Partial Differential Equations
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Lecture – 2.2
First Order Partial Differential Equations
Geometry of Quasilinear Equations

Welcome, from now onwards, we will start analysing the equations first order partial differential equations for their solutions. Today's lecture is devoted to geometry of quasilinear equations.

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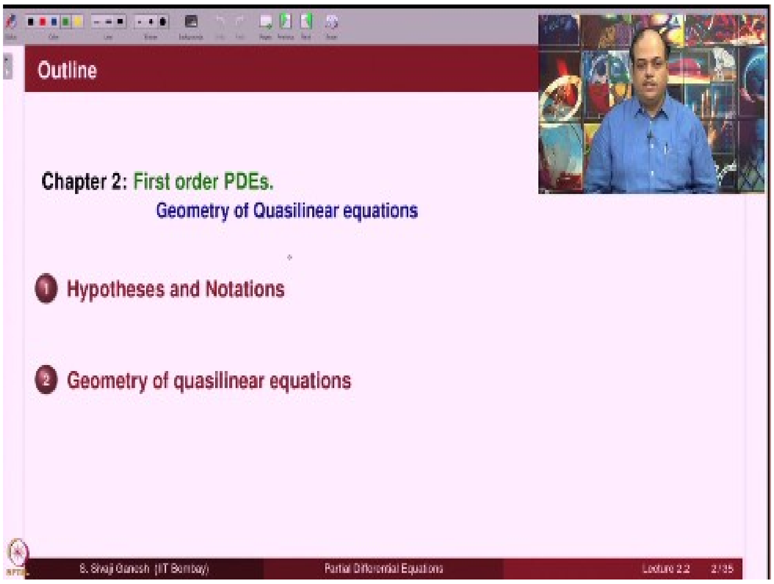

So, in this first we set up the basic hypothesis and notations which will be assumed throughout the analysis of the quasilinear equations. And then we discuss geometry of quasilinear equations, the geometrical objects which are associated to quasilinear equations.

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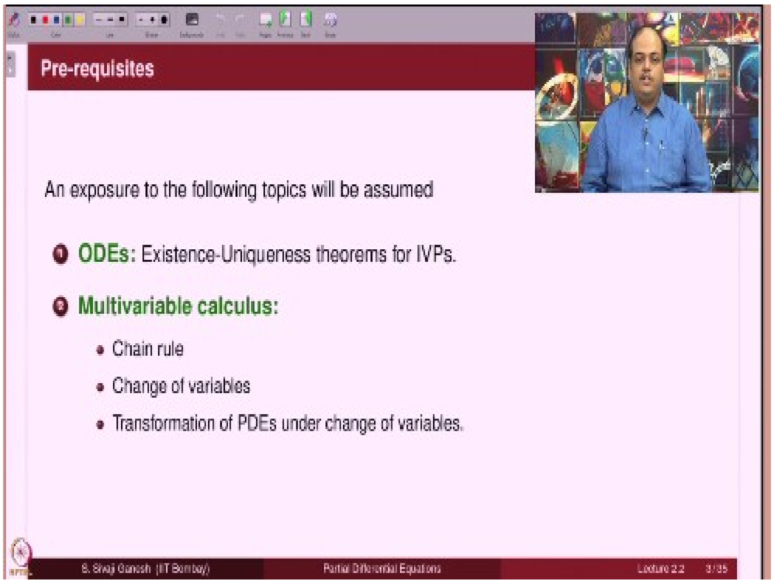

So, now, prerequisites is that from now onwards to follow this course, definitely for the first order PDEs what is very much required is an exposure to ordinary differential equation theory. Mainly existence uniqueness theorems for initial value problems, namely Picard's existence and uniqueness theorem, that is good enough. On the other hand, one more we require that is dependence of the solutions on the initial data.

That theorem will also be required, but not for this lecture. But in future lectures, please refer to some good books on ordinary differential equations, where you can find this material. Then, from multivariable calculus, the most important theorem, I would say is chain rule, and one should be very much expert with it. And change of variables, it looks quite easy, while computing certain integrals, which you would have done in plus 2.

But actually, it is not that easy, in the sense, you have to be very, very careful. If you are not careful, it can lead to a lot of confusion. And then how a partial differential equation changes under a change of variables to a new equation, how to derive that. That is also very important. So, quasilinear equations, hypothesis and notations.

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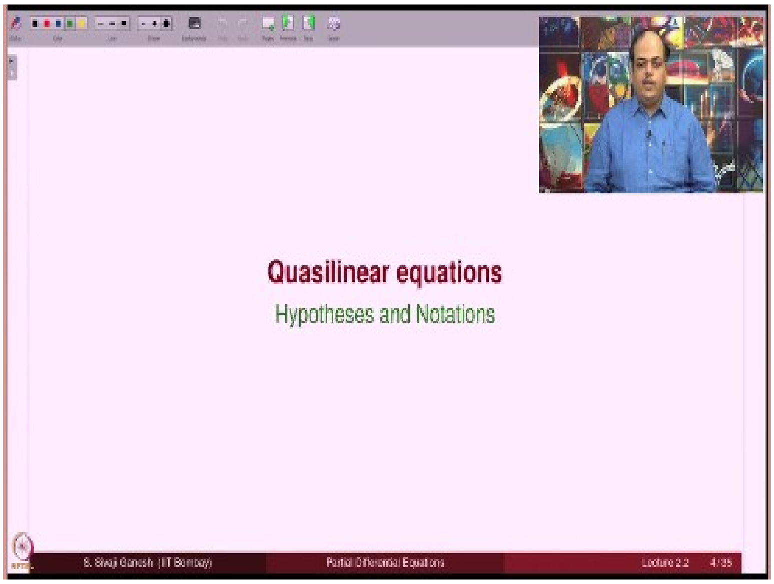

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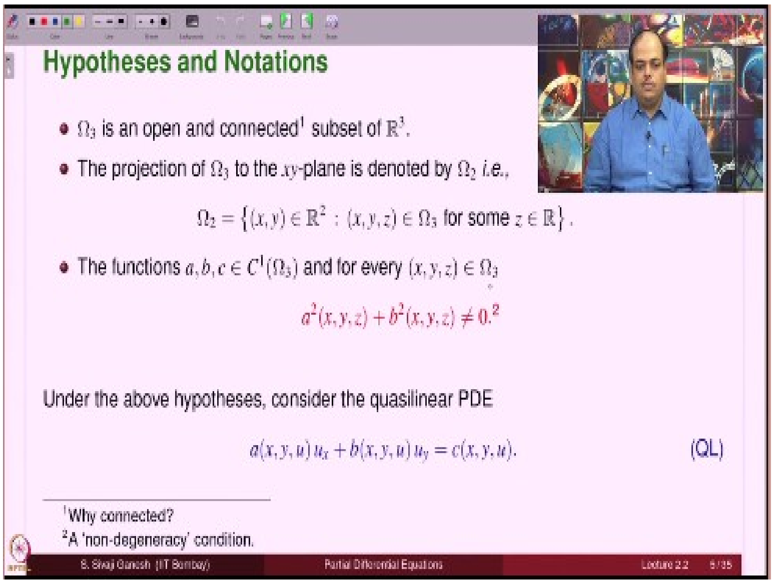

Omega 3 is an open and connected subset of $\mathbb{R}^3$ As I told you omega d will be an open subset in $\mathbb{R}^d$. But for the discussion of quasilinear equations, we have to take D equal to 3 because we are going to consider quasilinear equations in 2 independent variables. So, omega 3 is an open and connected subset of $\mathbb{R}^3$. Why do we need connected? It is because we are going to take some curves in omega 3.

Curve is quite connected quantity. I mean, there is no gaps in that, therefore, that curve should be in omega 3, otherwise also does not matter. That is why it is not that important at all here. It is enough it is an open set. There is nothing wrong in assuming it is also connected. There is no loss of generality. Then the projection of omega 3 to the xy plane is denoted by omega 2. That is consistent. Omega 2 is a subset of $\mathbb{R}^2$. So, how do we define that.

Those ordered pairs x, y in $\mathbb{R}^2$ such that x, y, z belongs to omega 3 for some z in R. Functions a, b, c which will be playing a role in defining quasilinear first order PDE they are assumed to be $\mathbb{C}^1$ functions on omega 3. That is continuously differentiable functions on omega 3, which is equalent to assuming that all partial derivatives of a, b, c with respect to the 3 variables exist and continuous on omega 3.

And we need this non degeneracy assumption, which says that at any point x, y, z in omega 3, at least one of the 2 functions a and b should be nonzero. A way to express that is a square plus b square is not equal to 0 at every point x, y, z. Another way to write the same thing would be this a of(x, y, z) comma b of(x, y, z) is not equal to (0, 0). This is another way of writing the same thing.

Why do we need this assumption is because imagine now both a, b are 0. Then this equation which we are going to consider the quasilinear equation, there is no differential equation. Because at that point maybe a is 0; b is 0. So it is just c x, y = 0, some kind of degeneracy. So, we do not want that. That is the reason why we assume a and b simultaneously cannot be 0 at any point in omega 3.

