Partial Differential Equations
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Lecture – 5.6
Qualitative Analysis of Wave Equation
Generalized Solutions to Wave Equation

Welcome, in this lecture, we are going to introduce notions of generalized solutions to wave equation. The outline is as follows.

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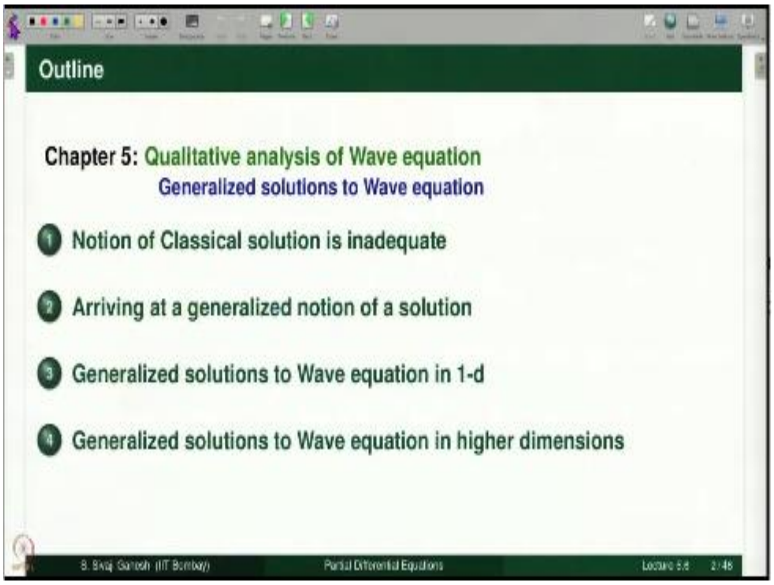

First, we mentioned that the notion of a classical solution is inadequate and thus, there is a need to generalize the notion of a solution and then in step 2, we show how to arrive at a generalized notion of a solution and then we demonstrate some generalized solutions to wave equation in 1D and in higher dimensions.

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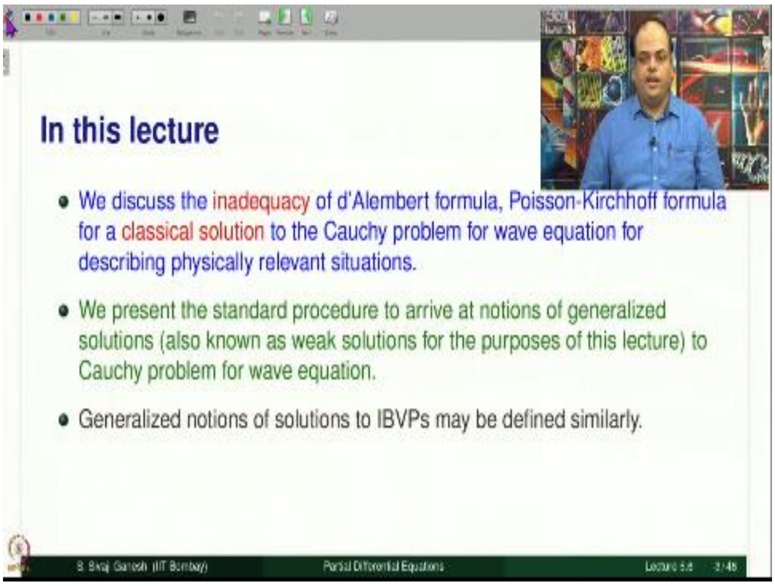

In this lecture, we discussed the inadequacy of d'Alembert formula, Poisson-Kirchhoff formula for a classical solution to the Cauchy problem for wave equation for describing physically relevant situations. We present the standard procedure to arrive at notions of generalized solutions also known as weak solutions for the purposes of this lecture to Cauchy problem for a wave equation. Generalized notions of solutions to IBVPs may also be defined similarly.

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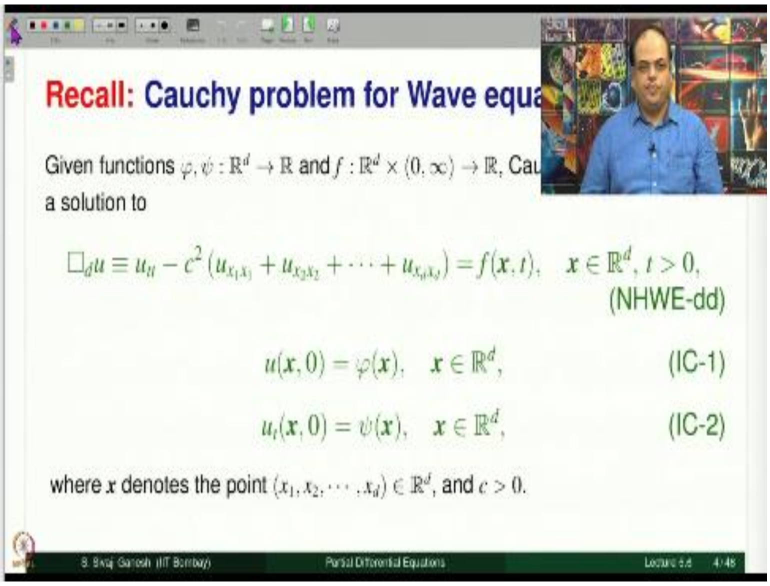

So, recall the Cauchy problem for a wave equation. Here, the data is given phi, psi and f; and we are required to solve non-homogeneous wave equation d'Alembert u equal to f and u x 0 is equal to phi x; u t x is equal to psi x for x in R d.

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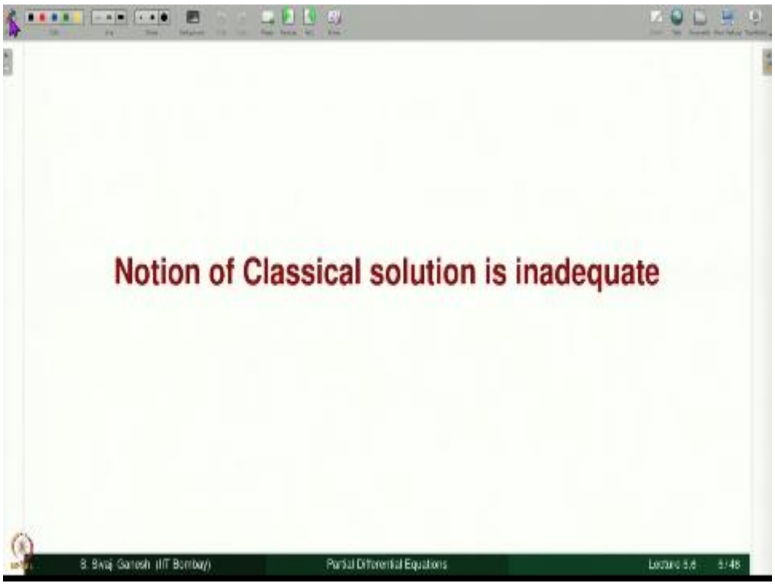

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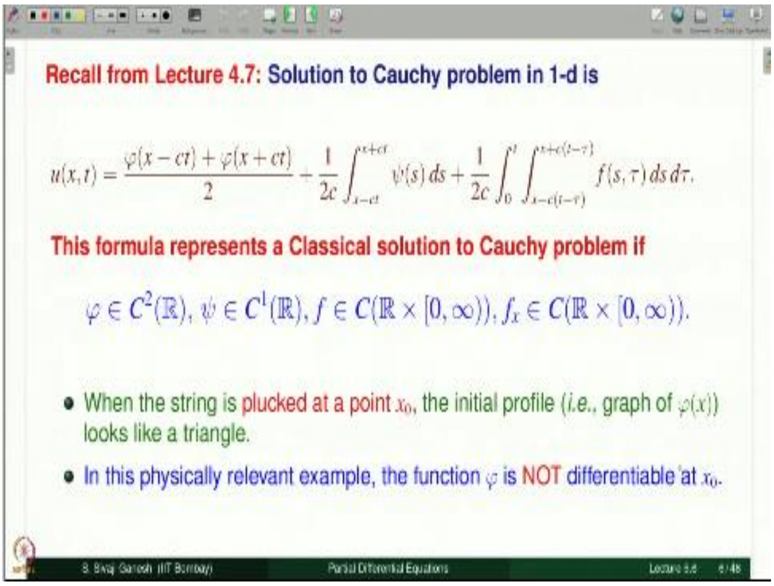

Recall from lecture 4.7, where we have derived this solution as given by this formula in 1D. This formula represents a classical solution if phi C 2, psi C 1 and assumption on f is that f x is continuous apart from f itself being continuous. So, when the string is plucked at a point x 0, what does that mean is this; suppose this is a point x 0, so, string is pulled up. So, as a consequence of this string looks like this is lying up to here and then let us say like that, like that, like that.

So, at this point x 0, it is raised. This is a graph of u of x here. These are situation clearly such a function cannot be differentiable at this point x 0. The initial profile that is a graph of phi of x looks like a triangle as we have seen in this physical irrelevant example, the function phi is not differentiable at x 0. Therefore, this formula does not make sense as a classical solution because the function will not be, u of x t will not be C 2 because phi is not C 2. It is not even differentiable. So, therefore, this formula is inadequate in this scenario.

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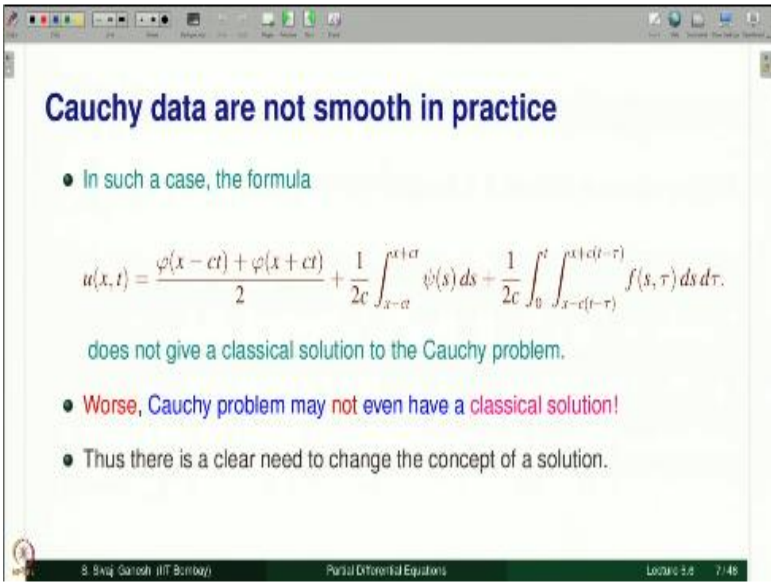

So, in such a case where the Cauchy data is not smooth, this formula let us call it d'Alembert formula even if we have a right hand side the source term here, right hand side in the wave equation. So, this, let us still call it by the name d'Alembert formula. So, this does not give a classical solution to the Cauchy problem. Worse, Cauchy problem may not have classical solutions; not only that this formula is not a classical solution.

It may not even have a classical solution. Thus, there is a clear need to change the concept of solution.

