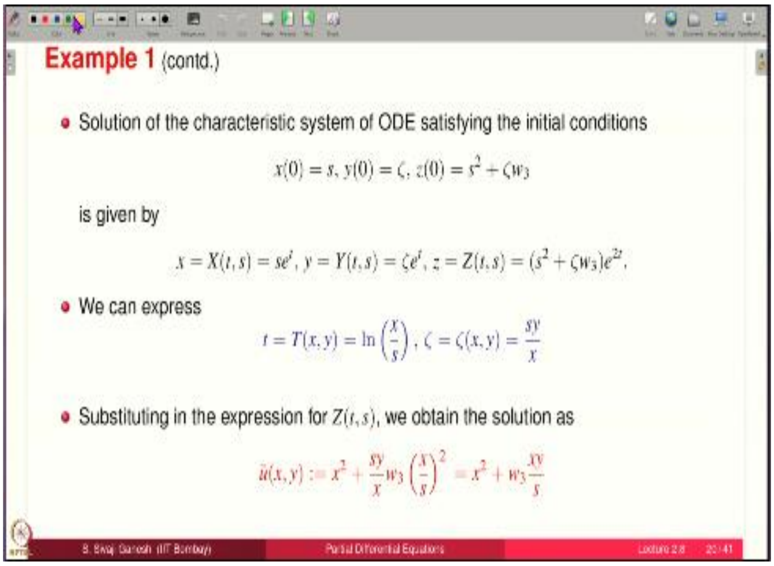


So, take a point on gamma say this point P, I do not really need to put s 0. So, I just put s, all of us know that s is fixed in this. The point P 0 is fixed. These how a typical point looks on gamma s 0 s square. Fix a w such that the w 1, w 2 and 1,0, what is 1,0? It is that v 1, v 2 that is 1,0 in this example that is linearly independent done. Now, take gamma tilde as this. This, if you see just a straight line, passing through the point P have the direction w.

P, I have used zeta here w; if you write in component form is what do you get? Yes, P is what? s 0 s square + zeta w 1, w 2, w 3. So, if you expand, you will get this. So, it is a straight line passing through P having the direction w. In fact, we may take w = 0,1 w 3 that means I am setting w 1 = 0, w 2 = 1. Why is that? Because what do you want to show infinitely many solutions even now, suppose I show for each such w, there is a solution.

Do we still have infinitely many? Yes, because w 3 is still arbitrary. w 3 is in R. One can do much more, but I am already showing infinitely many solutions. There are many more solutions also.

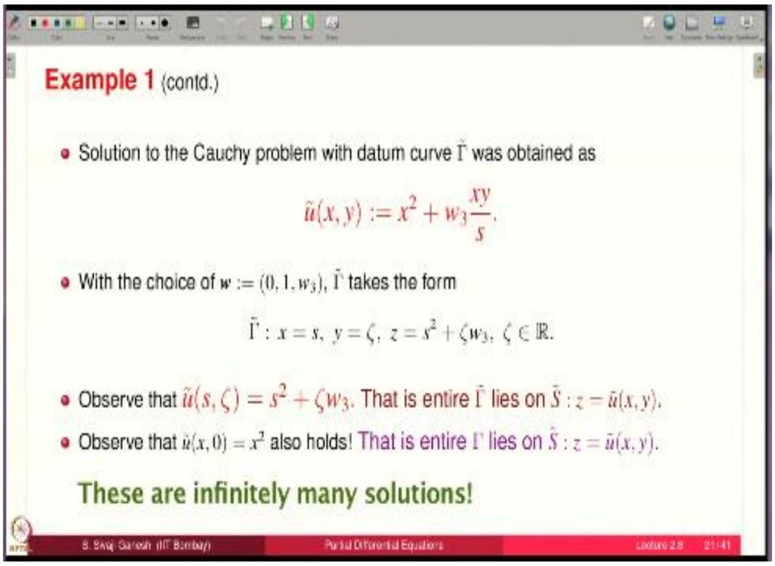
(Refer Slide Time: 22:24)



Solution of the characteristic system of ODE satisfying these initial conditions which are now based on gamma tilde is given by x = s e t, s e power t, y = zeta e power t, z = s square + zeta w 3 into e power 2 t. Now, we can express t and s in terms of x and y as follows t = T of x y = log x by s and zeta is equal to s y by x. Please note that s is fixed as far as gamma tilde is concerned.

Substituting in the expression for Z t s, we obtain the solution as u tilde of x y = s square that is s square into e power 2 t is x square + s y by x that is the zeta here into w 3 into e to the power 2 t which is x by s whole square. It comes from here. Some simplification, the solution u tilde become x square + w 3 x y by s.

(Refer Slide Time: 23:45)



Solution to the Cauchy problem with datum curve gamma tilde was obtained as this on the previous slide u tilde of x y is x square plus w 3x y by s. Please not, there is not one solution, this is infinitely many solutions, because there is a arbitrary w 3 in the formula. So, with the choice of w = 0,1 w 3 gamma tilde takes the form; we introduce gamma tilde with respect to w 1, w 2 w 3, but we made a choice that w will be 0,1 w 3.

Therefore, gamma tilde looks like this x = s, y = zeta z = s square + zeta w 3; zeta belongs to real numbers. Observe that u tilde of s zeta that is substitute x = s and y = zeta, you get x square + zeta w 3. But, what is s square + zeta w 3? It is this, the z coordinate. What is s and zeta? Their x and y coordinates respectively. It means that the entire gamma tilde lies on the surface s tilde.