So, quasilinear PDE we often use this notation QL. Regularly we use QL. By QL we mean this. Here a u, x plus b u, y equal to c. a, b, c are $\mathbb{C}^1$ functions in omega 3 and a squared plus b squared is not equal to 0 at every point in omega 3. So by just QL, it stands for all these properties.

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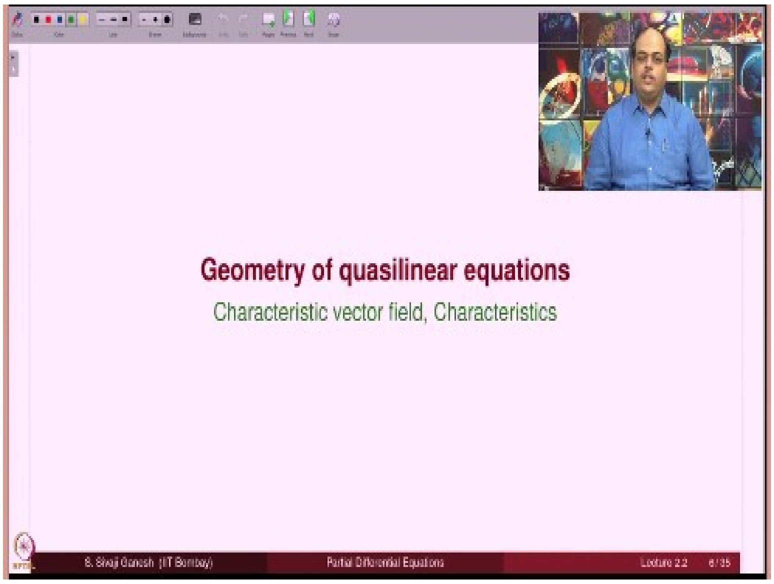

Now, let us discuss the geometry of quasilinear equations. That means, this equation says something something about the geometry of what of a solution surface. If you have a solution of this equation, let us say u of x, y defined for x, y in some domain D, then z equal u x, y will be a surface. And this equation says what kind of restrictions are imposed on the surface. Geometrically what this PDE means for a solution surface. That is what we are going to discuss.

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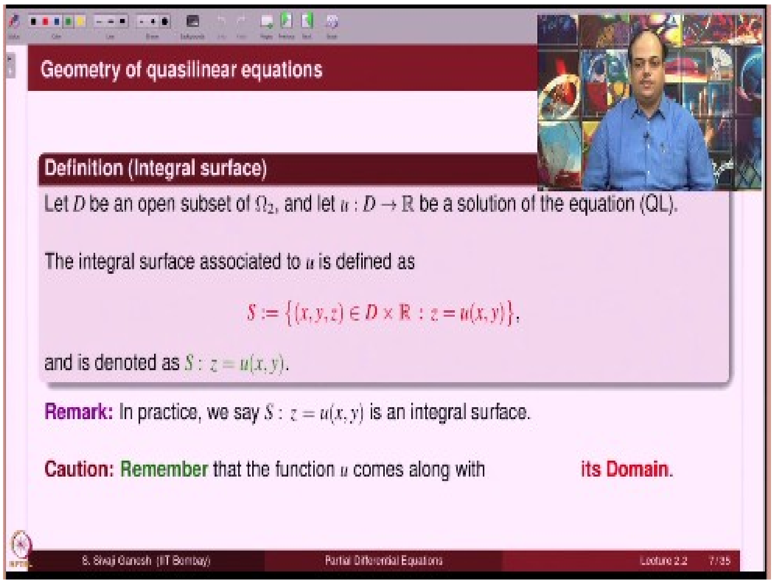

So, integral surface, the definition of an integral surface. Let D be an open subset of omega 2. What is omega 2? It is a projection of omega 3 to xy plane. And let u be a solution of the equation QL. u is defined on this domain D is the solution of the equation. What does it mean? The equation a of x comma y comma u of x, y, u x of x, y plus b x, y b x, y u x, y u y of x, y equal to c of x, y, u of x, y e satisfied at every x, y in D that is the meaning of solution.

So, the integral surface associated to u is defined as set of all triples x, y, z in x, y belongs to D and z belongs to R. So, x, y, z says that belonging to D cross R such that subset of $\mathbb{R}^3$ such that the third coordinate z is u of x, y. It is also denoted often in short notation as S equal S colon z = u of x, y. In practice we say s colon z = u of x, y is an integral surface. Caution: very important caution: Remember that the function always comes with its domain.

Function always comes with domain. So, if you say that S z = u x, y is an integral surface it means that you have a function u defined on some domain. All this needs to be mentioned. Or you know it, therefore, you are not mentioning. But, if you change the domain to some other domain, it can be a still a solution, but will be a different integral surface. We will come to these aspects in future lectures.

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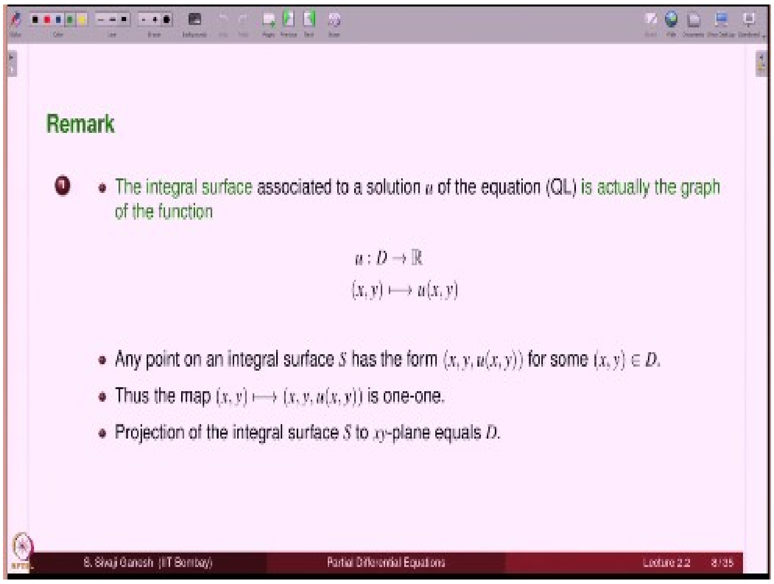

The integral surface associated to a solution of the linear equation is actually the graph of the function. We are saying set of all x, y, z. So, as z = x, y just means the third coordinate is a function of first 2 coordinates, which means it is a graph of a function u. And any point an integral surface looks like x, y, u x, y for some x, y in D. Therefore, this map is 1, 1. x, y going to x, y, u x, y is 1,1. Graph is always 1,1.

The moment x, y is different from x dash, y dash because the first 2 coordinates x, y and x dash, y dash are different the triples x, y, u x, y will be different from x dash, y dash u of x dash, y dash. They are different if x, y is different from x dash, y dash. Because this itself is different, that is why the function is 1 to 1. Now, projection of the integral surface to xy plane equals D.

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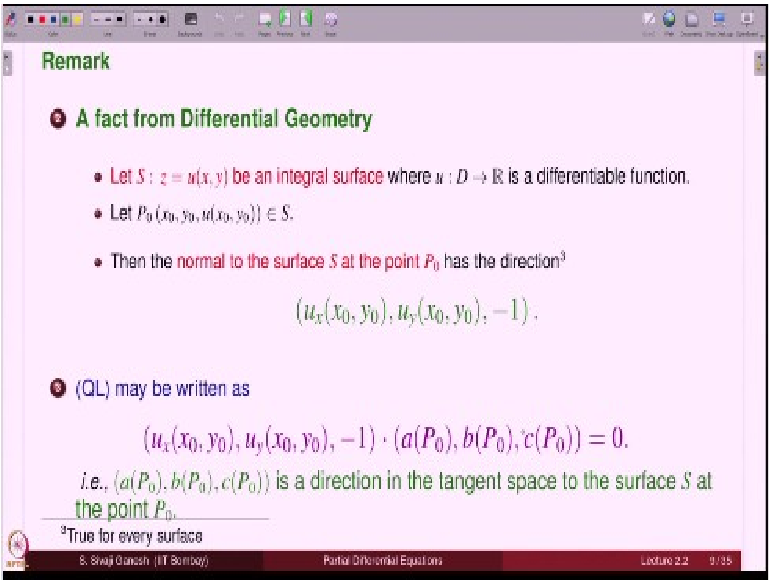

Now, fact from differential geometry. Let S be an integral surface where u is a differentiable function. Use a solution. Therefore, it must be a differentiable function. Actually in this fact we need not have integral surface written. It is true for any surface. That is why I wrote the let this be a surface and use a differentiable function. And why we wrote it? Write integral surface here is because we are going to discuss.

We are worried about only integral surfaces in this lecture. In fact, in this PDEs we want to find solution that we are we are interested in. So, surfaces described by solution of the equation. So, take a point P_0 on the surface. Then, a normal to the surface S, a normal to the surface at the point P_0 has this direction u x, u y minus 1. u x u y minus 1. That is a more general result.

I will write only roughly phi of x, y, z = 0. If this is a surface then what you have is grad phi. What is that namely phi x, phi y, phi z add that will represent a normal. Of course, they should be nonzero. So, all the conditions will be there. And in our case what is phi? phi of x, y, z is equal to let us write in this form. u of x, y minus z and its gradient is nothing but u x, u y and minus 1. So, that is true at every point.