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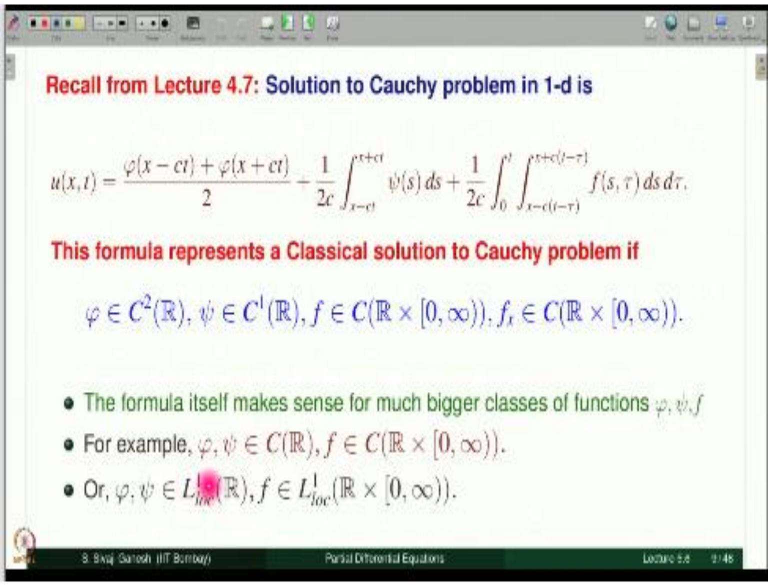

So, can we recover some lost ground? Say, this is a formula we have developed in 1D for the solution of Cauchy problem. It is a classical solution if these conditions are satisfied by the data phi C 2 psi C 1 f should be continuous on this domain and f x should also be continuous on this domain. The formula itself makes sense for much bigger classes of functions.

Because what all you need for this phi can be any function because there is no differentiability requirement on this. It can be any function this makes sense. Here I have a psi, I need to integrate. Therefore, psi should be integrable. But, this is for every x and t, it should be integrable on the interval x – ct and x + ct. Psi need not to be integrable and whole a part, it is enough.

It is integrable on every interval of this form or more generally any interval of the type a, b, closed interval a, b. Similarly f; f has to be integrable on certain as we already observed this domain is nothing but a triangle, this integral integration is done on a domain which is a triangle which is called characteristic triangle. So, we need f to be integrable on that. So, let us see some assumptions on phi psi f, which guarantee that the right hand side is meaningful.

Therefore, it defines a function. The question then is: is it a solution or in which sense is a solution etcetera? For example, if phi psi are continuous functions as I told you, they can be any functions, f is continuous and hence, it will be integrable on any triangle that you take. Psi is continuous therefore, this integral is meaningful for every x and t that is what I have written here phi and psi are L 1 loc. L 1 log simply means that this function is integrable on every compact set or equivalently on every bounded set.

So, similarly, f is assumed to be L 1 loc, locally integrable. These are notation for that. Please note that we are using this notation L 1 loc just to mean, these are locally integrable functions. It means functions are integrable on every compact set. Please do not confuse this with the Libic function spaces of L 1, L 1 loc because these kinds of things, the point wise evaluation do not make sense if I say that this is the Libic spaces.

But, when we see the weak formulation, their L 1 loc, it makes sense. So, there is no problem. So, whenever we see point wise evaluation like this, L 1 loc stands simply for those functions which are integrable on every compact set. And when we see in the weak formulations, this stands for the usual Libic L 1 loc spaces.

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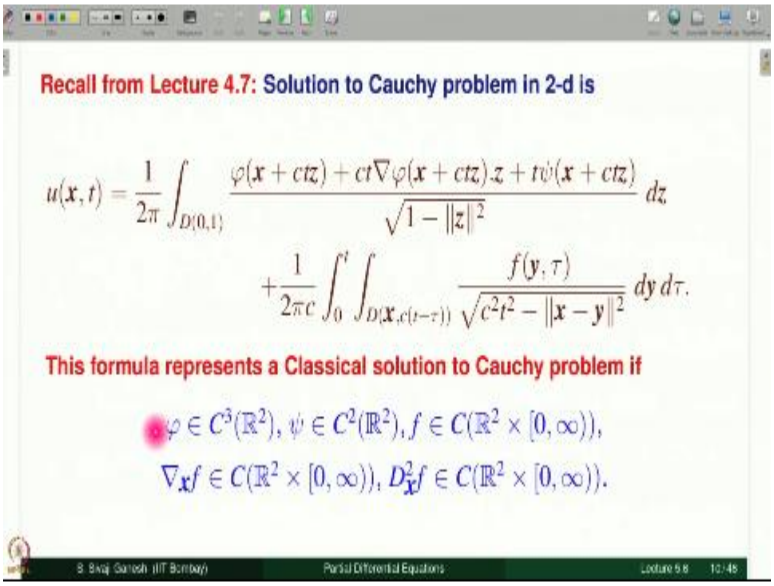

Now, look at the formula in 2D. 2D formula is slightly more complicated than the formula in 1D because here, phi comes along with the derivative also. Psi appears like before. So, it is not a problem, but phi has a derivative. Now, these are classical solution to the problem where phi C 3 and psi C 2 and f should have this kind of smoothness properties.

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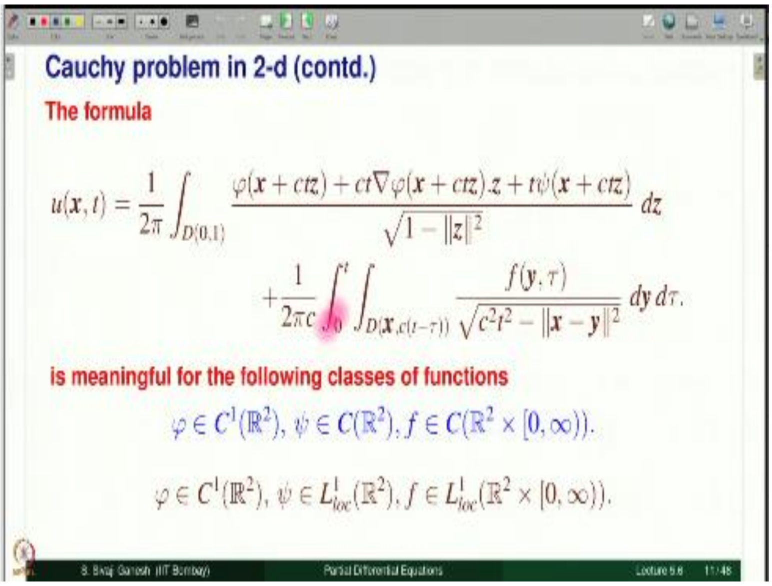

But the formula itself is meaningful for the following class of functions when phi C 1. So, these are continuous functions and they are integrating on a closed disk. So, it must be fine, integral religious similarly, psi continuous. So, integration is not a problem and f continuous. We can further weaken the smoothness requirements on phi psi f by saying phi should be C 1 that seems to be there, seems to be no alternative because grad phi appears in the formula.

Now, we are looking for conditions on the data so, that this integral makes sense. So, phi should be C 1 because grad phi is there and psi is L 1 and f is L 1. Of course, this is not the exhaustive list of classes or functions. There are much bigger class of function for which this is meaningful and discussion of such things is beyond the scope of this course.

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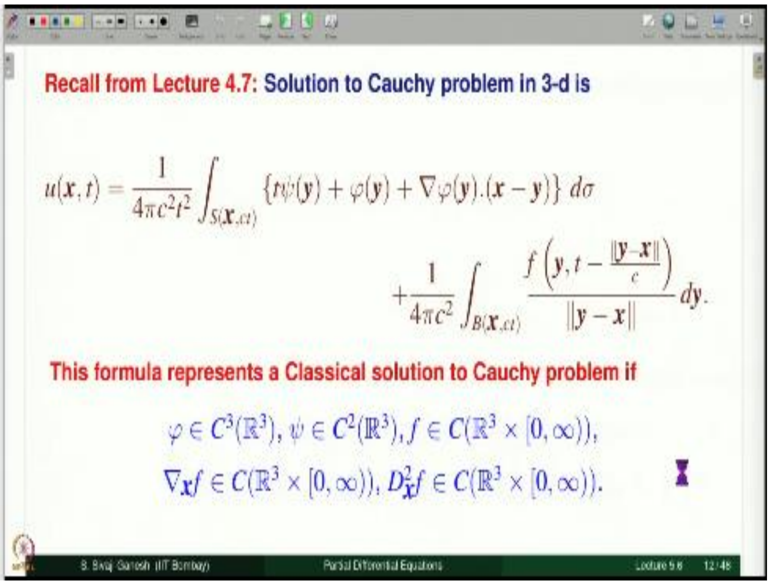

Now, solution in 3D. The same issue here because phi comes with the derivative. So, the formula represents a classical solution to Cauchy problem if phi is C 3, psi is C 2 and f has the regularity or smoothness properties, which is given here.

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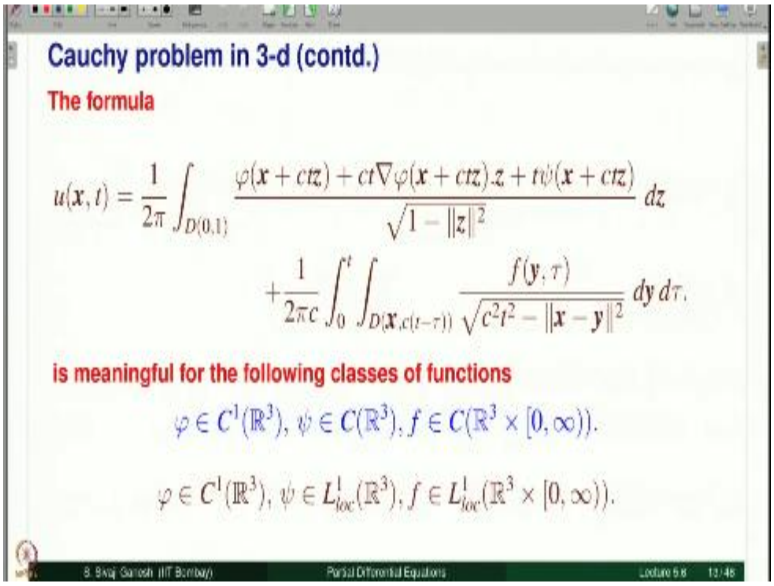

Formula itself is meaningful when phi C 1 psi is continuous and f is continuous or more generally psi, you can allow it to be local integral function and f also local integral function.