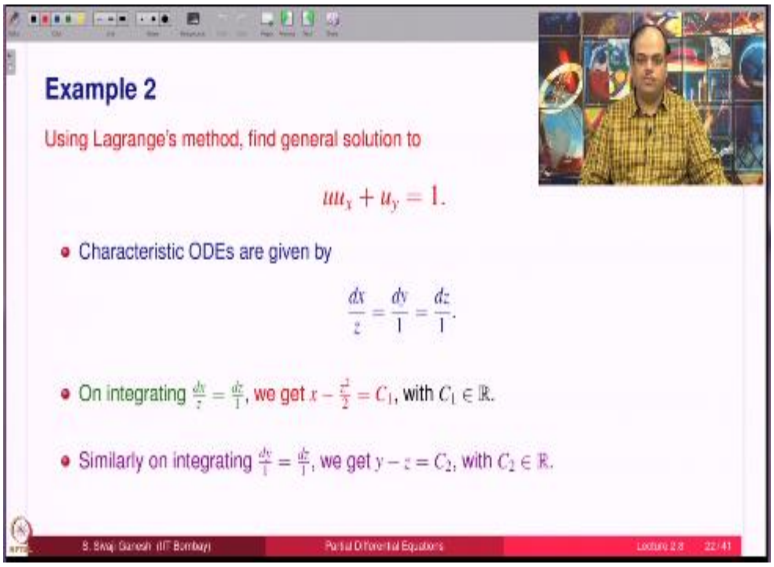
Does that surprise you? Because the applying existence and uniqueness theorem gives us only for zeta nearby 0 that gamma tilde lies on s tilde. It should not surprise because the existence and uniqueness theorem is a very general one. It is applicable for all sorts of Cauchy problems and what we are working with here is a specific Cauchy problem, therefore, the solution can behave better.

Next question is actually: can we apply the theorem? Of course, we have solved and got this expression for u tilde of x y is the theorem itself applicable. Recall our original problem is a characteristic Cauchy problem, where we had this determinant. This s 0 corresponds to a and b and this corresponds to original f prime and g prime and that was 0 for all s. So, it was a characteristic Cauchy problem.

Then what we did is that this continues to be 0. Now, we change to gamma tilde. When we change the gamma tilde, what is the corresponding f dash g dash? That is w 1, w 2 which is 0,1 which is equal to s and which is not equal to 0 because we are in the region s > 0. So, the theorem is applicable and that will give you as you know local solutions with respect to the datum curve, but however, we have obtained here by the computations, we see that we indeed have a global solution with respect to the datum curve.

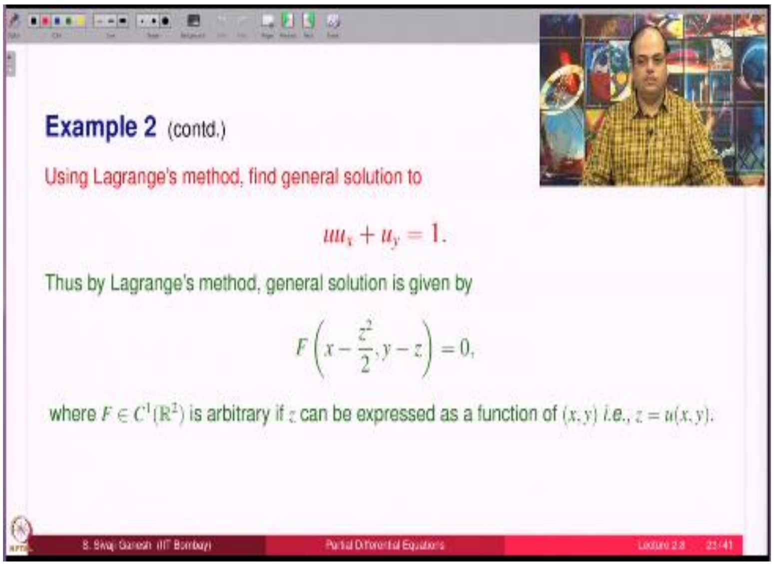
Observed that u tilde of x 0 = x square also holds. What does that mean? That means the entire gamma lies on s tilde that means, our original datum curve itself lies on s tilde. So, note all the solutions are also global with respect to domain. They are in fact defined on R 1. They are defined on R 2. So, there are infinitely many solutions.

(Refer Slide Time: 26:57)



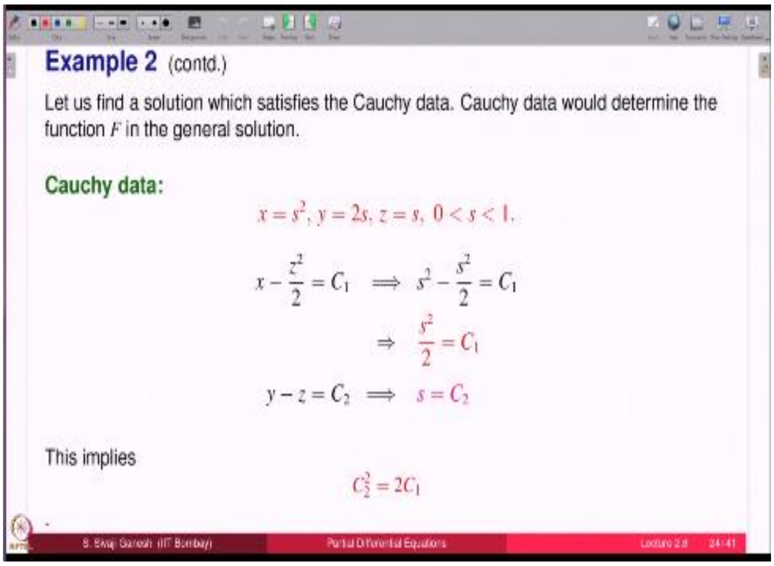
Now, let us look at the second example. This, I think, we already solved using Lagrange's method. u x + u y = 1 if not, let us do again. Let us do now. So, the characteristic system of ODE is dx by z = dy by 1 = dz by 1. So, integrating this set of equations, we get this x – x square by 2 = C 1. Other one will give you y – z = C 2.