So, this is a more general fact that this is normal. Therefore, quasilinear equation is nothing but the dot product of this u x, u y minus 1 with a, b, c = 0. What you get is a u_x + b u_y = c. That is the meaning of saying it satisfies the PDE at the point 0 by 0. In other words, the a, b, c which defined our quasilinear PDE is a direction in the tangent space to the surface S, at the point P_0.

Always you have to remember this phrasing. Tangent space or tangent plane if you are in plane to something at some point. So, it is a tangent direction. We caught hold of a tangential direction in a what would have be a search for it, integral surface. We know that I do not know that surface in the sense that I do not know solution, but I know a vector which is in the tangent space at a point on such a possible surface. This idea will be useful in finding solution later on.

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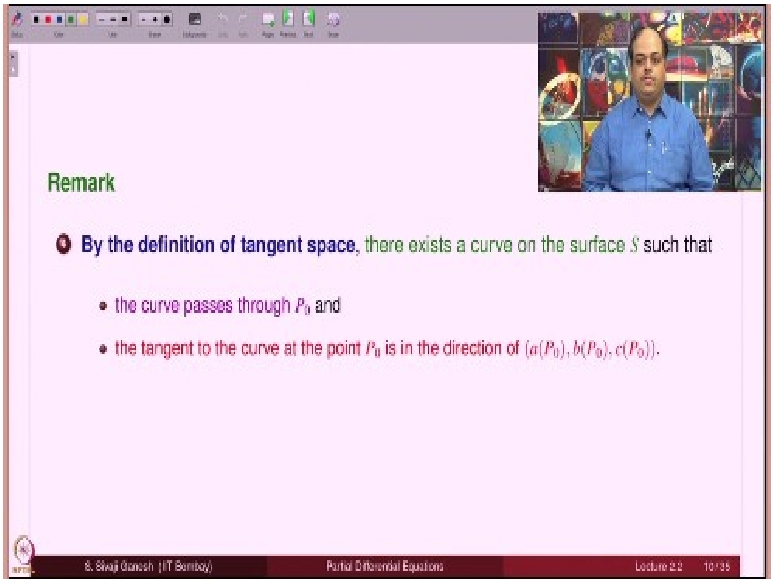

So, by definition of tangent space, there is a curve on the surface with what property? So, imagine this is a surface. There is a point P_0 then through this you have a curve gamma passing through that point and at this point the tangent vector is a of P_0, b of P_0, c of P_0. That is the meaning of somebody being a direction in the tangent space. So, tangent to the curve at the point P_0 is in the direction of (a(P_0), b (P_0), c (P_0))

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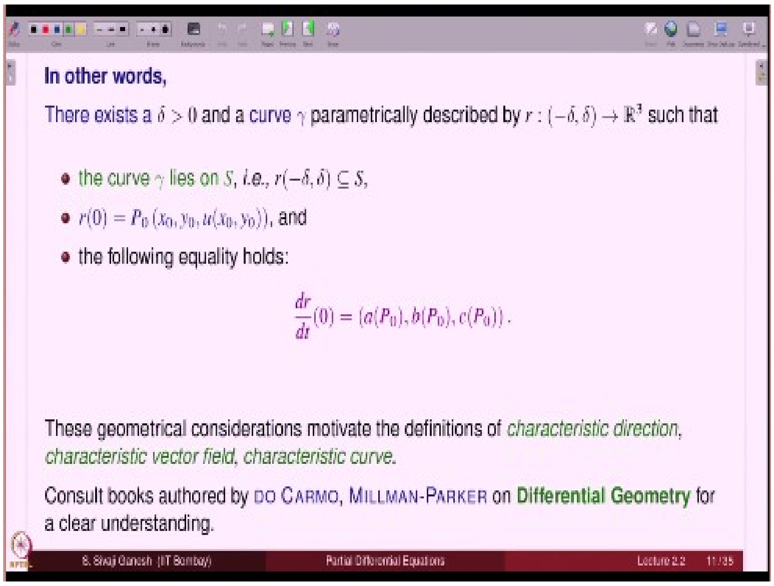

Now, what do you mean by that? There is a curve with this property, which is the meaning. There is a delta positive and a curve gamma described by a function R from some small interval minus delta, delta. What all you need is 0 should be there in the interval such that gamma lies on the surface, this curve is on the surface. And it passes through the point P_0 and the tangential direction is this.

This is a tangential direction that coincides with a, b, c at that point P_0. These geometrical considerations motivate definitions of characteristic direction, characteristic vector field and characteristic curve. For clear understanding of these geometrical objects, you may consult books on differential geometry written by do Carmo or another book by Millman and Parker, where these are very clearly explained.

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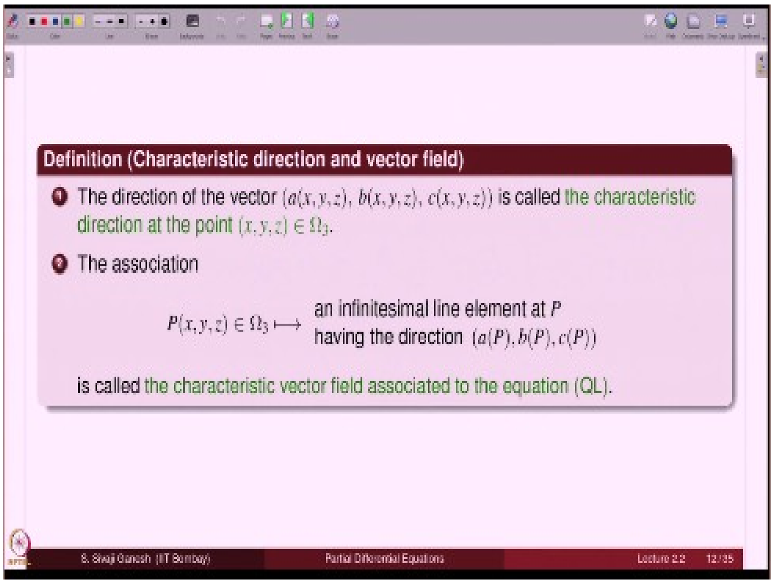

Now, we are going to define characteristic direction and vector field. The direction of the vector a, b, c is called the characteristic direction at the point x, y, z in omega 3. Now the association to every point in omega 3, take a point in omega 3, associate at that point and infinitesimal line element having this direction a, b, c. This association is called a characteristic vector field.

Basically vector field means you are associating a lot of you are associating a vector to each point. So it would look like I mean we cannot write things in 3 dimensions but let us see these imagine these are omega, I will not write on this, maybe you can write 3 any point you just put some lines like that. These are the directions of a, b, c at that point. So of course it varies from point to point and so on.

So this is called characteristic vector field associated to the quasilinear equation QL. If you change the equation, a, b, c will change therefore, this association will also change.

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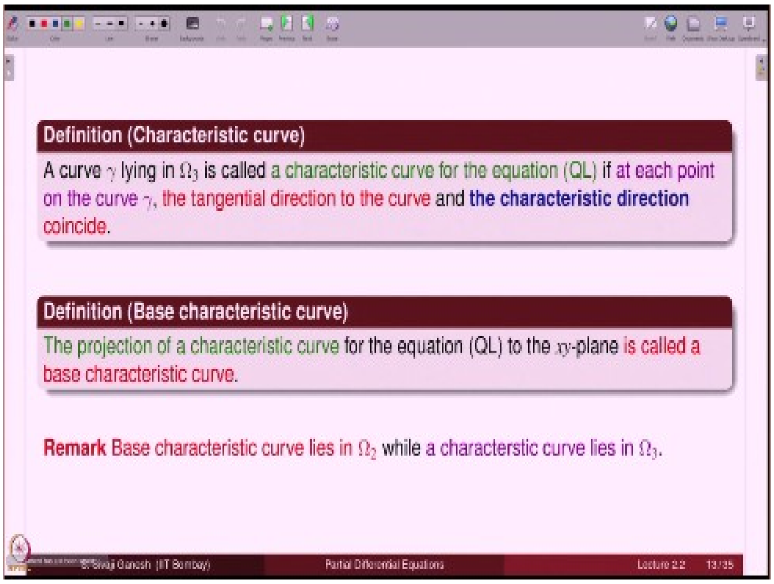

Now characteristic curve: A curve gamma lying in omega 3 is called a characteristic curve for the quasilinear equation QL if at each point on the curve gamma, the tangential direction to the curve is same as the characteristic direction, both directions coincide. That is called a characteristic curve. Now, if you project the characteristic curve gamma, which is in omega 3 to xy plane that will be called base characteristic It is called a base characteristic curve.

So, base characteristic curve lies in omega 2 which is the production of omega 3 to xy plane. Characteristic curve lies in omega 3.