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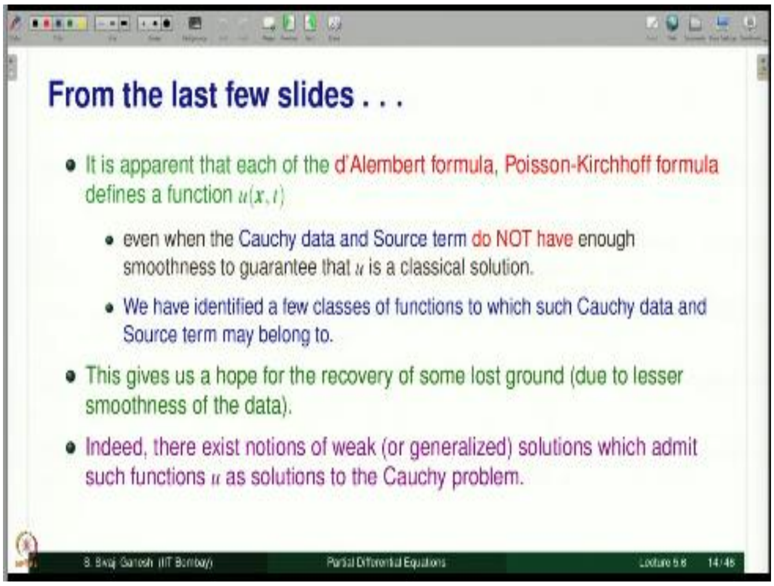

So, in the last few slides, it is apparent that each of these formulae d'Alembert formula, Poisson-Kirchhoff formula defines a function even when the Cauchy data and source term do not have enough smoothness to guarantee that the function is a classical solution. We have identified a few classes of functions to which such that Cauchy data and its first term may belong to in order that the expression for u of x t is meaningful or makes sense.

This gives us a hope for the recovery of some lost ground due to lesser smoothness of the data. Indeed, there exists a notion of weak or generalized solutions which admits such functions u as solutions to the Cauchy problem.

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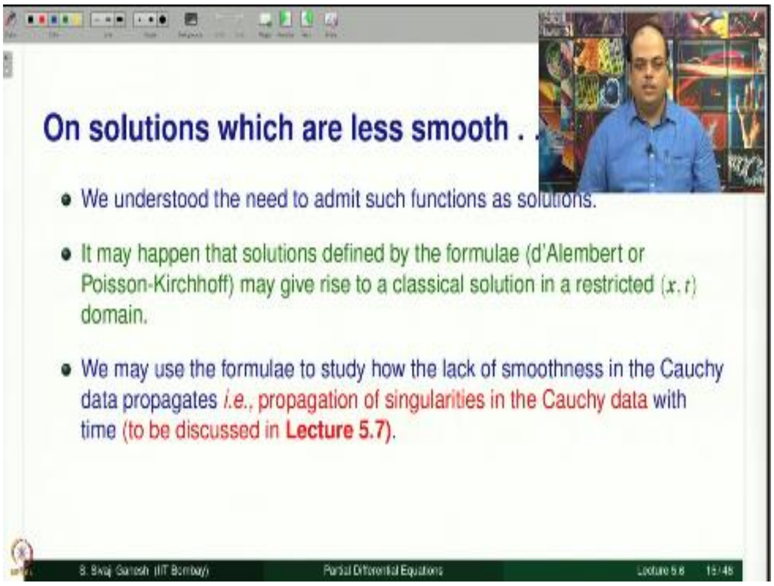

We understood the need to admit functions which are less smooth as solutions because Cauchy data itself, we have to admit which are lesser smooth. It may happen that solutions defined by these formulae dAlembert or Poisson-Kirchhoff may give rise to a classical solution in a restricted x t domain. We have seen that such things happened in the case of Burgers equation some conservational loss.

We may use a formula to study how the lack of smoothness in the Cauchy data propagates with time that is propagation of singularities. Lack of smoothness means it is called a singularity; something is not smooth, at some point that point is called a point of singularity. So, propagation of singularities in the Cauchy data with time. This, we will discuss in lecture 5.7.

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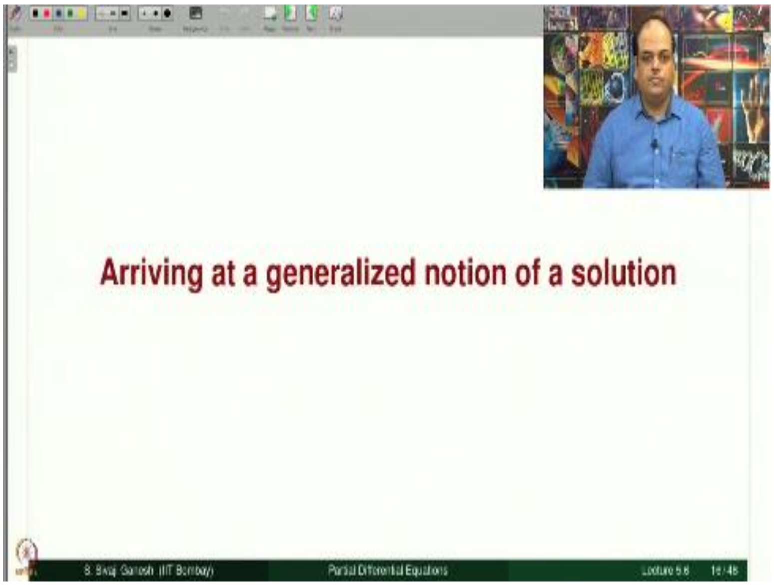

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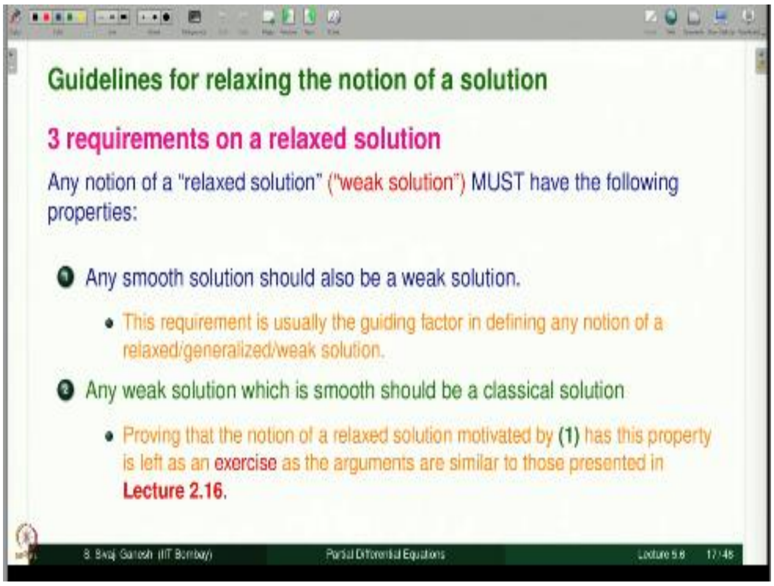

Arriving at a generalized notion of a solution: How to do that? So, there are guidelines for relaxing the notion of a solution. We have seen this in lecture 2.16 where we have introduced in the context of Burgers equation and consideration form of that. Three requirements on a relaxed solution: any notion of a relaxed solution or weak solution or generalized solution, whichever word you may use, must have the following properties.

What are they? Any smooth solution should be a weak solution. This requirement is usually the guiding factor in defining any notion of a relaxed or generalized or weak solution. Any weak solution which is smooth enough should be a classical solution; proving that the notion of a relaxed solution that we are going to introduce soon motivated by 1, the point 1 that is a requirement 1 has this property is left as an exercise because ideas are very similar to what we did in lecture 2.16.

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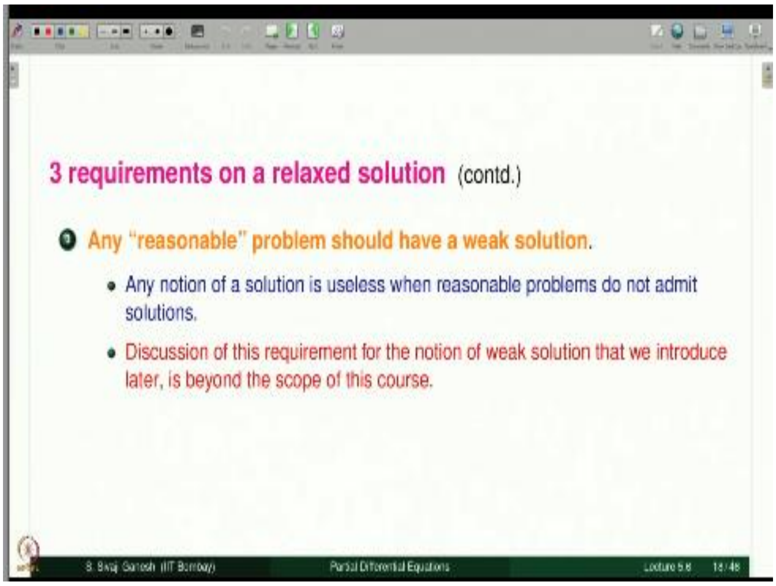

Now, the third requirement. Any reasonable problem should have a weak solution or a relaxed solution. Any notion of a solution is useless when reasonable problems do not admit solutions. Discussion of this requirement for the notion of weak solution that we are going to introduce soon is beyond the scope of this course, we will not discuss them.

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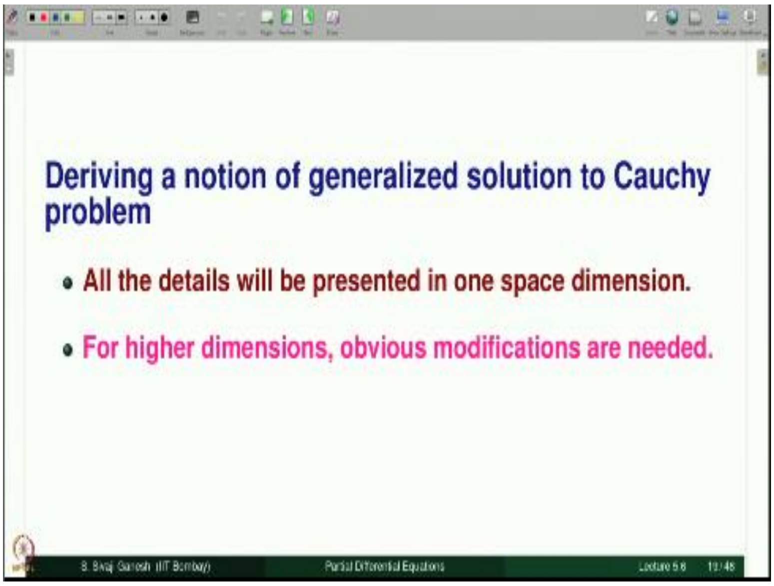

So, deriving a notion of generalized solution to Cauchy problem; all the details will be presented in 1 space dimension. For higher dimensions, obvious modifications are needed and we get a similar formulation. We are going to show that formulation.

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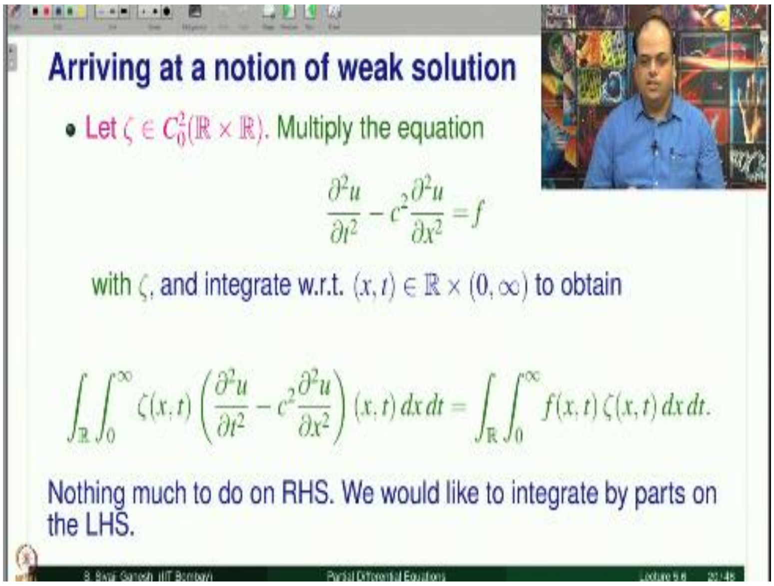

So, let zeta be a C 2 function on R cross R; this 0 stands for compact support. That means the support of zeta is a compact set. So, support is contained in a big enough ball equivalently. So, multiply the given equation non-homogeneous wave equation with zeta and integrate with respect to x t over this domain R cross 0 infinity. What we get is this. It is simply integrating both sides after multiplying with zeta, you can see equation.