(Refer Slide Time: 27:28)



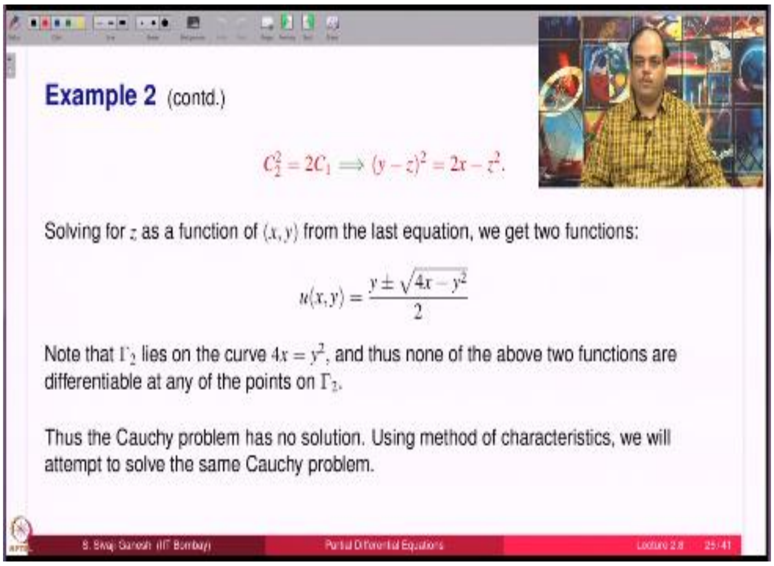
Therefore, Lagrange's method says f of C 1 C 2 = 0 that means take any arbitrary function f and this equal to 0. Of course, it remains that this is valid for only those x y z for which this kind of quantity, this tuple lies in the domain of f that is always there. Second thing is that you should be able to solve for z in terms of x and y, only then I would like to call this a solution that is done.

(Refer Slide Time: 27:57)



Now, we will substitute C 1 C 2. So, now, I think, we will find the f using the Cauchy data that should fix the f, then we will get the solution that is the idea. Whether it will be successful or not, we will see. So, x = s square, y = 2 s, z = s, this quantity equal to C 1, substitute for x = s square and z is equal to s, you get this. That will give you, x square = 2 C 1. Now, from the other one, y – z = C 2, when you substitute the values for y and z, you get s = C 2. Therefore, there is a relation between C 2 and C 1 that we get C 2 square = 2 C 1.

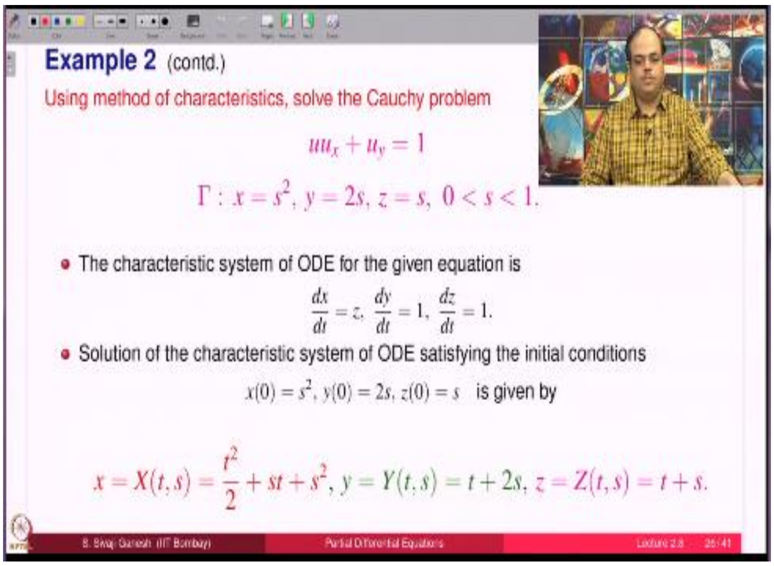
(Refer Slide Time: 28:40)



So, we are going to substitute for C 2 whatever y – z and C 1 this, that gives quadratic equation for z. When solved, we get this expression y plus or minus root 4x – y square by 2. Now, gamma 2 the projection of datum curve lies on this curve on 4x – 4x = y square. Therefore, this function is not differentiable, because whenever 4x – y square is 0. There is trouble, square root function is not differentiable.