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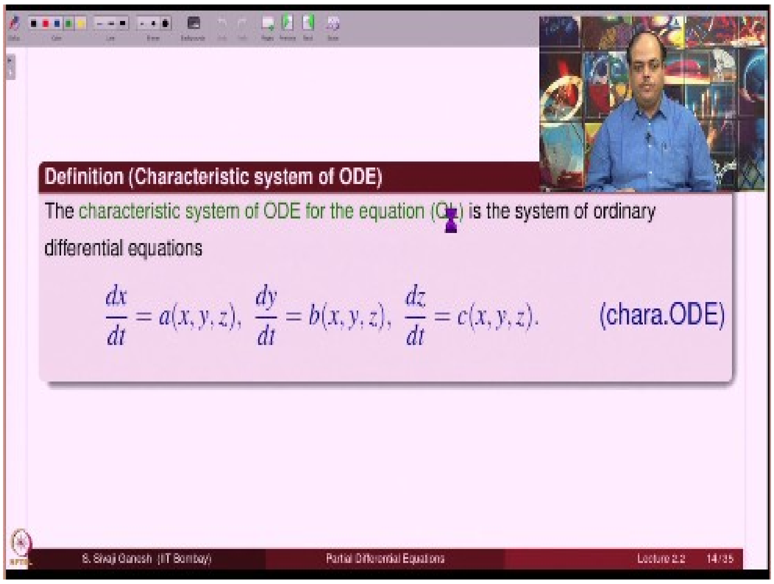

Now, we have a characteristic system of ODEs. The characteristic system of ODE for the equation QL is the system of ordinary differential equations dx by dt equal to a, dy by dt equal to b, dz by dt equal to c. So, it is an autonomous system of ordinary differential equations with the right hand side a, b, c were coming from the equation QL.

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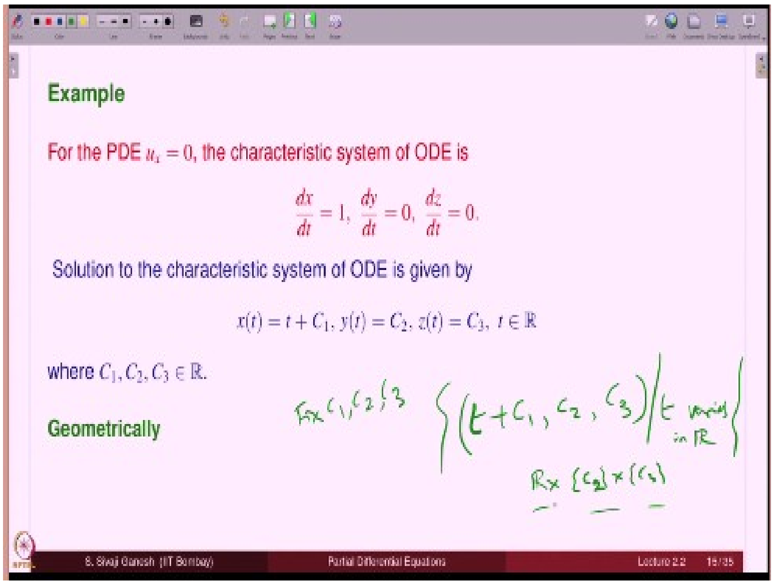

Now, let us look at an example for this PDE u_x = 0, very simple PDE. Let us say PDE in 2 variables x and y. The characteristic system of ODE is dx by dt equal to a. In this example, a is 1, b is 0 and c is 0. Therefore, dy by dt is 0, dz by dt equal to 0. We can integrate them. x of t equal to t plus constant, y of t equal to another constant z of t equal to another constant, where C_1, C_2, C_3 are real numbers.

Now, what is it geometrically, what we have to look at is t plus C , this 3 tuple, t + C_1 comma C_2 comma C_3 as t varies in R, t belongs to R. This is what we have to look at. Of course, to write down this fix C_1, C_2, C_3. Fix C_1, C_2, C_3 C and look at this. This is a line. Second coordinate third coordinate are fixed, first coordinate will give your entire R. So, this is precisely what you get is R cross singleton C_1 cross sorry C_2 cross C_3.

This is precisely that second coordinate is fixed third coordinate is fixed first coordinate can be any real number. So, that is what we get.

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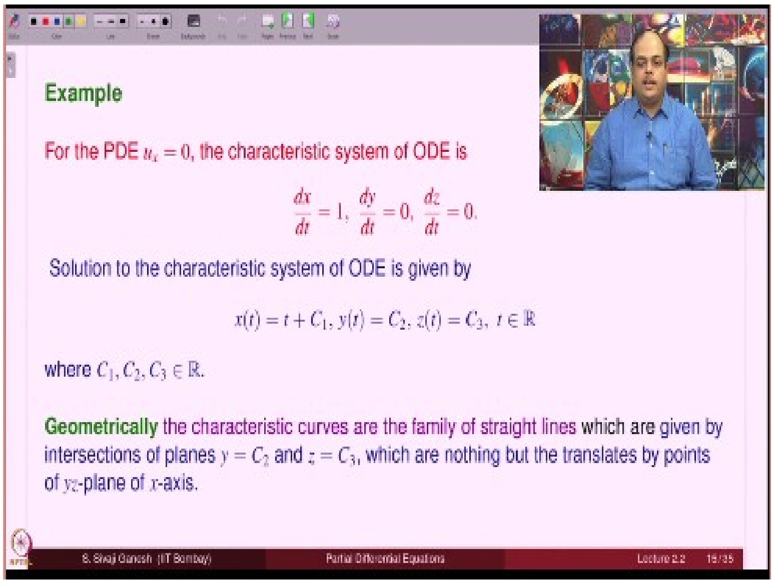

So, characteristic curves are the family of straight lines which are given by the intersection of the plane y equal to C 2 and z equal to C 3 which are nothing but the translates by points of the yz plane of x axis.

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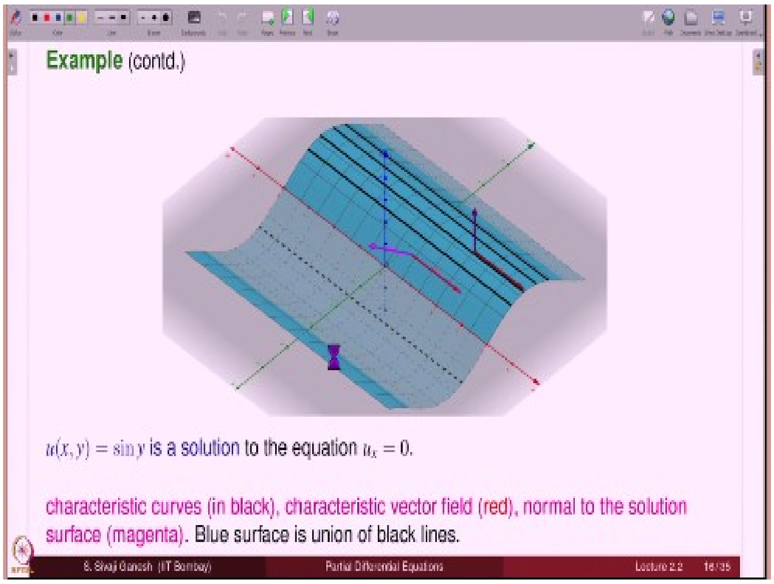

So, this is a picture specific to some particular quasilinear equation u_x = 0. Equation is u_x equals 0 and u (x, y)equal to sin y is a solution given. You can verify. It does not depend on y. u_x = 0 means it is constant with respect to x. So, it is a function of y only. I have taken that to be sin y. Now, this is a graph the sin wave is going like that. The curve you can see the sin curve.

Now in this red axis red is always x axis green is Y axis blue is z axis. So, this is x axis, green is Y axis y axis blue is z axis. Now, characteristic curves are in black, which are straight lines we know that straight lines. And characteristic vector field is in red colour which is this. Because characteristic vector field is 1, 0, 0 at every point. So it is in the direction of e 1at every point. And normal to the solution surface is in magenta.

So, it keeps changing the normal from point to point. So blue surface this is just an observation. And this is in fact also a general situation we will show the theorem later on. Blue surface, that is what integral surface corresponding to the solution u (x, y)= sin y. So z = sin y is this. That is union of these black lines. So you can think one line, this line keeps on moving and you get the surface.

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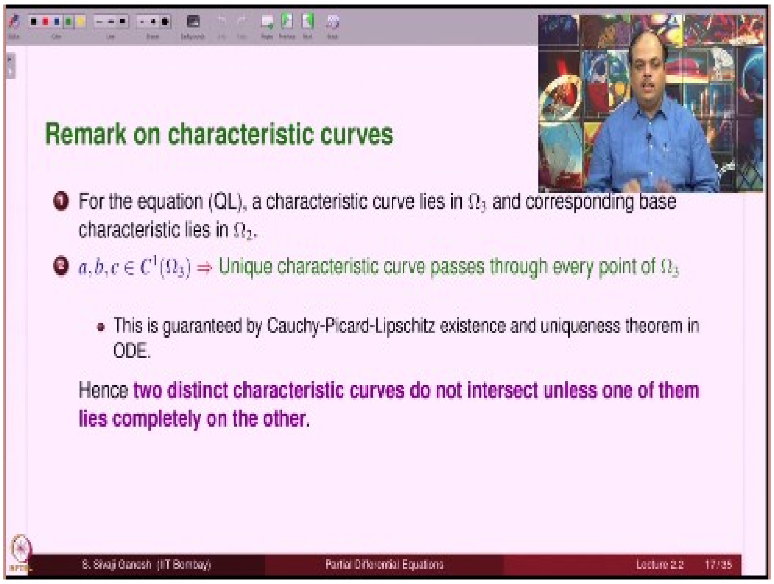

Remark on characteristic curves for the quasilinear equation characteristic curve lies in omega 3 and base characteristics lies in omega 2. This is very important or many times this word is blurred. So, people refer to both of them as characteristics. So, you have to be very careful. Therefore, to clear this problem or confusion, we use the word base characteristic curve.