There is nothing much to do this side because you really do not f to do anything. Here, we can do something. Here, we see that there are derivatives on u and we are not looking for smooth solutions to the wave equation. Therefore, we would like to relax this requirement and that can be done by transferring these derivatives to zeta which is integration by parts. So, nothing much to do on RHS. We would like to integrate by parts on the LHS.

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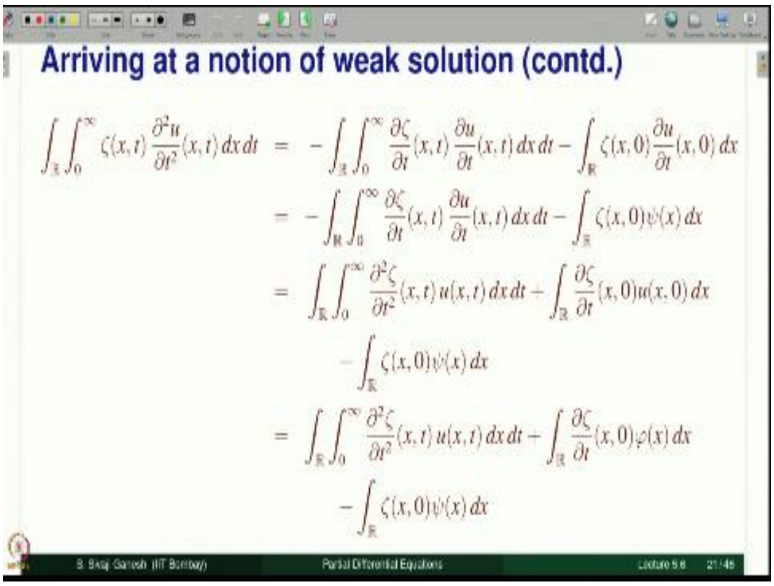

So, this is the LHS. First term in the LHS because there is also a dou 2 u dou x square term. So, this on integration by parts, you get this one derivative shifts to zeta and a minus sign, welcome and there will be a boundary term with respect to 0 infinity, t 0 infinity, because t = 0 is a lower limit, you will get another minus sign, minus zeta x 0 dou u by dou t x 0. Upper limit will not contribute because zeta has compact support. So, this is what we have.

But, what is dou u by dou t of x, 0? That is psi x so, we have to apply. So, we are done one integration by parts. So, let us do once more transfer this dou u by dou t to this one. That will make this minus as plus and the transfer is done. And similarly as you had this term coming out of the integration by part, we have one more term coming from this. So, that is this. And the other term, which we had here is written here. But now, what is u of x 0? It is phi x. So, this is what we get by integrating by parts in the first term on the LHS.

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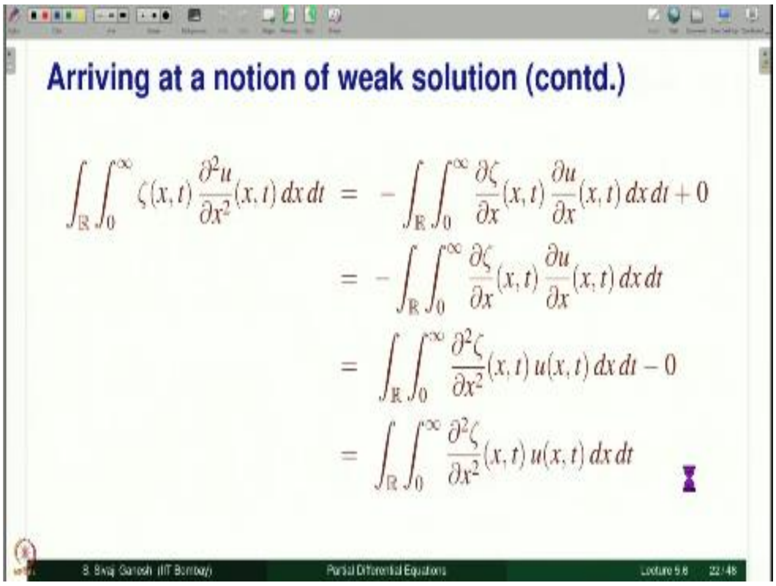

Now, let us take up the second term, which involves zeta dou 2 u by dou x square. Again integration by parts, here there will be no boundary term because with respect to x, zeta is supported once again compactly. And we do not have this situation of 0 infinity in when we are doing with respect to x. That is why there will be no boundary terms that is why 0. So, it is this.

Once more integration by parts, we get this, no boundary terms because integration by parts, when you do, you get an integral on the domain and you get an integral on the boundary. So, this is what we have.

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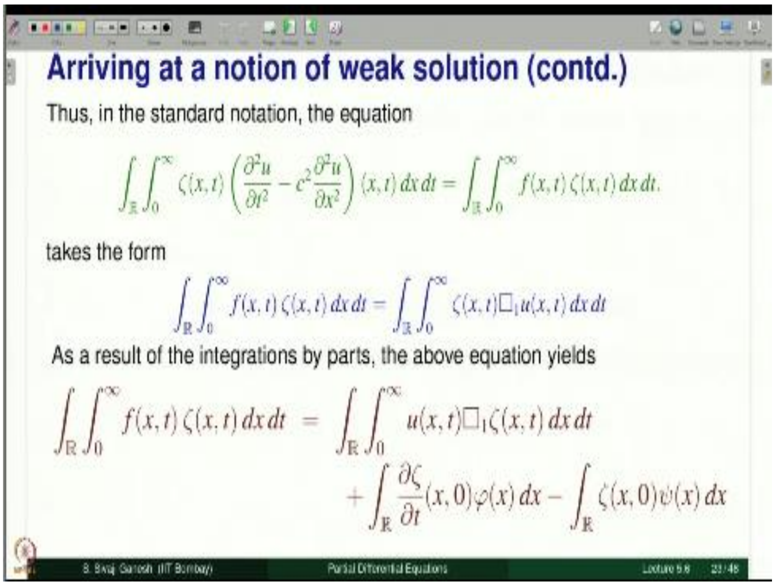

So, this equation, we are going to use the standard notation, this we call d'Alembertion one, square one. So, in that notation, I am going to write, so I have just interchange the LHS and RHS because this is where we are going to write expression for this. So, expression for this that we derived using integration by parts. That is why I am writing here. So, this is what we get. In fact, what we got by integration by parts is this term on RHS equal to this term on the RHS.

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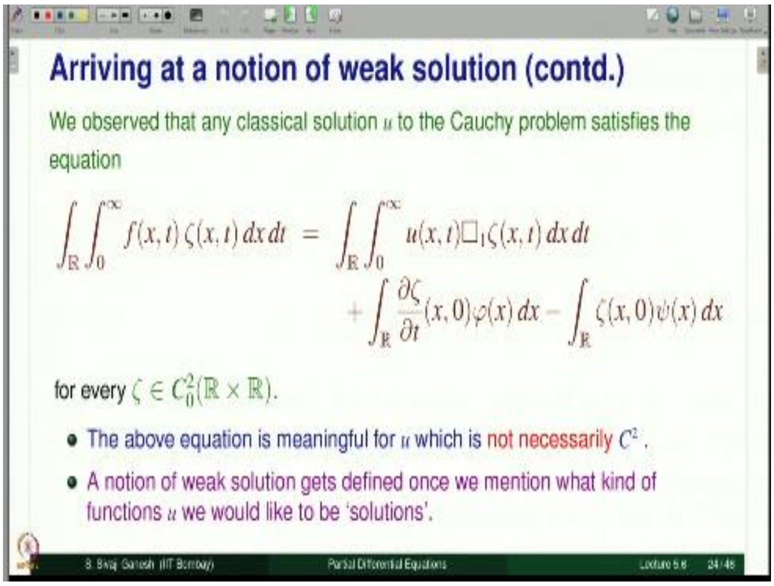

So, what we did is that any classical solution to the Cauchy problem satisfies this equation which we have derived. So, this is the requirement 1 in fact, that was a guiding principle now, we are going to define what is the weak solution based on what we have got here. So, the above equation is meaningful for u, which is not necessarily C 2 because nowhere there is a derivative one u; u is only here nowhere else.

Zeta is a smooth function with compact supports; C 2 function is compact support. So, these are continuous function with compact support essentially, you need to integrate and a compact set. So, if u is L 1 loc locally integrable, this is meaningful. So, a notion of weak solution gets defined once we mentioned what kind of function we would like to be solutions.

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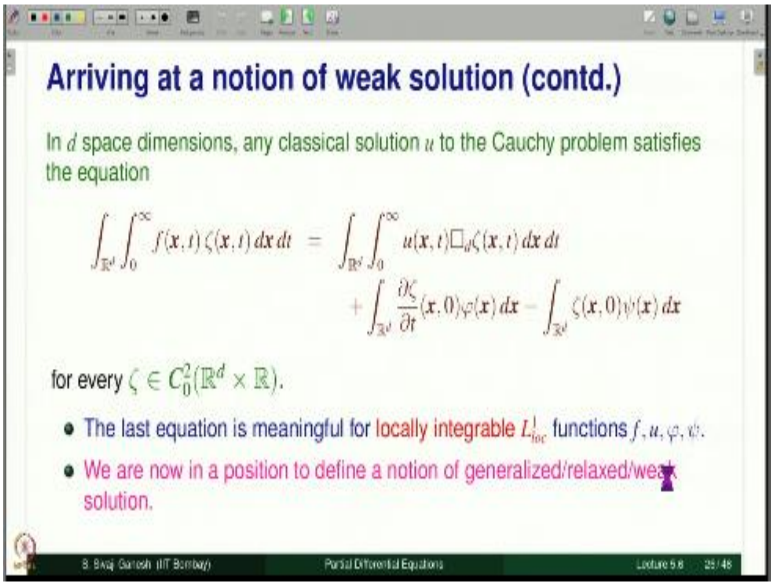

So, in d space dimensions, any classical solution satisfies this equation. Derivation is exactly the same, we are shown the derivation when d = 1, but exactly the same step will give us this. The last equation is meaningful for locally integrable functions, not only u, for phi psi f also. So, we are now in a position to define a notion of generalized or relaxed or weak solution.