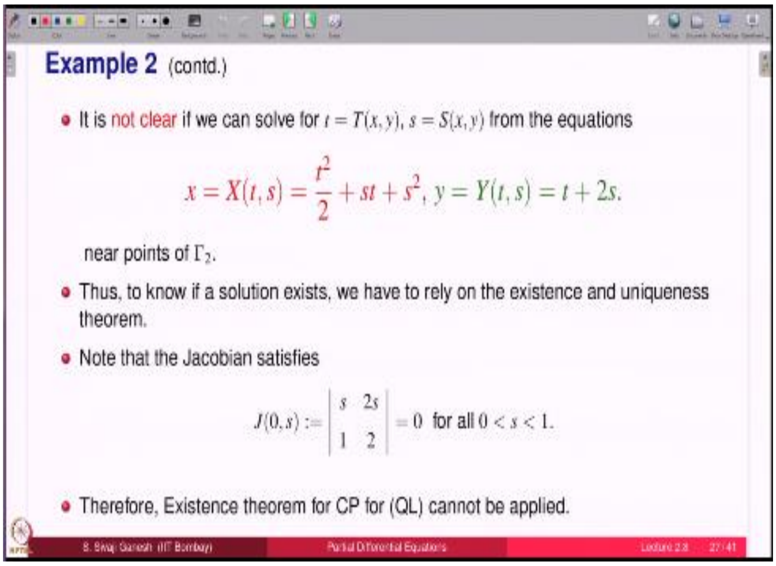
So, u is not differentiable at any point of gamma 2. Therefore, Cauchy problem has no solution whatever we have obtained. Now, using method of characteristics, we will attempt to solve the same problem again.

(Refer Slide Time: 29:26)



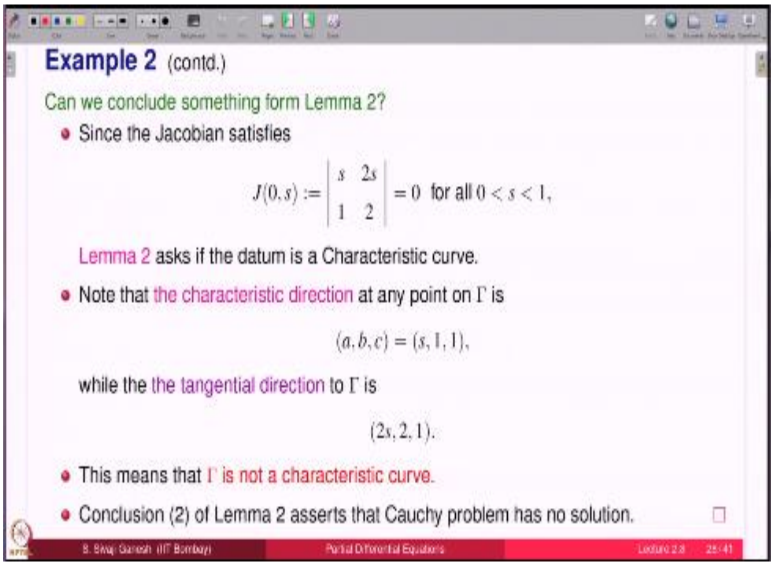
So, we have to write the parametric form of gamma, done. Then system of characteristic ODE is this. Now, we have to solve this with initial data this, so that at t = 0, the characteristic curve lies on the datum, done. When we solve these expressions, we get for x, y and z.

(Refer Slide Time: 29:52)



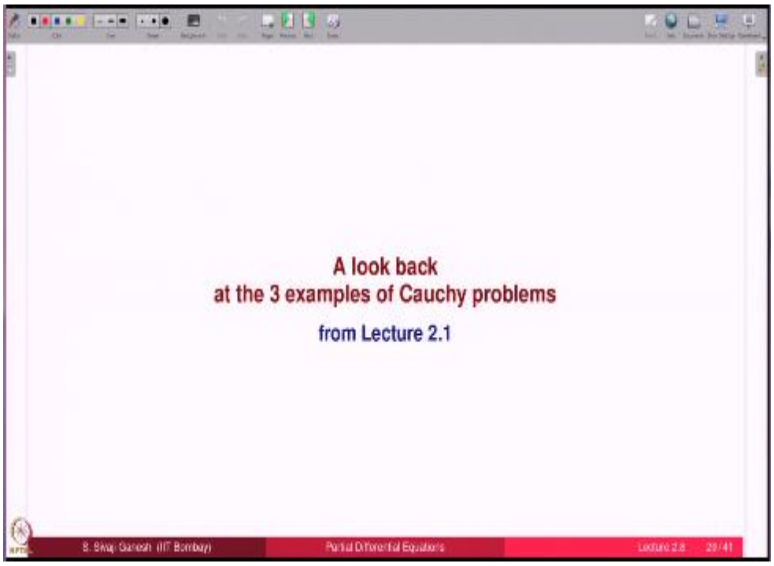
So, to know if a solution exists, we have to rely on the existence and uniqueness theorem. Because, it is not obvious whether I can use the equation for x and y and get an expression for t and s. So, let me do that. Jacobian is this. It is 0. So, Jacobin is 0. Therefore, existence theorem cannot be applied.