Now, if a, b, c are C_1 functions, then a unique a characteristic curve passes through every point of omega 3. What is the characteristic curve? It is the solution of x dx by dt = a, y dy dt = b, z dz by dt = c. Its imaged. This is guaranteed by Cauchy-Picard-Lipschitz existence and uniqueness theorem in ODEs. Therefore, 2 distinct characteristic curves do not intersect, otherwise uniqueness will be violated if they intersect, unless one of them lies completely under the other unless one is on the other.

In other words, we may ask what do you mean when one lies on the other. So, it may be that you have a solution which is that long if you look at a small piece of the dissolve solution, so, these are only when uniqueness is violated. So, therefore, we do not consider such things as different solutions. So, therefore, 2 distinct characteristics do not intersect unless one of them lies completely on the other.

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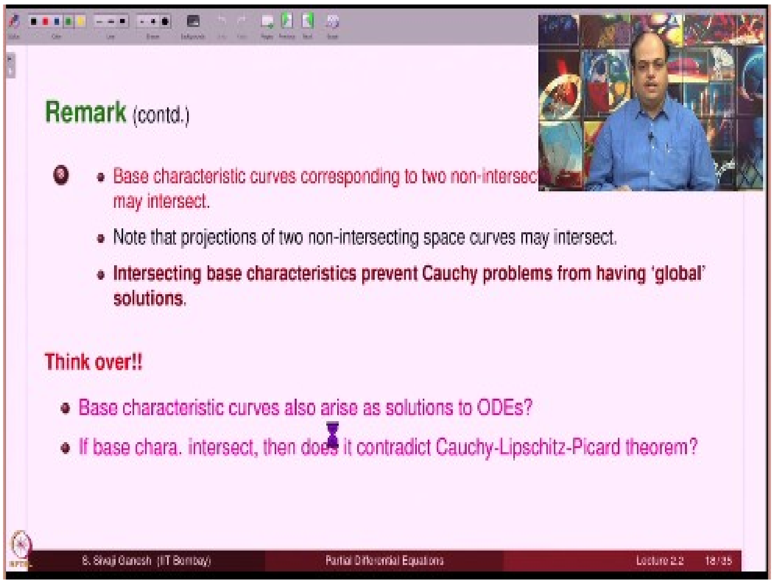

Base characteristic curves corresponding to two non intersecting characteristic curves may intersect. It can happen. Characteristic curves do not, but their projections can intersect. Intersecting base characteristic curves prevent Cauchy problems from having global solutions. We will discuss them further but this is a fundamental obstacle. Now, this is an exercise for you to think over.

Base characteristics: They also arise as solutions to ODEs. If base characteristic intersect, does it contradict Cauchy Lipschitz Picard theorem does it contradict a theorem of course, things are working well it should not contradict, but then this confusion is there because they are also solutions to initial value problems of ODEs, we will clear that later.

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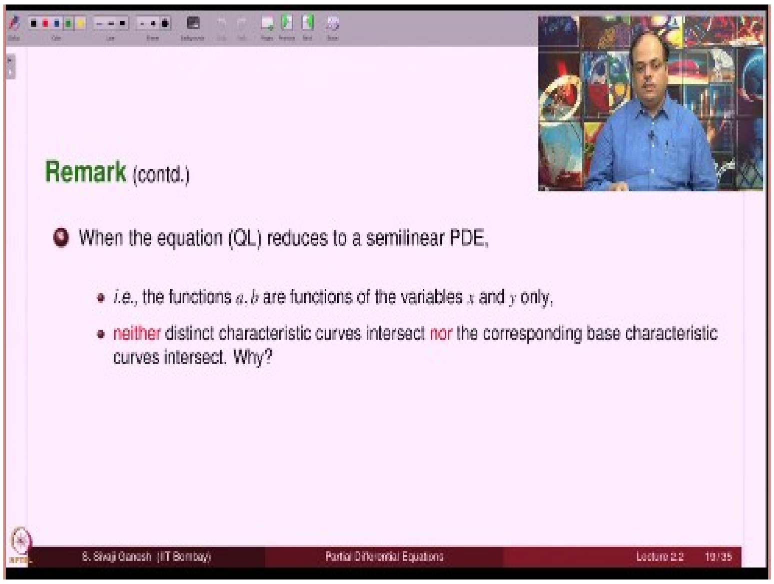

So, when the quasilinear equation reduces to a semi linear equation, what happens? a, b are not functions of the third variable anymore, they depend only on the x and y variables. Then, neither district characteristic curves intersect that we know even for quasilinear equations nor the corresponding base characteristic curves intersect. Because what is the base characteristic curve? It is a projection of the characteristic curve to omega 2.

But solutions of which equations, because it is semi linear the equations are dx by dt equal to a x, y, dy by dt equal to b x, y and the a, b are C_1 functions still. So, by Cauchy Lipschitz Picard's theorem there is exactly one base characteristic curve passing through every point of omega 2. That is why. Semilinear base characteristic do not intersect because the equations govern governing the base characteristic curves, the right hand side is a and b and they are functions of x, y only.

If they are also functions of x, y, z you cannot claim uniqueness because you cannot solve. The problem is not does not make sense.

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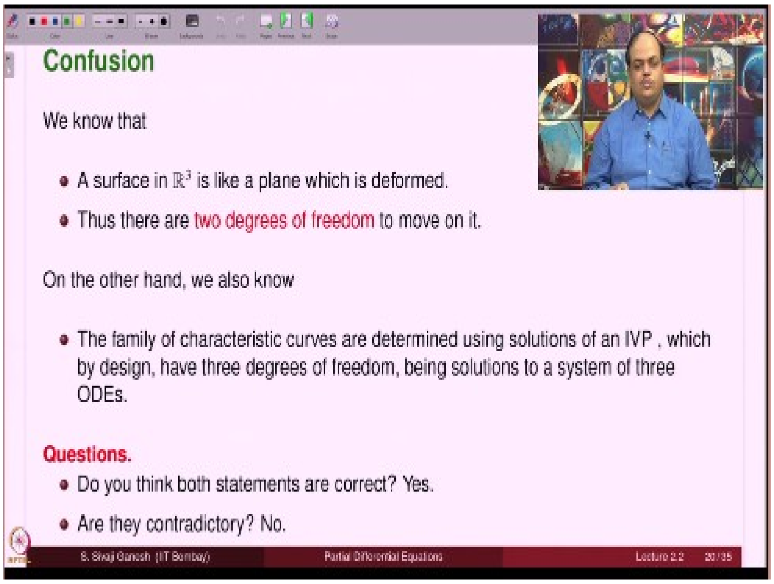

Now, a surface in $\mathbb{R}^3$. How does it look? It looks like a plane but slightly deformed. That is a feeling. Therefore, it looks like there are 2 degrees of freedom to move on a surface. If you are a surface like this, you can move like this, 2 degrees of freedom. It is not like one single straight line. Like a road on which you cannot overtake. On the other hand, we also know the following.

The family of characteristic curves are determined using solutions of initial value problem which by design have 3 degrees of freedom because a system of 3 equations being solutions to a system of 3 ODEs. Now some questions. Do you think both statements are correct? Answer is yes. They are both are correct. Are they contradictory? Because you are saying on one hand there are 3 degrees of freedom for these curves, which are lying on the surface, surface itself is 2 degrees of freedom, are they contradictory? Answer is no.

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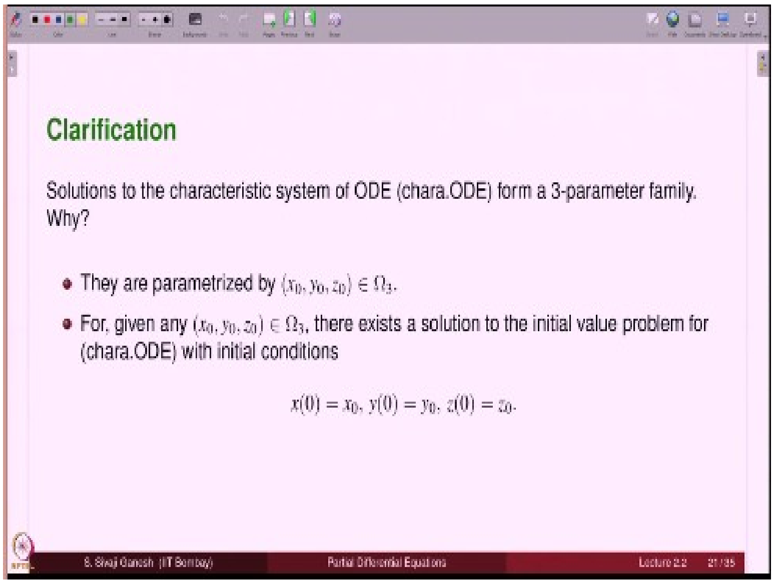

Clarification to this is the following. Solutions to the characteristic system of ODE, they form a 3 parameter family. Why? Because they are parametrized by the points of omega 3. So, take a point in omega 3 x_0, y_0, z_0 and then for any given x_0, y_0, z_0 in omega 3 there is a solution to the initial value problem, which we call Chara ODE. This stands for dx by dt = a, dy by dt = b and dz by dt = c. With these initial conditions at t = 0, x_0, y_0, z_0 is $x_o y_o z_o$.