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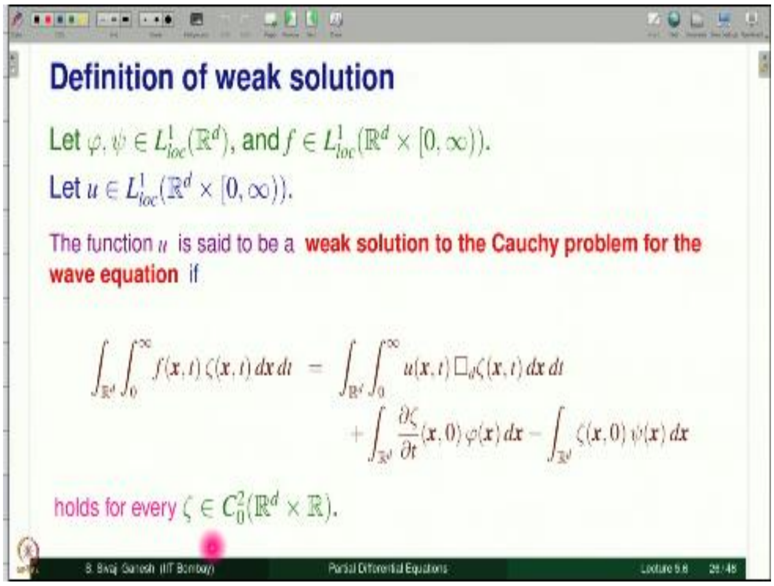

Let phi and psi be local integral functions and f be local integral function on this domain R d cross 0 infinity. Let u be also a L1 loc function. The function u is said to be a weak solution to the Cauchy problem for the wave equation if this equation is satisfied by every zeta which is C 2 un-compactly supported in R d cross R.

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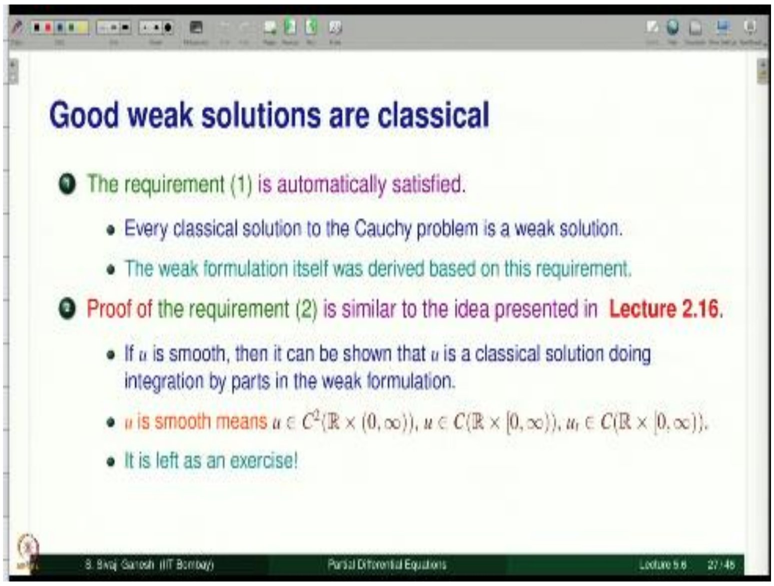

Requirement 1 is automatically satisfied. Every classical solution to the problem is a weak solution. The weak formation itself was derived based on this requirement. Proof of the requirement 2 as I told before, it is similar to the idea which we presented in lecture 2.16 if u is smooth, then it can be shown that u is a classical solution by doing integration by parts in a weak formulation.

So, in the definition of the weak solution, we had an equation valid for all zeta in C 0 2, C 2 0 functions that is called weak formulation of the Cauchy problem. So, u is smooth means this because this is what we need for a classical solution. If all those conditions are met by our weak solution, then it is actually a classical solution. And that is left as exercise.

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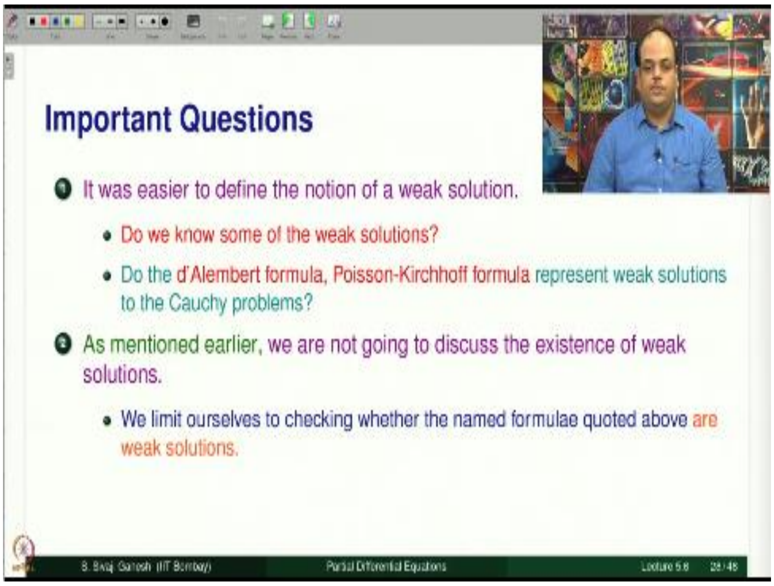

Important questions: it was easier to define the notion of week solution is always easy to define something that is not a difficult job. Do we know some of the weak solutions? Do the d'Alembert formula, Poisson-Kirchhoff formula represent weak solutions to the Cauchy problems? These are the questions. As mentioned earlier, we are not going to discuss the existence of weak solutions.

We limit ourselves to checking whether the named formula quoted above namely the d'Alembert formula or Poisson-Kirchhoff formula are week solutions.

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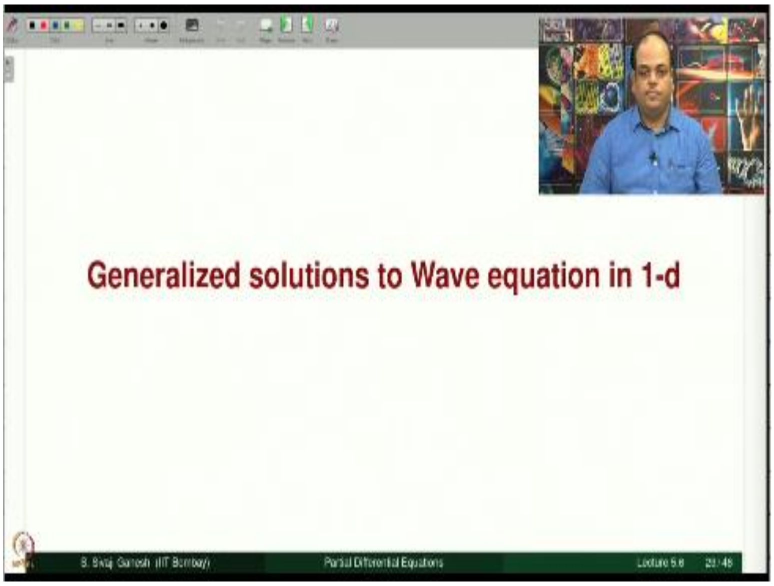

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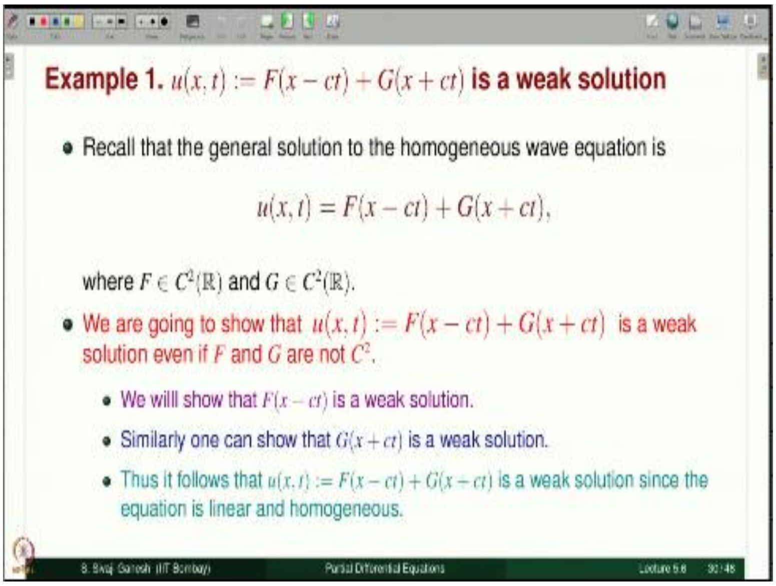

Let us look at some of the generalized solutions to wave equation in 1D. Recall u of x t = F of x – ct + G of x + ct, we have obtained this as a general solution to the wave equation whenever F and G are C 2 functions. Now, we are going to ask the obvious question. Is this a weak solution when F and G are not C 2 functions? So, we are going to show that this formula F of x – ct + G of x + ct is a weak solution even if F and G are not C 2.

We will show that F of x – ct is a weak solution. Similarly, we can show that G of x + ct is a weak solution. Therefore, it follows some of 2 weak solutions is a weak solution because equation is linear and homogeneous. Here, what do we require if you want to say F of x – ct a week solution? Definition of week solution wants local integral functions. So, we can as well take F may be local integral function.

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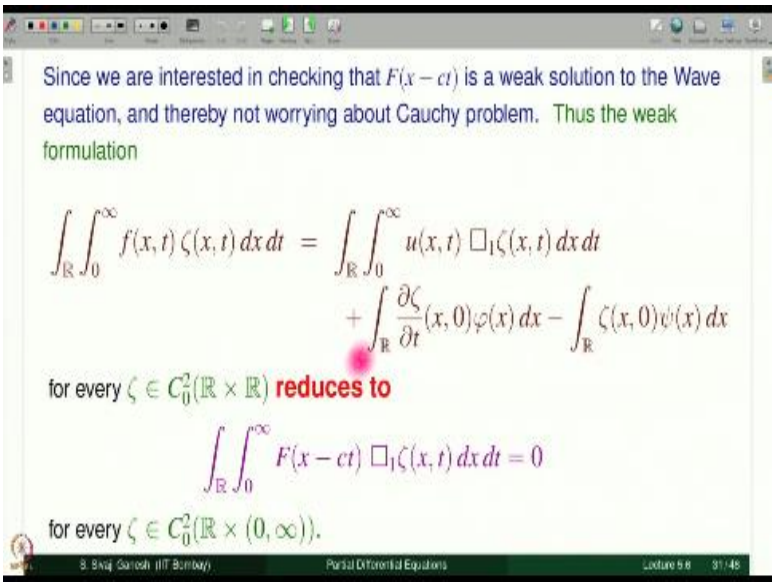

Since, we are interested in checking the F of x – ct is a weak solution to the wave equation. And thereby, we are not worried about the Cauchy problem. So, this week formulation where we insisted that zeta is C 2 0 R cross R now, it reduces to exactly same equation without these terms. And zeta, we can take to be open 0, infinity. Then automatically these terms are not there.

So, essentially, F is not their homogeneous wave equation. So, F is 0 and phi and psi are not there because zeta is compactly supported in open 0 infinity. Therefore, what we have is just this and I want u of x t, the candidate I am proposing is F of x – ct. So, we want to show this equal to 0. We need to show this.