(Refer Slide Time: 30:20)

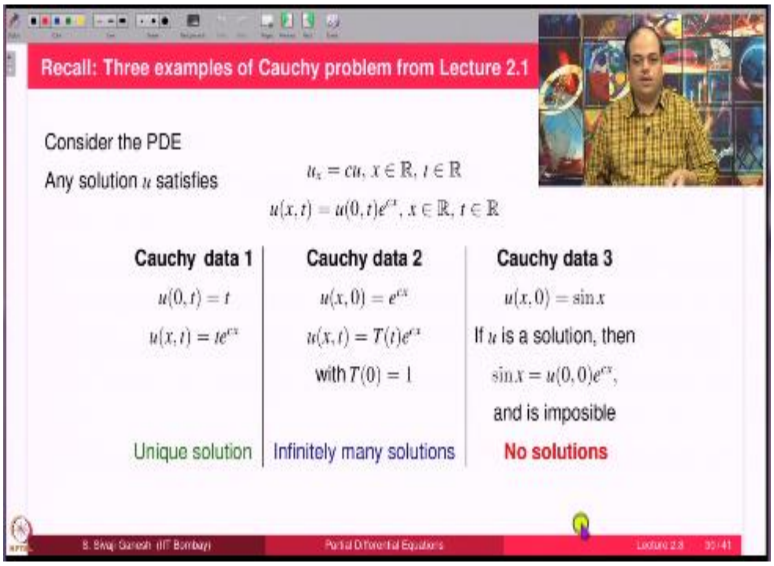


Now, what does lemma to ask? It asks whether it is a characteristic curve. If it is a characteristic curve, answer is infinitely many solutions. So, we check what is it. Is it a characteristic curve or not? The characteristic direction at any point and gamma is the a b c that is s 1,1 but tangential direction is 2 s 2,1. So, both are not proportional. They are not parallel. So, therefore, gamma is not a characteristic curve. Therefore, the second part says, you do not have solution. Conclusion 2 of lemma 2 says, it has no solution.

(Refer Slide Time: 30:59)

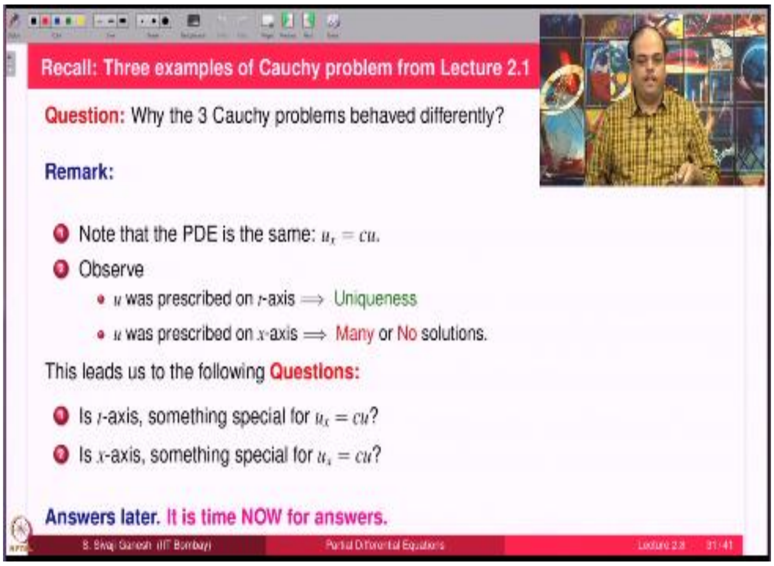


(Refer Slide Time: 31:05)



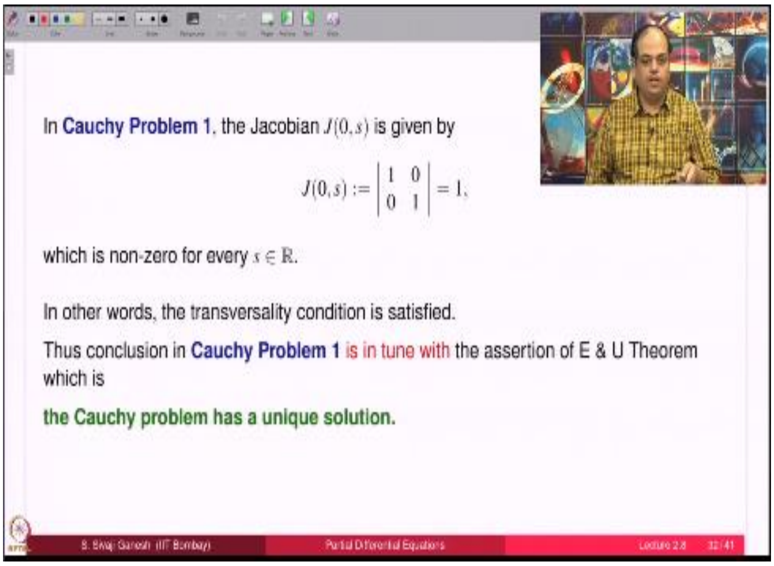
Now, let us go back to the 3 examples that we were discussing in lecture 2.1. That is this. Recall the PDE was u x = c u. In fact, It is like ODE. There is no t dependence and then Cauchy data 1, 2, 3. Cauchy data 1 had a unique solution. Note where is it prescribed? It is prescribed on the t axis while Cauchy data for 2 and 3 are prescribed on the x axis. In one case, you have infinitely many solutions. In other case, there are no solutions.