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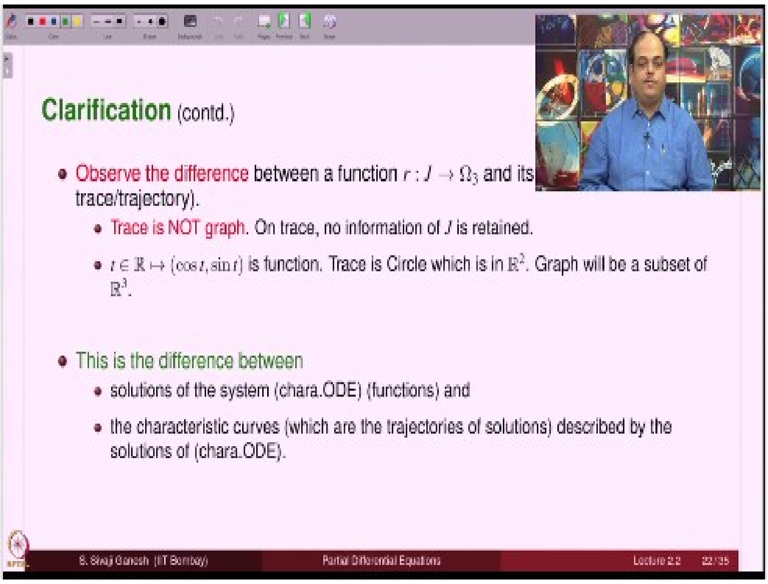

Now, observe the difference between a function and its image. Image is often called try trace our trajectory. It is just image. Trace is not a graph. On trace no information of J is retained. We will look at an example. Look at this t belongs to R going to cos t sin t. It is a function.

Trace is a circle. That means set of all cos t sin t as sin t varies, this is a circle which is in $\mathbb{R}^2$.

Graph will be a subset of $\mathbb{R}^3$.

Graphs look like t comma cos t comma sin t. That is not a circle. So, this is a difference between solutions of the system chara ODE and the characteristic curves, which are images of this are trace or trajectory of solutions described by solutions of the chara ODE

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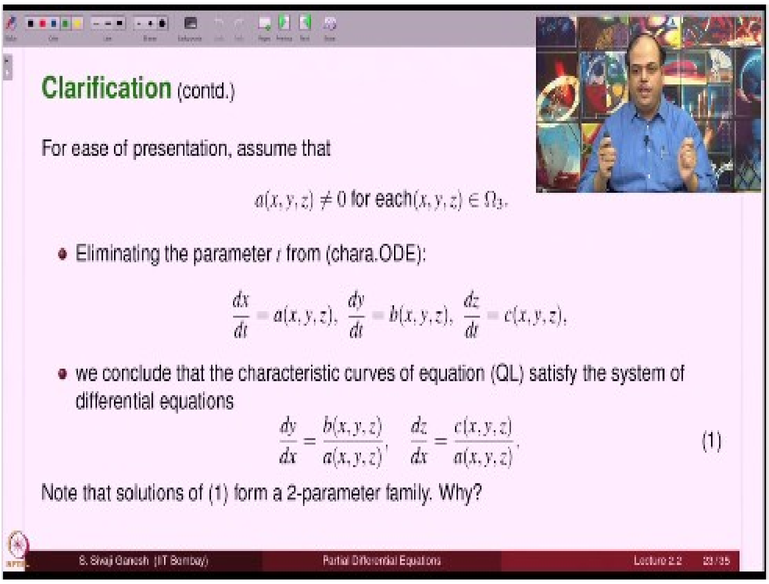

For ease of presentation, let us assume that a is never zero in omega 3. Once it is never zero you know what I am planning to do. I am planning to divide. Now I will eliminate the parameter t from this chara ODE system. t will go then I will get dy by dx equal to b by a and d z by d y equal to c y dz by dx I think. We will see. We conclude that that is a satisfying. Yes dy by dx and dy by dx, I eliminate t.

It means that I am assuming something about it. Let us not discuss that right now. So note that solutions of 1 forms a 2-parameter family. System of 2 equations. So 2 parameter family. Justification exactly same that we gave for system of characteristic ODE. System of 3 equations 3-parameter family. Similar explanation gives you the exact 2-parameter family.

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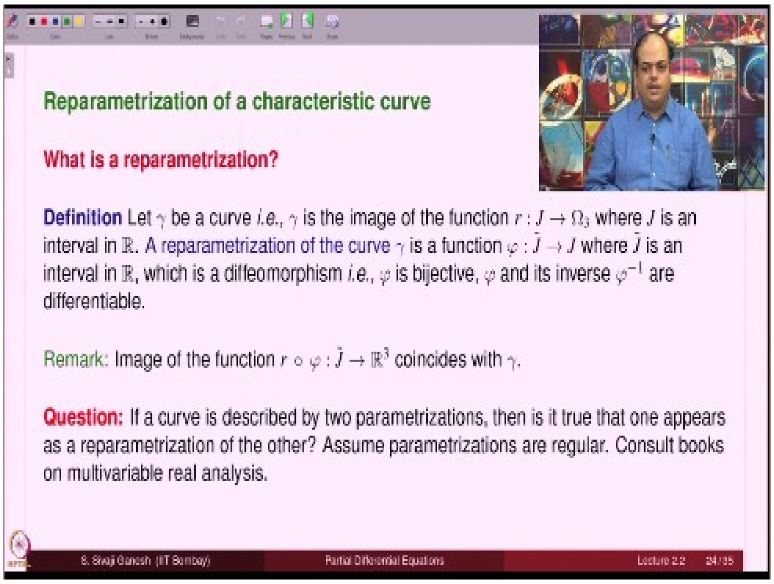

Now, you see here the t has vanished in this, there is no t in this. Reparametrization of a characteristic curve: What is a reparametrization? Let gamma be a curve. That is gamma is the image of a function r from some interval J to omega 3, where J is an interval in R. This is a curve given. Now, we are going to define what do we mean by reparametrization of this. It is a function phi defined on some interval J tilde to J.

J tilde to J, where J tilde is an interval. That is it. With what property? It is a diffeomorphism. That means, the function is 1 to 1 and onto. So, that is function is bijective. And therefore, the function and its inverse makes sense. Both of them are differentiable. Now, image of this function R circle phi which is now defined on J tilde to $\mathbb{R}^3$ coincides with gamma. So, the image of R circle phi and R are the same. Both are gamma.

So, that is a re parametrization. Now, if a curve is described by 2 parametrizations then is it true that one appears as a reparametrization of the other. In other words, you are given R from J to omega 3 and R tilde from J to omega 3 such that images are same. Now, the question is, is there a function phi from J tilde to J which is a diffeomorphism? Answer is almost true but you would assume some more properties about the parametrization being regular.

So, this you can once again look up books of Do Carmo or Millman Parker on differential geometry. But our question is not about this we are not asking this question.

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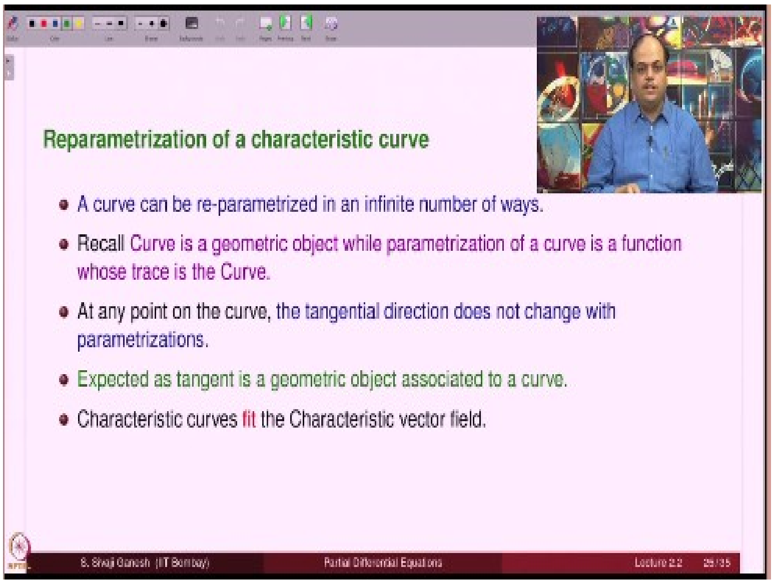

Now we are interested in reparametrization of a characteristic curve. So a curve can be parametrized in infinite number of ways. Simply because the functions that we have here phi tilde is upside the function phi from J tilde to J. You can take any interval J tilde. And you can always define any interval means the open interval J tilde. Then you can always find a diffeomorphism between J tilde and J.