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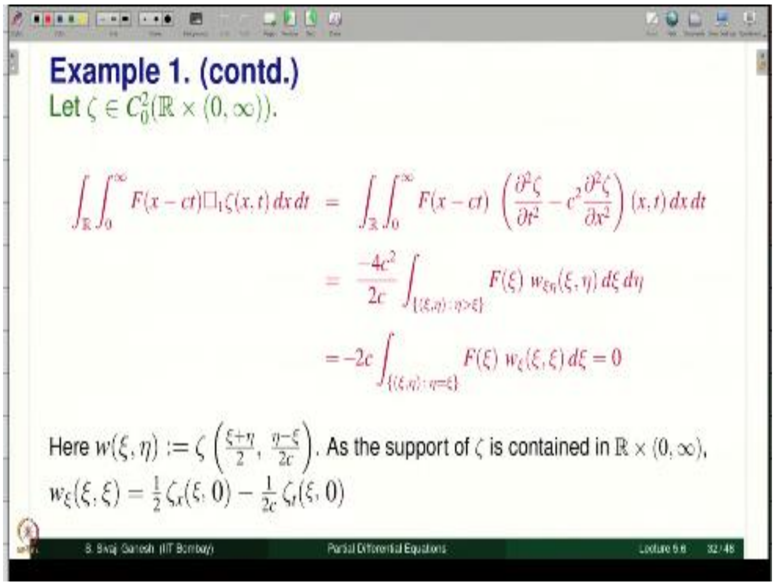

So, let zeta be a compactly support function C 2 in defined on this domain R cross 0 infinity. We would like to do integration by parts here. But unfortunately, we cannot do that, because we are assuming F is not a C 2 function. Therefore, I cannot transfer all the derivatives to this by integration by parts. So, we have to do something else. In particular, we are looking for F to be a weak solution.

Therefore, this will be a L 1 loc function. We want to show that this integral is 0. Therefore, there is no way we can do integration by parts, but here somebody helps us we namely change the variables. Let us set x – ct = psi and x + ct = eta, then this integral becomes this integral; this is F of psi and this wave operator dou 2 zeta by dou t square – C square dou zeta by dou x square becomes – 4 C square w psi eta of psi eta.

w is nothing but zeta expressed in psi eta coordinates. Then dx dt become d zeta deta by 2 C and the integration domain is the upper half plane is given by eta bigger than psi. Now, we can do integration by parts in this formula with respect to eta because F is L 1 loc depends only psi. So, there is no eta dependence. So, once we do integration by parts here, the domain integral will be 0 and only boundary contribution remains.

On the boundary of this domain is precisely eta = psi, which is nothing but the x axis. So, therefore, we should know what this quantity is and this is equal to 0. Here, w of psi eta is nothing but zeta of psi + eta by 2, eta – psi by 2 c. So, we will differentiate this with respect to psi and evaluated eta = psi. As a support of zeta is contained in R cross 0 infinity, w psi of psi psi is nothing but zeta x derivative of psi with respect to x at the point psi, 0 into derivative of this with respect to psi which is 1 by 2 – 1 by 2 c zeta t at the point psi, 0.

And that is equal to 0 because the support of zeta is contained in the upper half plane. And here, evaluation is happening because of the t = 0, we are in x axis, therefore it is 0.

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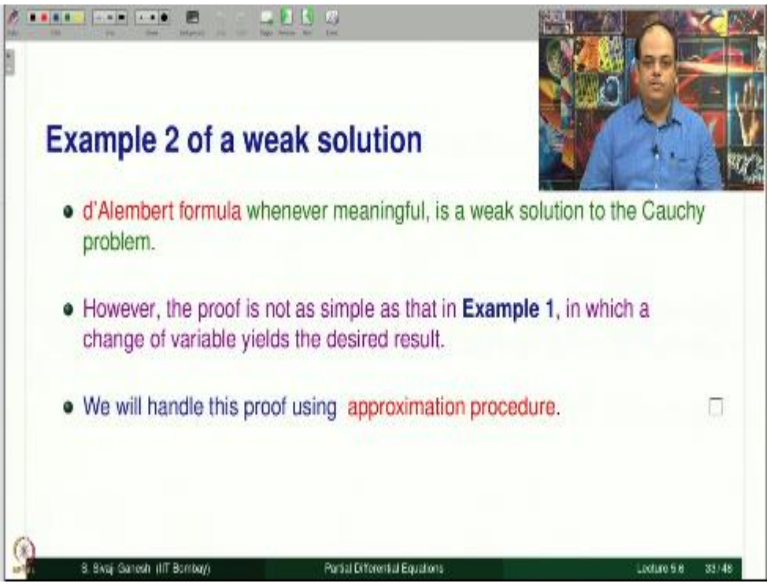

Let us look at the example 2 of a weak solution. d'Alembert formula whenever meaningful is a weak solution to the Cauchy problem. However, the proof is not as simple as the one we saw an example 1 in which a change of variable yields the desired result. We will handle this proof using approximation procedure.

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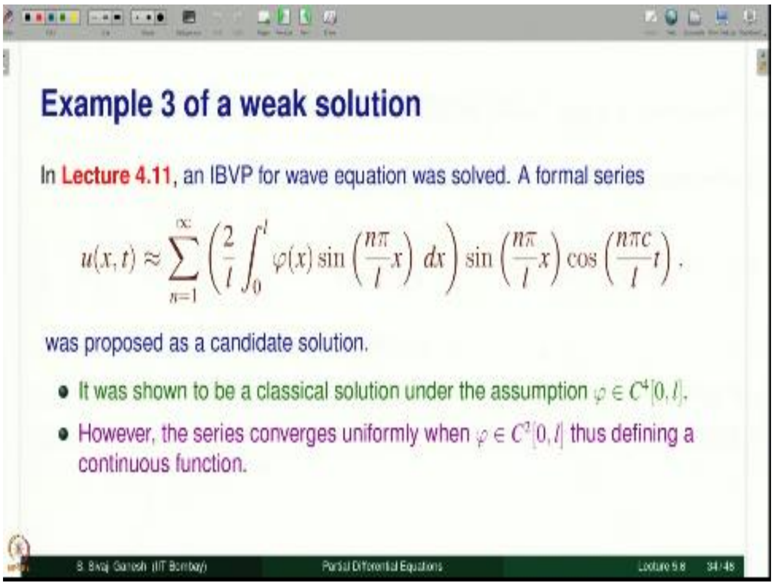

In lecture 4.11, an IBVP for wave equation was solved. A formal series was proposed as a solution. It was shown to be a classical solution if phi was C 4, which is unreasonable, we saw the example of clustering is unreasonable. However, if phi is C 2, the series nevertheless converges uniformly and it defines a function that function, let us call it u of x t, because the series converges, this is meaningful.

So, that will be only be a continuous function, not more than that, so it cannot be a candidate for classical solution. But it can be candidate for week solution.

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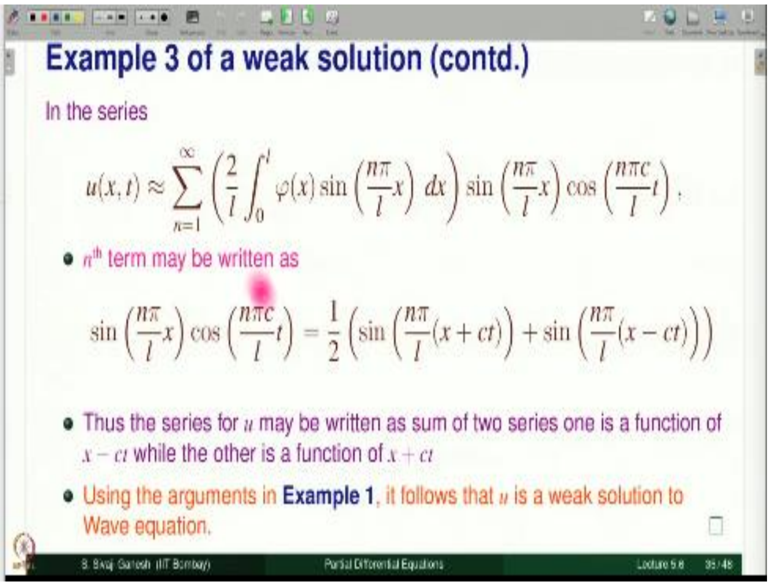

So, the nth term may be written as forget about this, these are numbers, look at this term that can be written as this using sin A cos B formula. We get this. Now, something interesting is happening here. This is a function of x + ct; this is a function of x – ct. So, each of the terms here is sum of 2 terms, which is a function of x + ct and other one is a function of x – ct.

So, we can split the series into 2 series, then what we have, one series will be a function of x – ct, one series will be a function of x + ct. And whenever you see a function of x – ct alone, it has a weak solution to the wave equation, homogeneous wave equation. Similarly, other one x + ct, so and therefore, it is a weak solution. This represents a weak solution to wave equation.

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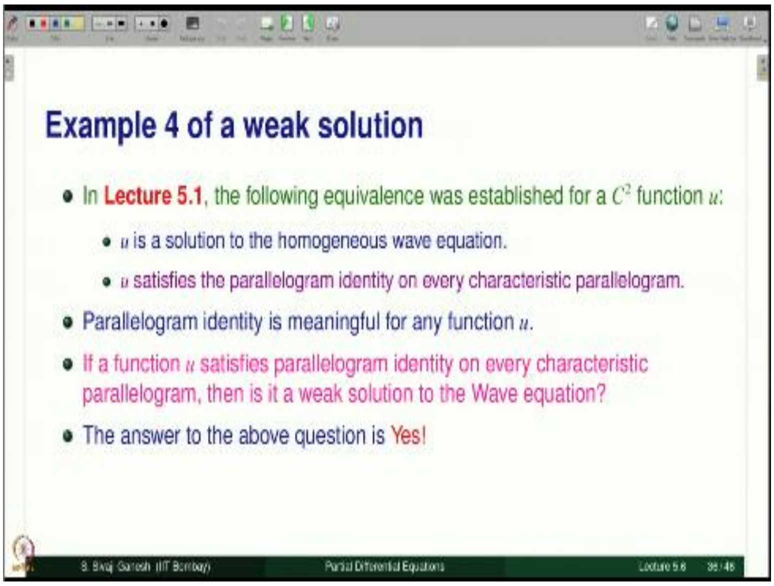

Let us look at another example. In lecture 5.1, the following equivalence was established for a C 2 function. What was that? u is a solution to the homogeneous wave equation if and only if u satisfies the parallelogram identity on every characteristic parallelogram. Parallelogram identity is meaningful for any function, it just involves values of the function at the 4 vertices of the characteristic parallelogram a relation between the values of the function.

So, if a function u satisfies the parallelogram identity on every characteristic parallelogram, then easily weak solution to the wave equation. That is the question. What we are shown is that if u is C 2, then it is a classical solution to the wave equation. Now, the question is, u is any function which satisfies parallelogram identity nothing more than that. No extra hypothesis on u will be being C 2.

Therefore, we asked this question: is it a weak solution? Of course, to define the concepts of weak solution, you have to start with L 1 loc function. So let us assume u is L 1 loc function, can I do this? The answer to the above question is yes.