(Refer Slide Time: 31:30)



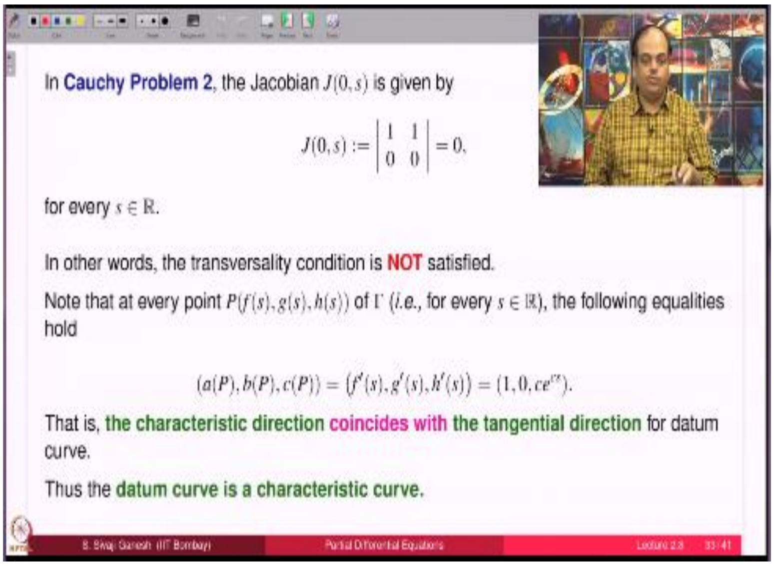
Then we asked this question why they behave differently PDEs are same and t axis is uniqueness; on x axis, you may have many or no solutions. Then we had this question who is special is a t axis or x axis? And we said, answers later. Now, it is a time for answers.

(Refer Slide Time: 31:52)



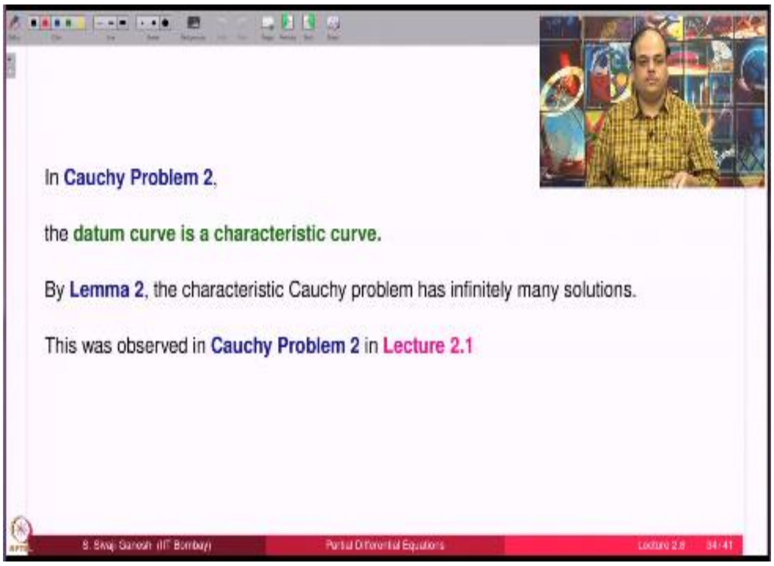
Cauchy problem 1: J 0 s. So, we are trying to use lemma 2 and try to answer. J 0 s is 1, therefore, we have existence uniqueness theorem. Therefore, uniqueness is what you expect, done. It is in tune with the assertion of existence uniqueness theorem.

(Refer Slide Time: 32:11)

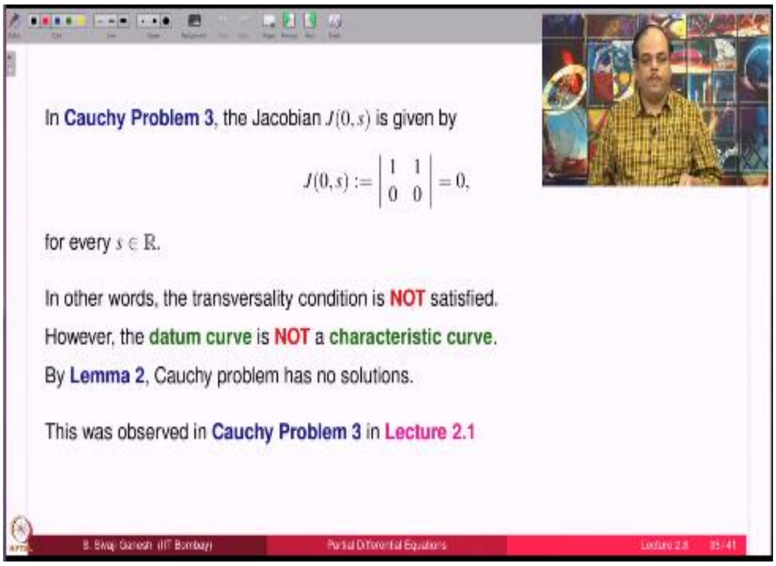


Now, second Cauchy problem, we had J 0. Now, the question is: is it a characteristic curve or not? So, transversality condition is not satisfied, fine. At every point on gamma, the characteristic direction is f prime g prime h prime which is equal to 1, 0, c into e for cs. And it coincides with the tangential direction. Therefore, it is a characteristic curve. Therefore, there are infinitely many solutions that is what lemma 2 also says.