So, therefore, parametrizations are infinitely many. So, curve is a geometric object. The image is a curve that is a geometric object. While parametrization is a function whose trace is the curve. So, at any point on the curve the tangential direction does not change with parametrization. It should not change because tangent is a geometric object associated with a curve. So, characteristic curves fit the characteristic vector field.

That means, you have a characteristic vector field. Characteristic curves are precisely those curves at each of those points. The tangent is the characteristic direction. This is not coming under reparametrization but we wrote this because we will see this. We will use this.

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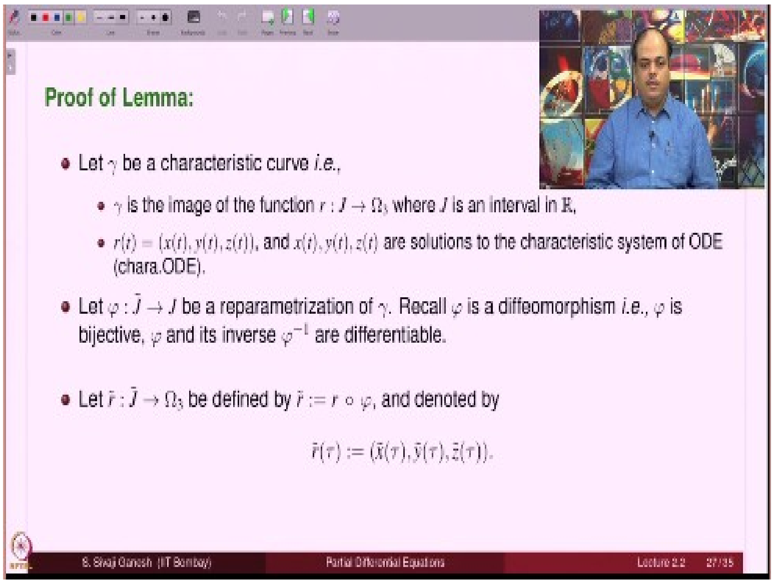

Yeah. So, the following lemma is a restatement of geometric fact discussed on the previous line in the context of characteristic curves. Any reparametrization of a characteristic curve is also a characteristic curve. Remember, it is not about a curve, it is a characteristic curve. It means a tangent should be the characteristic direction. In this context is this statement made here the last one.

You know that the characteristic vector field is the same and tangents are geometric objects, characteristic curves are somebody who fits this. Therefore, next theorem is expected. This lemma and therefore, one advantage of this is you take J tilde to be R. Again R to that particular J you can always define a diffeomorphism. Therefore, you can always re parametrize by R. It is very useful.

We will see that when we prove existence theorems for the Cauchy problem of quasilinear equations, and as well as general nonlinear equations. So, proof of the lemma, let gamma be a characteristic. That means, first of all, it is image of a function. Let us write because it is sitting in omega 3, there are 3 components. Let us write r(t) = x(t), y(t), z (t). And x (t), y (t), z(t) are solutions to the characteristic system of ODE.

Let phi be a re parametrization of gamma given to us. Phi is given, J tilde is given such that this phi is at diffeomorphism. These are meaning of diffeomorphism . Now, R tilde is r circle phi and R tilde, let us use a different running parameter in J tilde. For J we use t, for J tilde let us use tau. R tilde tau is x tilde tau comma, y tilde tau comma z tilde tau.

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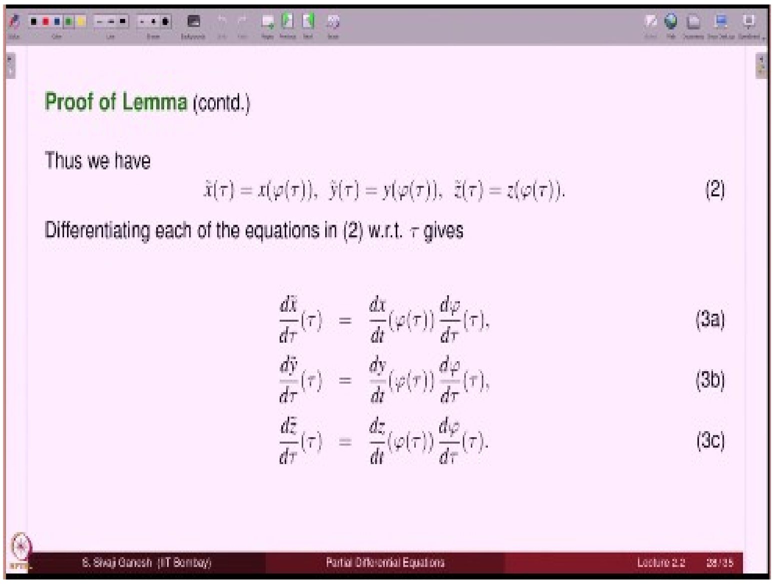

We want to show this is also a characteristic curve. It means we have to find the tangent to this curve and show that this characteristic has the characteristic direction. So, x tilde tau by definition is this. r tilde is r cirlcle phi. So, x tilde tau equal to x of phi of tau, y tilde tau equal to y of phi of tau, z tilde tau equal to z of phi of tau. So, differentiating each of the equations in 2 with respect to tau gives us the following equations in 3 which is actually chain rule.

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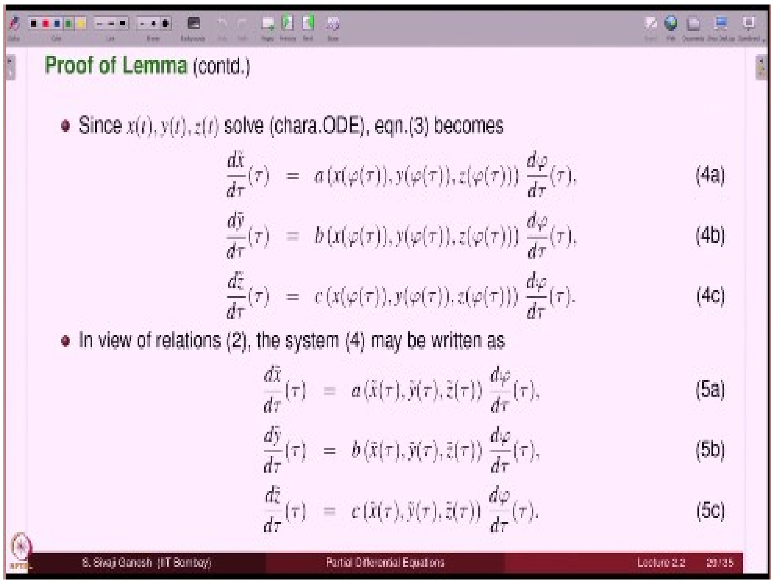

Now, x t, y t, z t solve chara ODE. Therefore, we know that dx by dt dy by dt dz by dt are a, b, c respectively at the point x of phi of tau, y of phi of tau, z of phi of tau. So, this is the equation we get. Now, when you have the relations 2, what R does in relation 2. It is the definition. x tilde, y tilde, z tilde gives us these equations.

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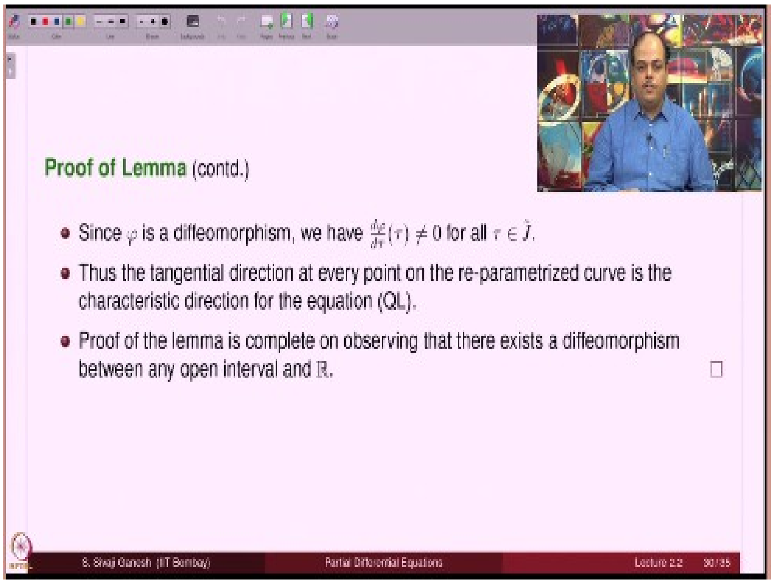

Since phi is a diffeomorphism we have dphi by dtau which is nonzero at every point. So, that means this quantity which is multiplying a, b, c is a nonzero quantity. Therefore, the direction of this right hand side is same either direction of this a, b, c, because it is nonzero. If it is 0, we cannot say. This is nonzero because of the diffeomorphism. So, proof of the lemma is complete.