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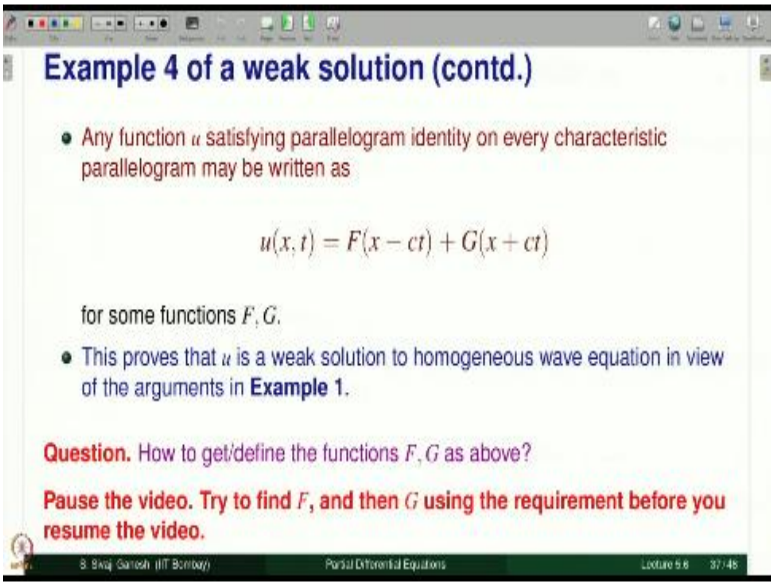

So, any function u satisfying parallelogram identity on every characteristic parallelogram may be written as u of x t = F of x – ct + G of x + ct for some functions F and G. This proves that u is a weak solution to homogeneous wave equation. In view of the arguments we gave an example 1 because the sum of 2 functions, one is a function of x – ct alone and other is a function of x + ct alone.

But, how do I get this F and G? I am given a function u, which satisfies the parallelogram identity on every characteristic parallelogram, how do I get F and G? How do I catch them? At this point, I would suggest you to pause the video; try to find F and then G using the requirement which is here is the requirement before you resume the video. It is fun.

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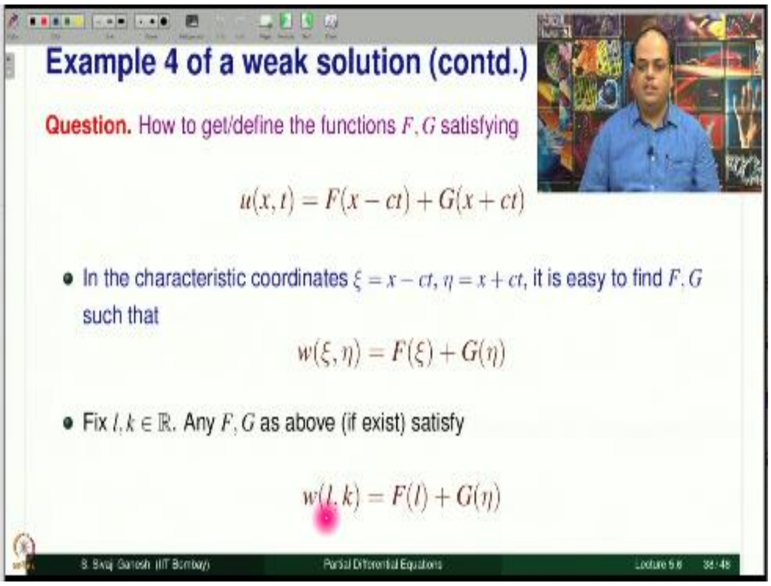

Let us see how to get these functions F and G. In the characteristic coordinates, it is always easy. It is easy to find F G such that w psi eta = F psi + G eta. Now, exactly do the same exercise as before, pause the video, try to find F and G from this requirement, first you get the F and then you get G. Once you try for some time, then even if you are not successful, does not matter, but you have tried.

So, now you can easily follow what is being done on the next slide. I have also tried like this, I was not successful, I tried to define a function F of psi. I have defined. Then G of eta when I am using this equation directly that is involving w psi eta – F psi that is not a function of eta alone because psi is also there. Then I also pause for some time thought about it and then I got answer. So, please try yourself.

Let us see now. Fix l, n, k in R because we have to use parallelogram identity. Without using that any formula that you try to do will not work. I am sure about that. Once you try, you will also be sure. So, fix l, n, k, any F and G as above should satisfy this. If they exist, w of l k must be F l + G k.

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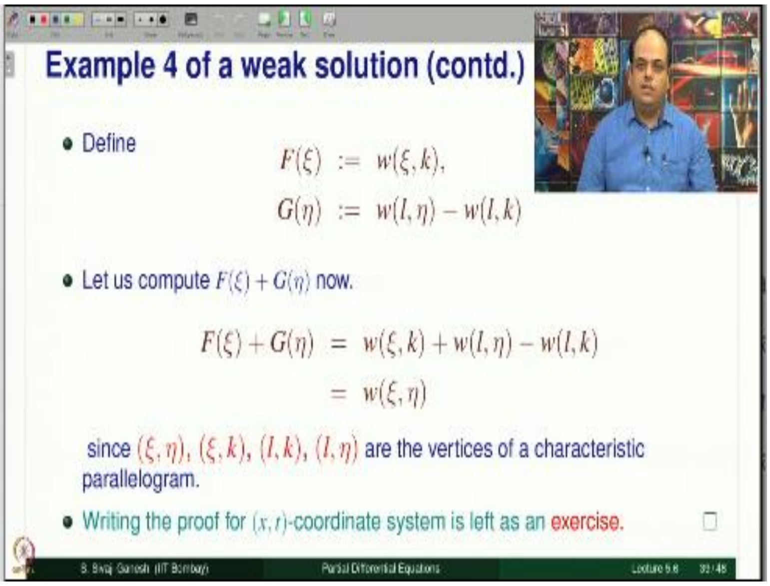

So, define F of psi = w of psi k, G of eta equal to this. Now, you see there is no psi in this expression that is a trick which works. So, let us compute F psi + G eta to show this is equal to w of psi zeta. So, this computation will give us this and that is equal to w psi zeta because these 4 points are the vertices of a characteristic parallelogram but in the characteristic coordinate systems, it is a characteristic not parallelogram but rectangle.

So, these are the vertices and you know the parallelogram identify wholes. Therefore, this all the combination of 3 terms is giving you a w psi zeta. So, we have shown what you want to show. But in characteristic coordinate systems, writing the proof for x t coordinate system is left as an exercise to you. If you recall in lecture 5.1, we are shown the equivalence of a function being a solution to the homogeneous wave equation and if it is satisfying parallelogram identity on every characteristic parallelogram.

In that proof, we have presented only x t coordinate systems. But if you think of a proof in psi zeta coordinates, it is very easy as I have presented here in this context. So, what we have done there, we have to imitate that here in x t coordinate systems. So, that is a hint for solving this exercise. So, please go back and watch lecture 5.1 again carefully; try the proof of the theorem in characteristic coordinate systems and see how we have written the proof in x t coordinates.

Now, exactly you should be able to do that by translating these ideas into a x t coordinate systems.

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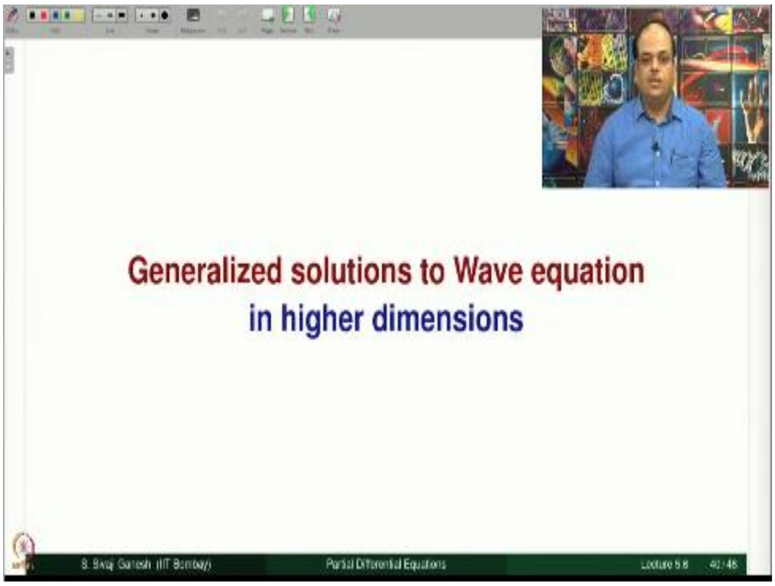

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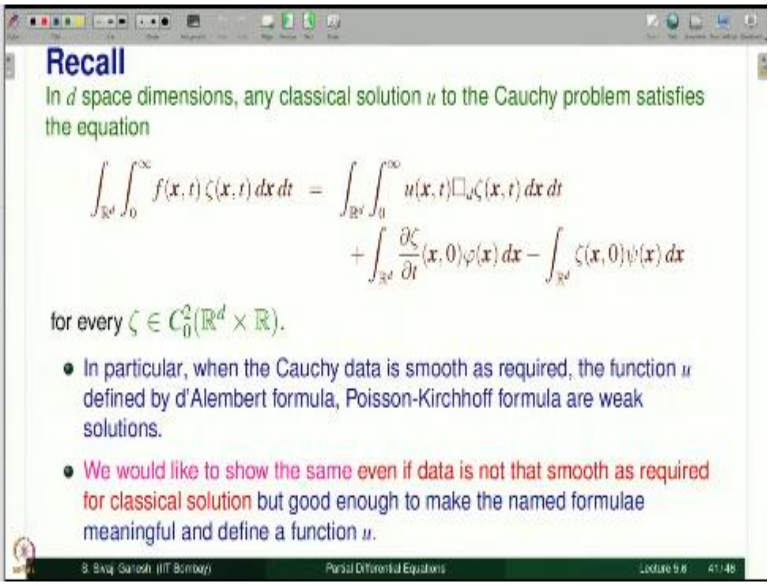

Now, let us look at some generalized solution in higher dimensions. So, in d space dimensions, any classical solution satisfies this that is how we have defined the notion of unique solution based on this. In particular, when the Cauchy data is smooth as required, the function u define by d'Alembert formula or Poisson-Kirchhoff formula or weak solutions, if you are thinking of d which is greater than or equal to 2, we need not mention about the d'Alembert formula.

We would like to show the same even if data is not that smooth as required for classical solution, but good enough to make these named formulae namely Poisson-Kirchhoff for d = 2 and 3 and d'Alembert formula for D = 1. They are meaningful and define a function.

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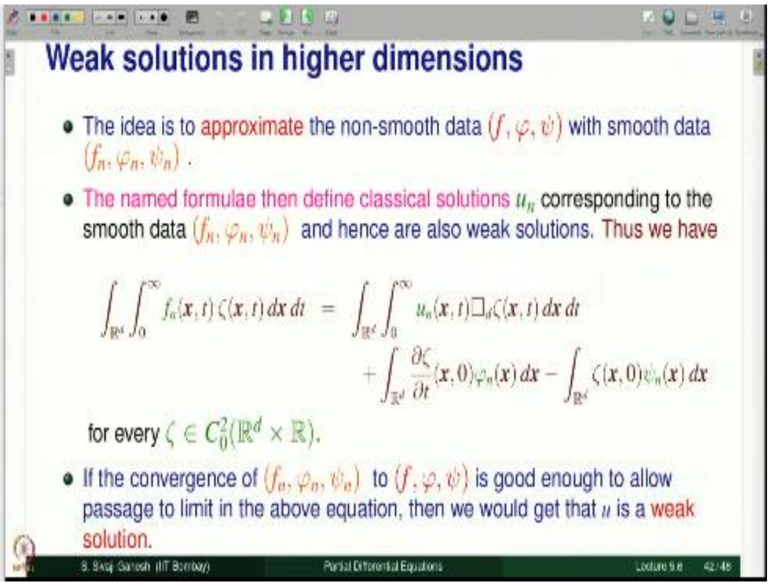

So, the idea is to approximate the non-smooth data F phi psi in the Cauchy problem with smooth data f n, phi n and psi n. What are the meaning of smooth data? Smooth data means it is that data which when you take the Cauchy problem, it has a classical solution and is given by those named formulae. Let the solution be denoted by u n; u n is a classical solution corresponding to this data.