(Refer Slide Time: 32:45)

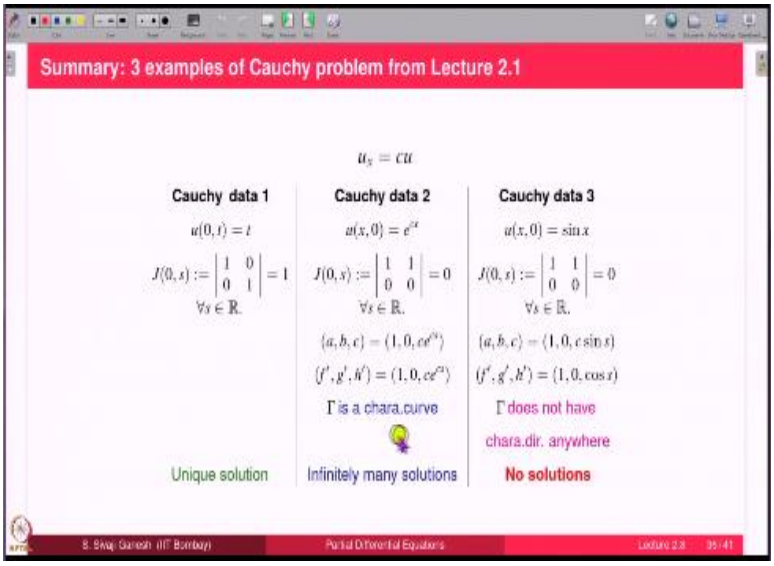


(Refer Slide Time: 32:53)



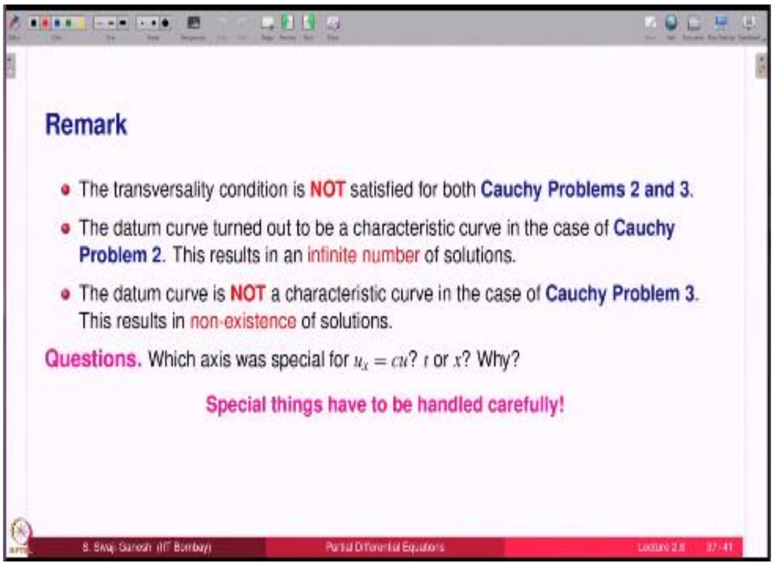
Cauchy problem 3: J is 0. So, we ask whether the curve gamma is a characteristic curve or not. It is not a characteristic. So, the conclusion 2 of lemma 2 says no solution. So, that is the thing.

(Refer Slide Time: 33:11)



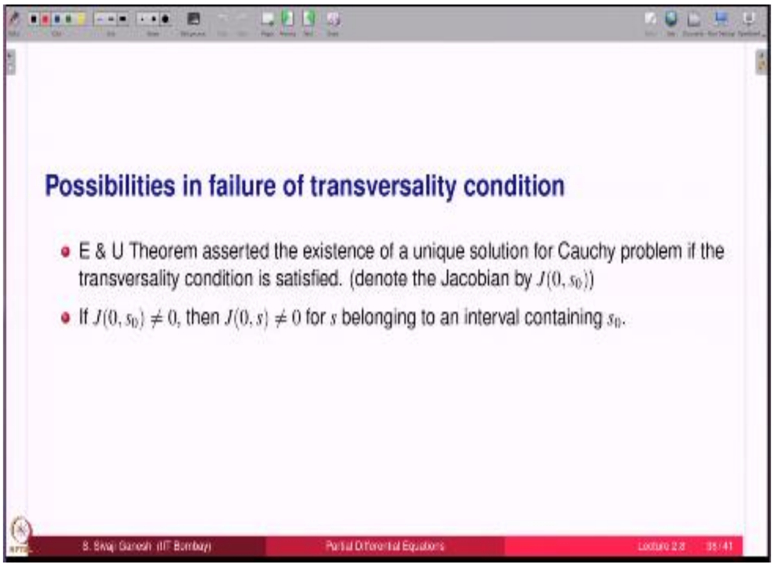
Here. transversality condition is satisfied. So, we do not go further. Here, we checked gamma is a characteristic curve because a b c is this f prime, g prime, h prime this and here, gamma does not have the characteristic direction anywhere, therefore, no solutions. This is a characteristic Cauchy problem, we have infinitely many solutions.

(Refer Slide Time: 33:33)



The transversality condition is not satisfied by both Cauchy problems 1 to 3. Datum curve turned out to be a characteristic curve in the case of Cauchy problem that is why we had infinitely many solutions. Datum curve is not a characteristic curve for the Cauchy problem 3, therefore, no solution. Now, let us answer the other part, which axis is special t or x and why? Special things have to be handled carefully.