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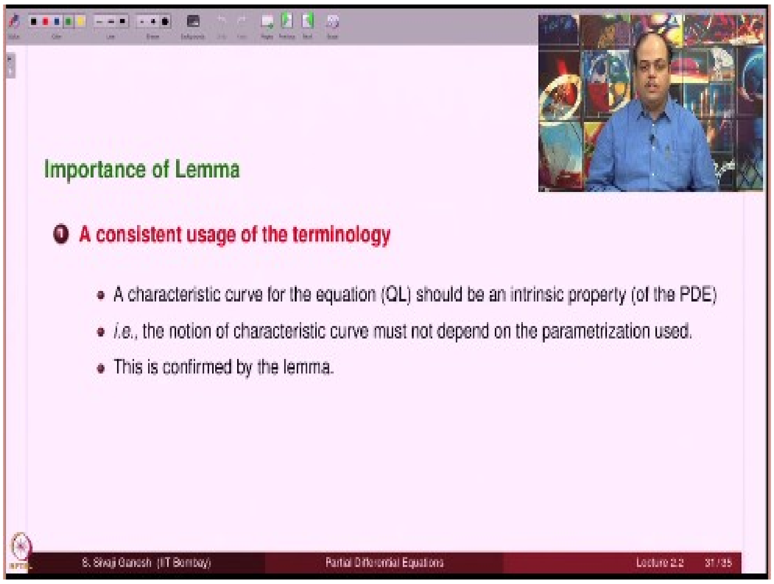

On observing that there is a diffeomorphism between any open interval and R, therefore, we can re parametrize a characteristic curve by R. Importance of this lemma is it gives us a consistence usage of the terminology, a characteristic curve for QL should be an intrinsic property of the PDE. It should not depend on the parametrization used. Remember the definition was using parametrization. This is confirmed by this lemma.

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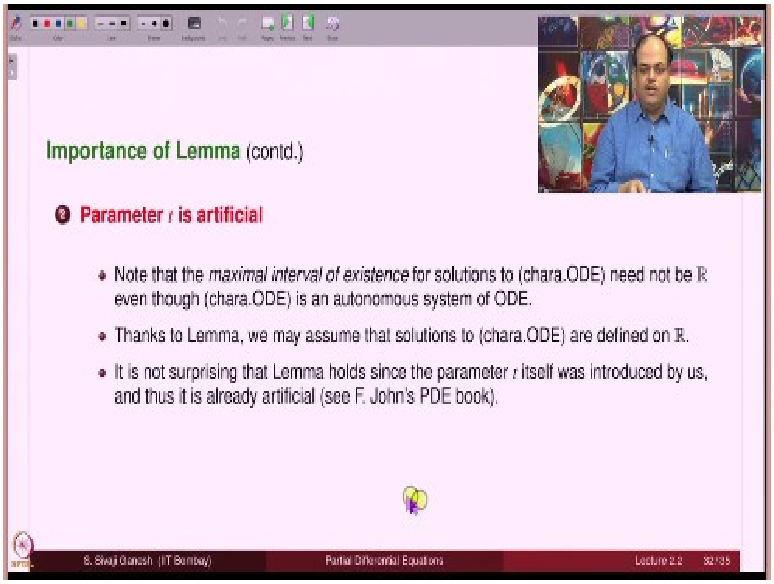

And parameter t is artificial. So, maximal interval of existence for any solutions need not be R even for autonomous systems. For example, look at dy by dt equal to y square and y of zero equal to any number you take y_0 . Solution will not be defined on R even though the ODE makes sense for all t 0. Therefore, there is a concept of maximal interval of existence, it will not be R even if it is an autonomous system as we just saw in this example.

But thanks to this lemma, we may assume that solutions are defined on whole of R. Equation is different. Our interest is only in the trace, right in the trace of the solution in the image of the solution. That is same. So, it is not surprising that the lemma holds since the parameter t itself was introduced by us by us. Equation gave us a, b, c. We said yes, there is a curve whose tangent is this. Therefore, we said curve means $r(t) = (x(t), y(t), z(t))$.

Its derivative equal to a, b, c. We did that, we introduced the T. Therefore, it is already artificial. It is confirmed in fact, this phi usage is there in free John's book.

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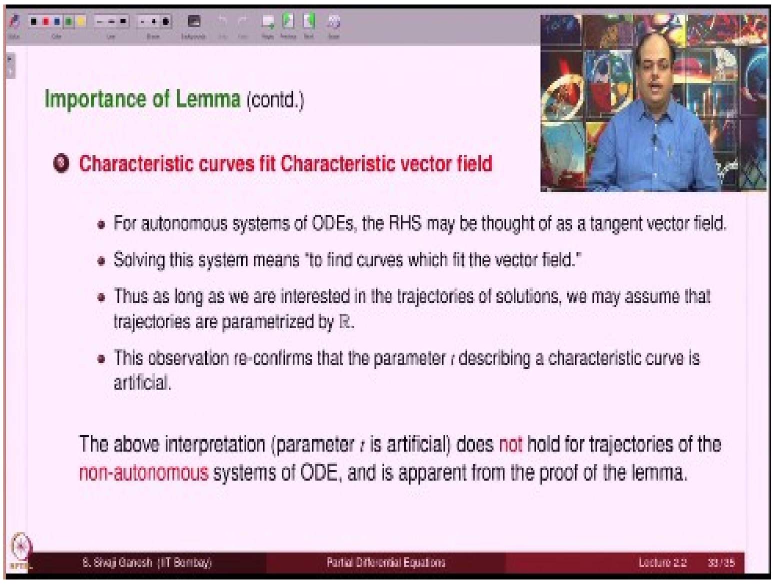

He also says this parameter is artificial, which is this lemma conforms. Now, characteristic curves fit the characteristic vector field. For autonomous system of ODEs the RHS may be thought of as a tangent vector field. Solving this system means, to find curves which fit the vector field. Thus as long as we are interested in the trajectories of solutions, we may assume that trajectories are parametrized by R.

This observation reconfirms that the parameter t describing a characteristic curve is artificial. The above interpretation that is parameter t is artificial does not hold for trajectories of the non autonomous systems of ODE. The proof of the lemma suggests where the proof will fail and hence that is the reason why this will not hold.

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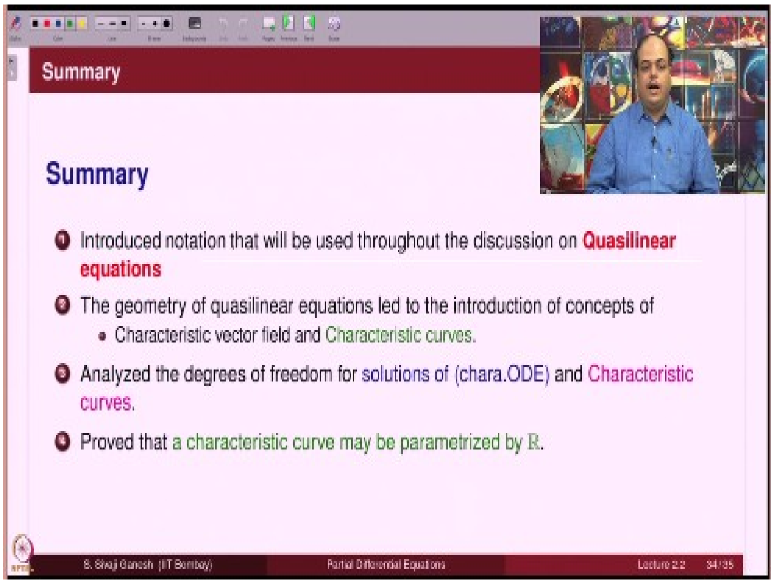

Let us summarise what we did now, today, we introduced the notion that can be used throughout the discussion on quasilinear equation notations. We have introduced the notations and then we discussed the geometry of quasilinear equations, which led to the introduction of concepts of characteristic vector field and characteristic curves. We analyzed the degrees of freedom for solutions of a chara ODE and characteristics curves. We proved that a characteristic curve may be parametrized by R.

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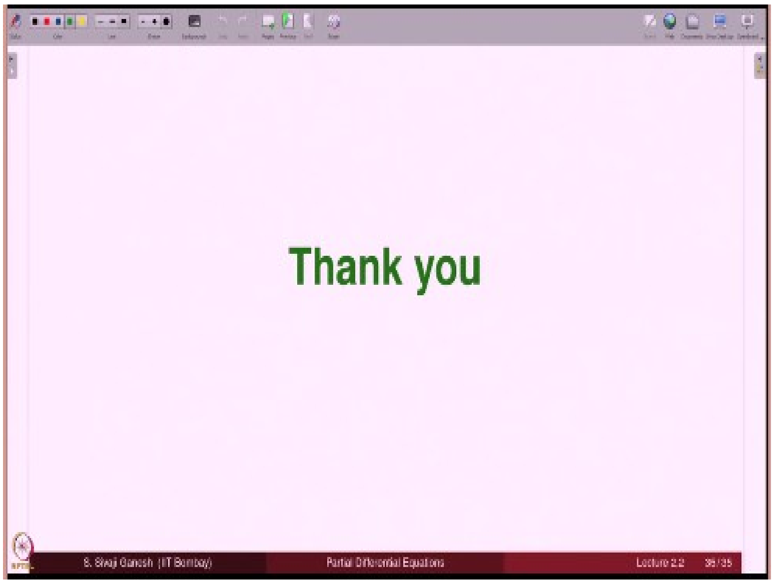

So, using this geometry, geometry of quasilinear equations, we try to solve a Cauchy problem for quasilinear equations in future lectures. Thank you.