Once it is a classical solution, it is also a weak solution. So, therefore, this is satisfied. Now, I have written this with the n everywhere, it will be very nice if you can pass to limit f n goes to f; u n goes to u; phi n goes to phi; psi n goes to psi, so that the corresponding integrals will go to; the corresponding integral where there is no end that is a dream always. If the convergence is good enough to allow passage to the limit in the above equation, then we would get that u is a weak solution.

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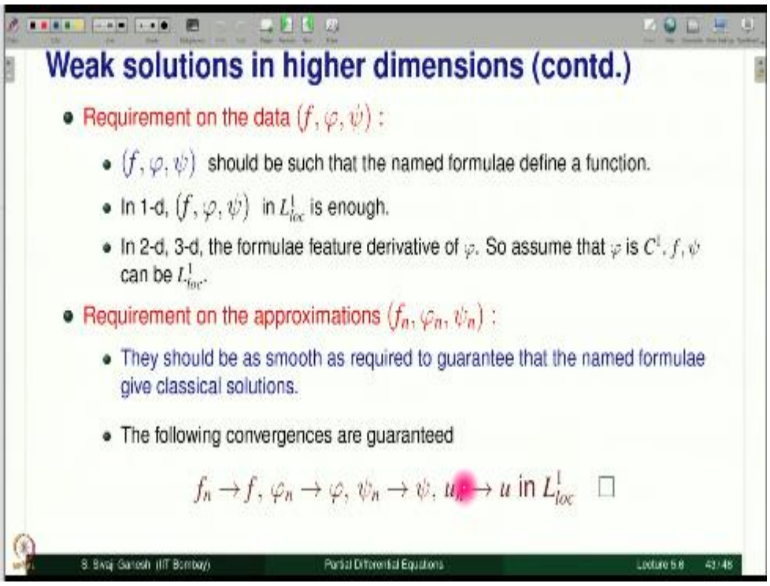

So, what is the requirement on the data? F, phi, psi should be such that the named formula define a function in 1D and define a function which is also L 1 loc. In 1D, f phi psi in L 1 loc is enough as we already observed. In 2D 3D, the formulas feature derivative of phi therefore, we assume phi or phi is C 1. F psi can be L 1 loc. So, requirement on the approximations; they should be as smooth as required to guarantee that the named formulae give classical solutions.

The following convergence is must be guaranteed also which is that f n goes to f; phi n goes to phi; psi n goes to psi and u n goes to u, because please note that u n, you have a formula in terms of f n, phi n and psi n. So, u n has a formula and that u n should go to some function u in L 1 loc, then we can pass to the limit in in this integral and get that u is a weak solution.

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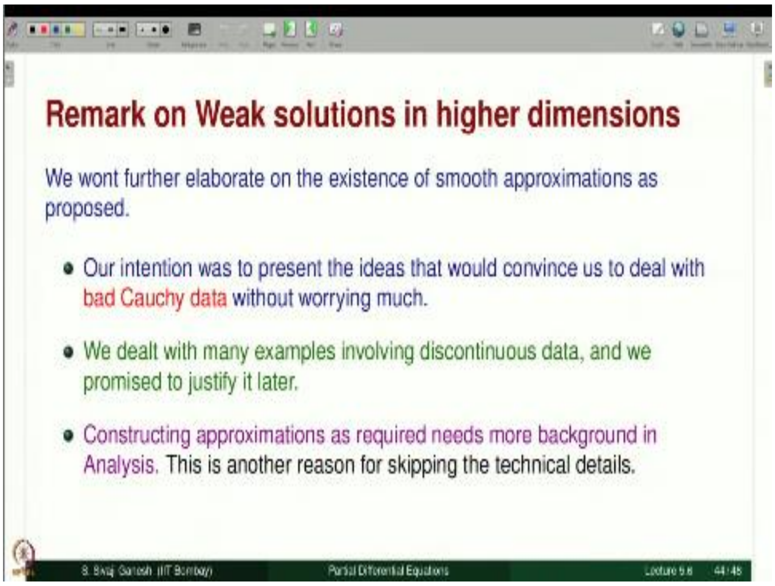

So, remark on weak solutions in higher dimensions: we will not further elaborate on the existence of smooth approximations as proposed. Our intention was to present the ideas that would convince us to deal with bad Cauchy data without worrying much. We dealt with many examples involving discontinuous data and we promised to justify it later and we have fulfilled to an extent that promise.

So, constructing approximation as required needs more background in analysis. And this is another reason for skipping the technical details. But the idea we have presented that is good enough.

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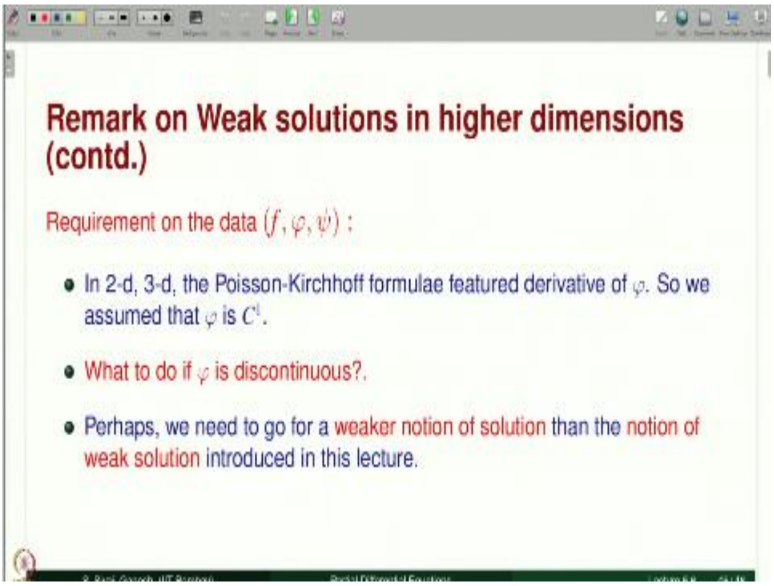

So, we had the requirement on the data f phi psi. In 2D 3D, the Poisson-Kirchhoff formula feature derivative of phi. So, we assume phi is C1. So, what to do if it is discontinuous? In fact, one of our examples in lecture 4.6 there, we have solved problems in higher dimensions. There, we have used a discontinuous function for phi. So, what happens? Perhaps, we need to go for a weaker notion of solution than the notion of weak solution introduced in this lecture.

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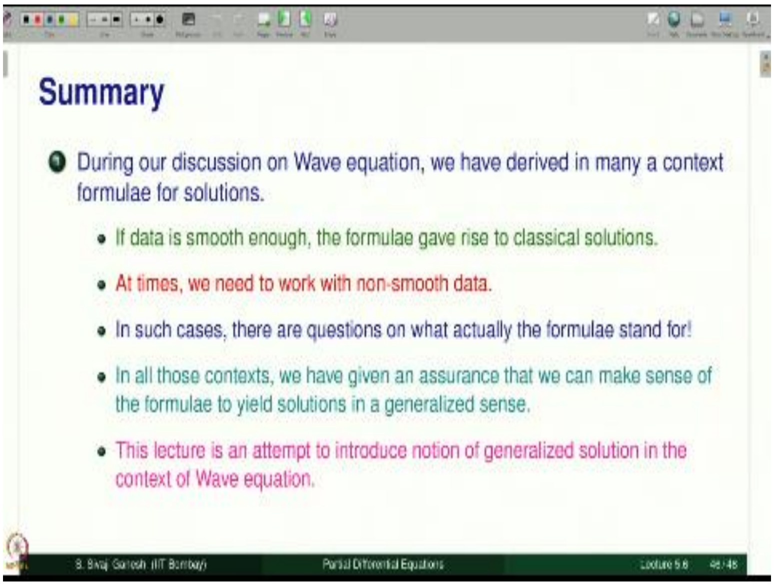

Let us summarize what we did in this lecture. So, during our discussion on wave equation, we have derived in many a contexts, formulae for solutions. If data is smooth enough, the formulae gave rise to classical solutions. At times, we need to work with non-smooth data. In such cases, there are questions on what actually the formulae stand for. In all those contexts, we have given an assurance that we can make sense of the formulae to yield solutions in a generalized sense.

This lecture is an attempt to introduce notion of generalized solution in the context of wave equation. So, this is at the expository level.

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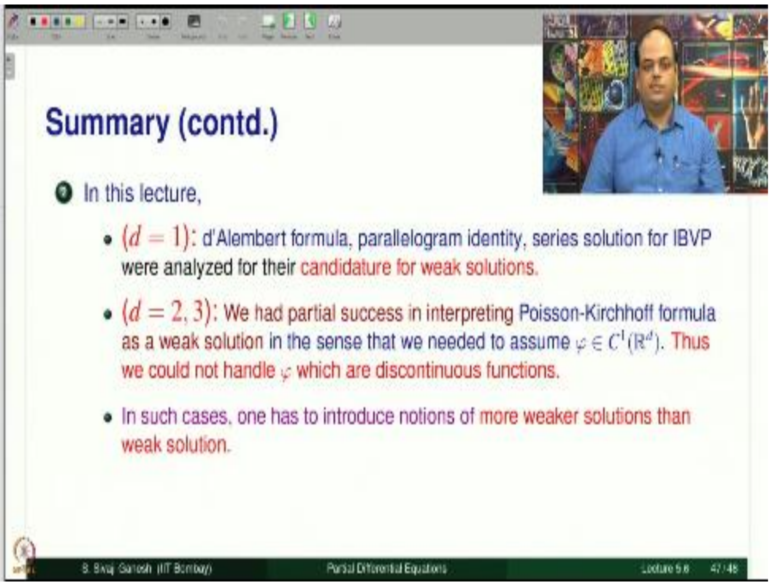

We are not presented really details. In this lecture, d = 1, d'Alembert formula parallelogram identity series solution for IBVP. They were analysed for their candidature for weak solutions. If you recall, wherever we use d'Alembert formula with discontinuous initial conditions, parallelogram identity and the series solution, we promise that we do not have to worry, there may be some new notion or a generalized notion of solution, which will admit them as solutions, go ahead and compute.

So, this lecture was an attempt to justify those statements. D = 2, 3, we had partial success in interpreting for some Poisson-Kirchhoff formula as a weak solution. In the sense that we needed to assume phi C 1; we assumed that. That is why only partial success. We could not handle phi which are discontinuous functions. In such cases, as I mentioned earlier, one has to introduce notions of more weaker solutions than weak solution. Thank you.

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