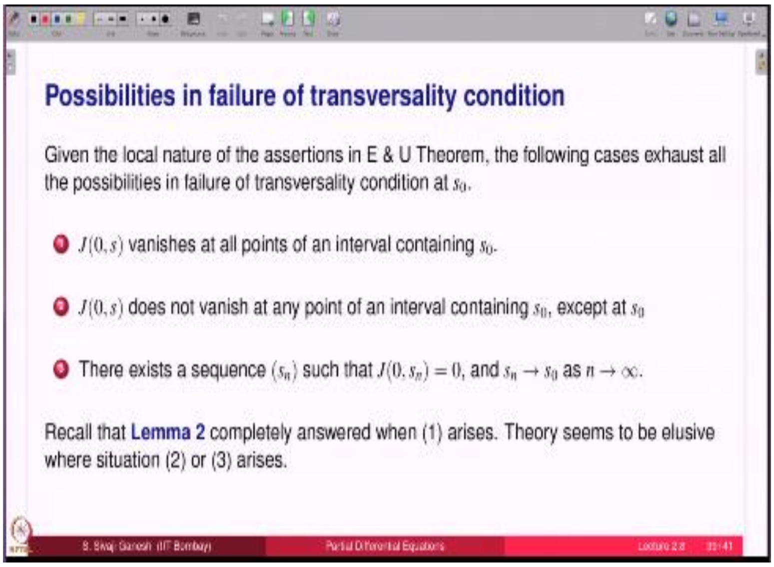
(Refer Slide Time: 34:12)



So, you answer for yourself this question because the answer lies in special things have to be handled carefully. Something not special does not matter, you give anything you want, you know, no problem. So, existence uniqueness theorem assert the existence of a unique solution for Cauchy problem if transversality condition is satisfied denote the corresponding Jacobina J 0 s 0.

If this is nonzero, then it will be nonzero for s belonging to an interval containing s 0 for the reasons that things are involved in the determinant continuous functions of s; undetermined itself is a continuous function.

(Refer Slide Time: 34:47)

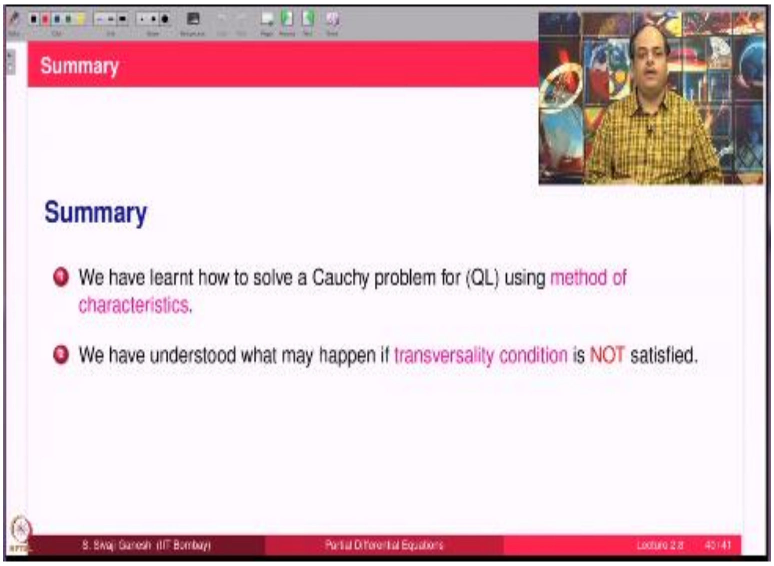


So, given the local nature of the assertions in existence and uniqueness theorem, the following cases exhaust all the possibilities in when the transversality condition fails at s 0. J vanishes at all points of some interval containing s 0. Nearby s 0 is what we are worried about and some interval, J is 0 or it does not vanish at any point of some interval containing s 0 excepted s 0 that is the isolated point in some interval.

In rest of the point of the interval, J is not vanishing, but only s 0 is vanishes. Or, it can happen like this kind of limit point. You find a sequence of s n, so, as the J is 0 at the point J 0 s n and s n goes to s 0, otherwise it is not 0. Imagine a function like x sin 1 by x, 0 is the limit point of zeros of this function, something like that. Now, lemma 2 answer the cases 1 and when the case one happens when J is identically equal to 0 on some interval containing 0.

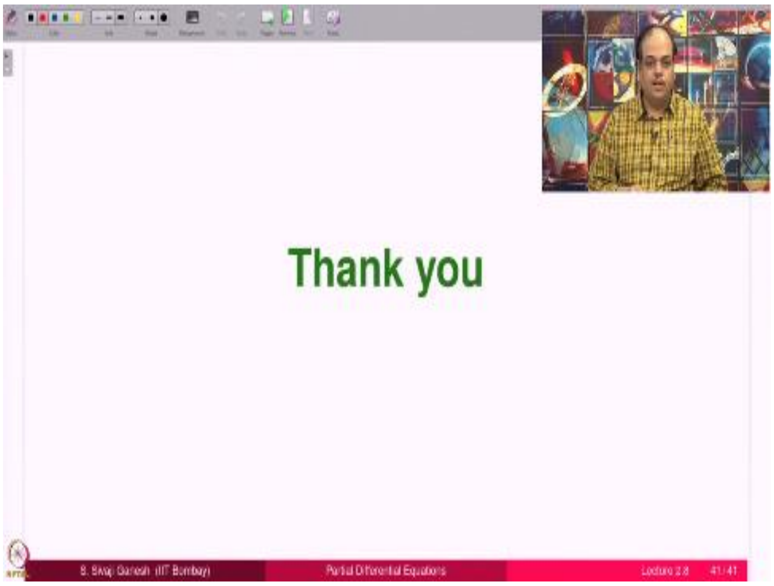
And t seems to be elusive where situation 2 or 3 occurs, one has to look from case to case and make conjectures and I do not think, there is any complete theory available written in a textbook.

(Refer Slide Time: 36:13)



So, let us summarise. We have learned how to solve a Cauchy problem using method of characteristics. We have understood what may happen if transversality condition is not satisfied.

(Refer Slide Time: 36:24)



And in the next lecture, we solve some more problems from first order partial differential equations upto Quasilinear equations and then we will move on to general nonlinear equations or general equations later on. Thank you.